Galaxy Bias Loops for Power Spectrum & Bispectrum

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ICC

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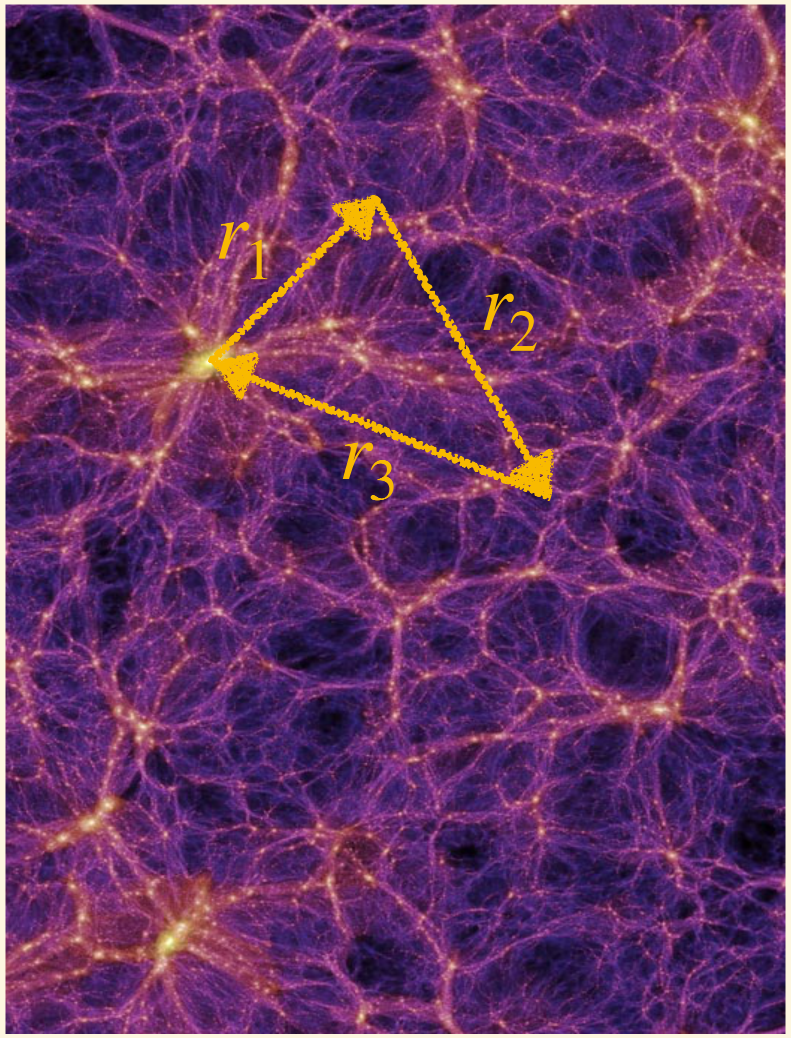
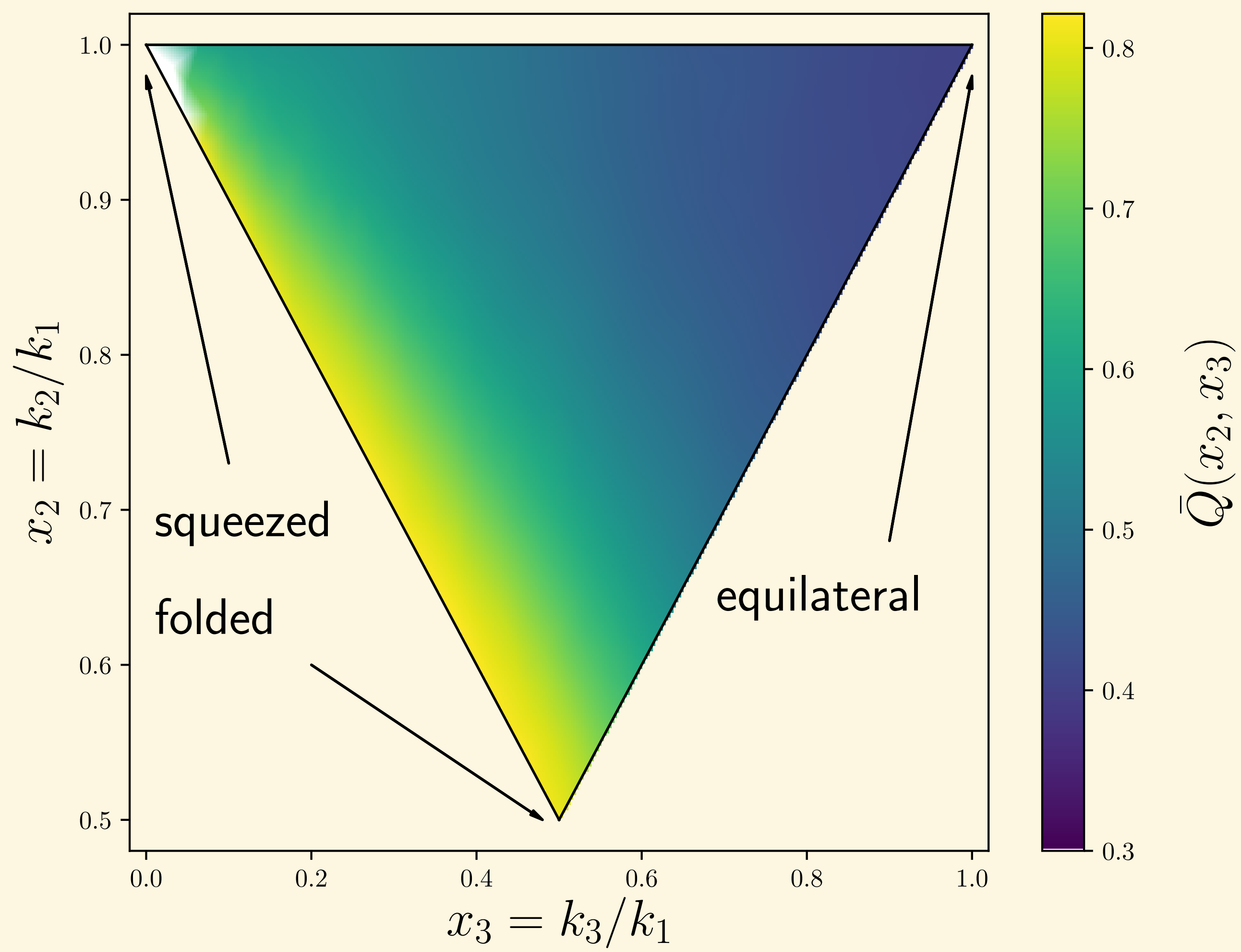
Three-Point Statistics at a Glance



Configuration Dependence



Reduced Bispectrum

$$Q_g(k_1, k_2, k_3) = \frac{B_g(k_1, k_2, k_3)}{P_{g,1}P_{g,2} + P_{g,2}P_{g,3} + P_{g,3}P_{g,1}}$$

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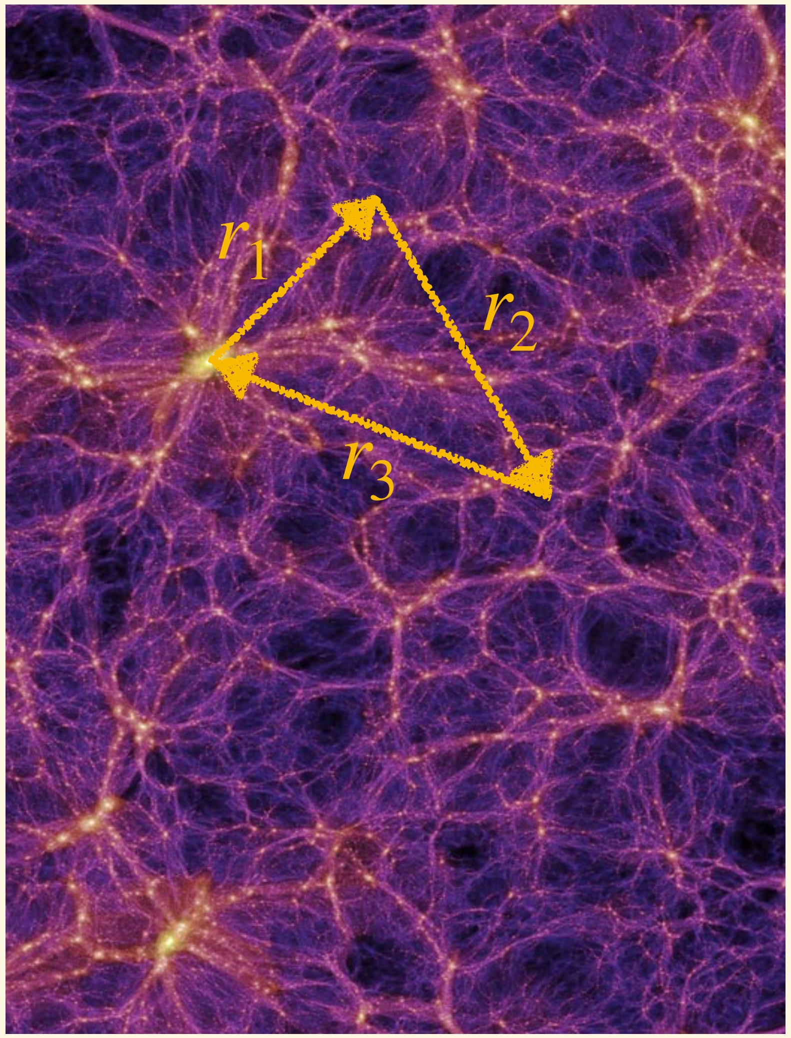

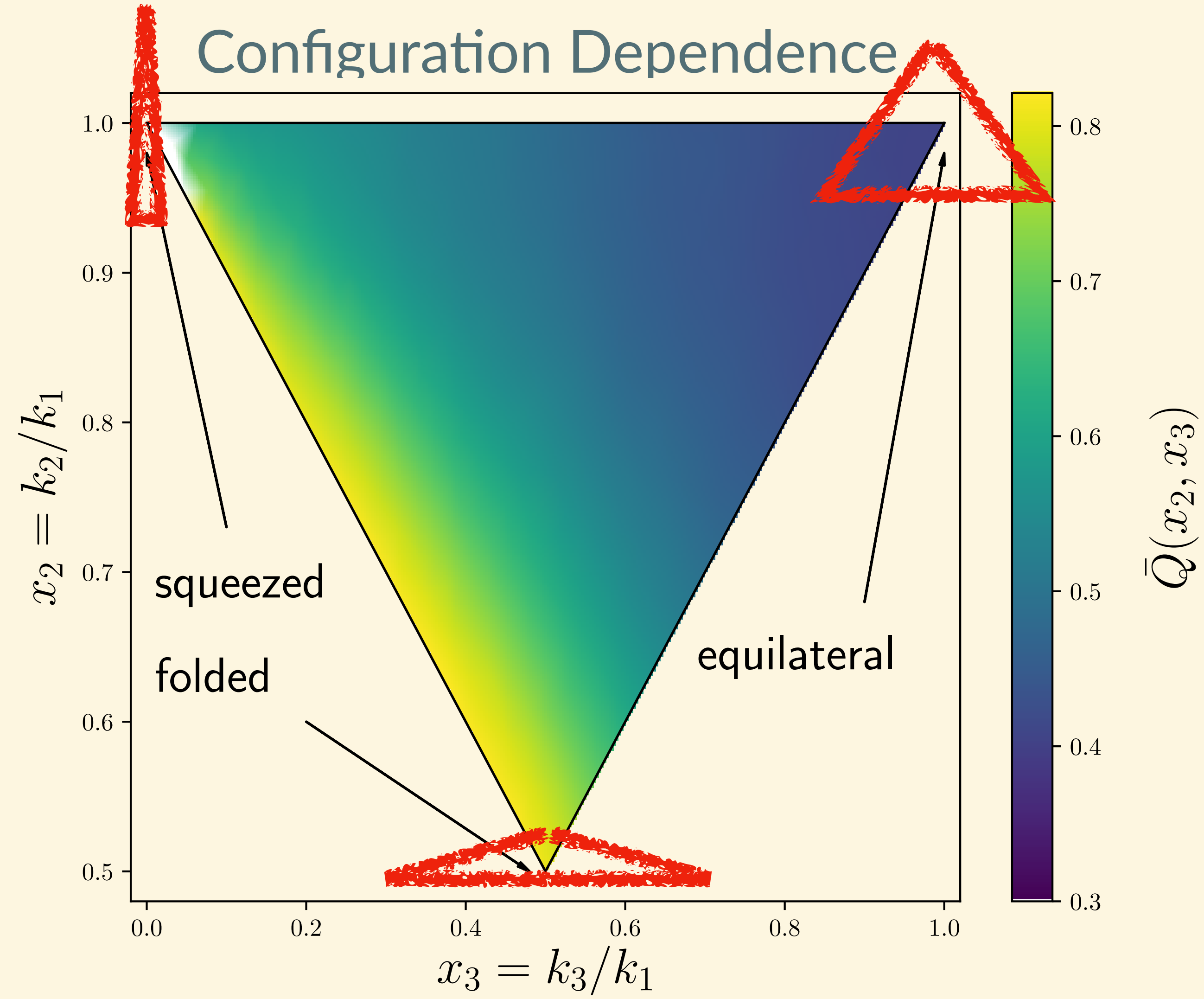

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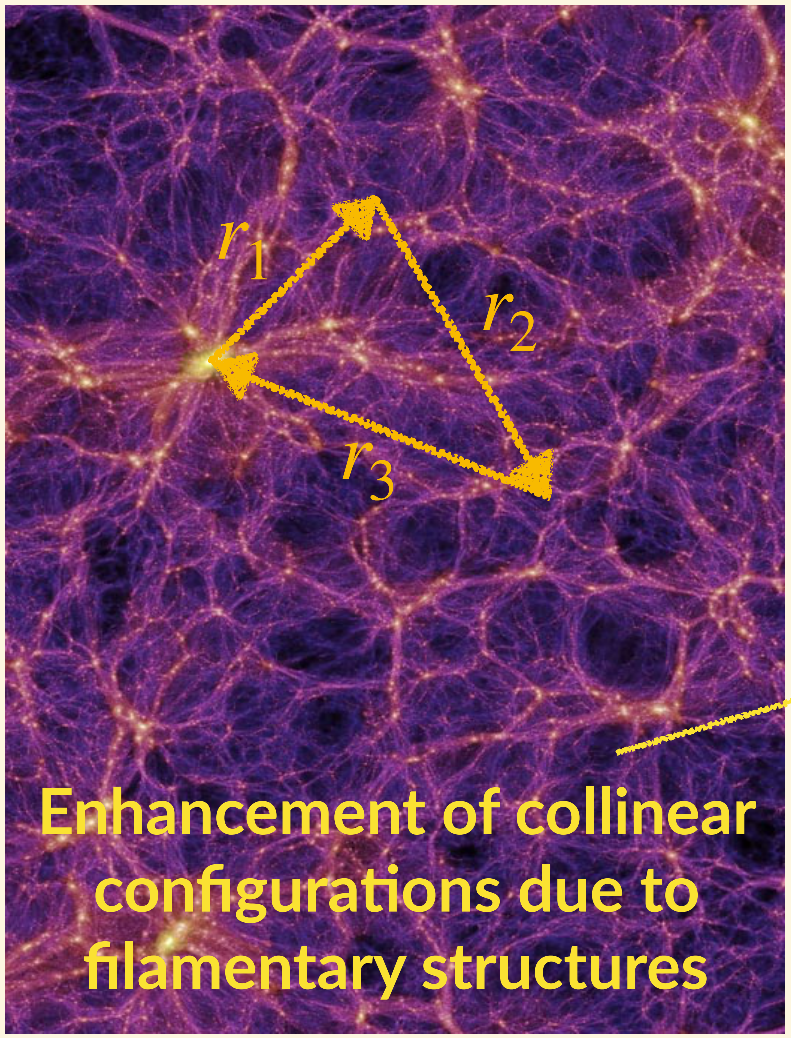

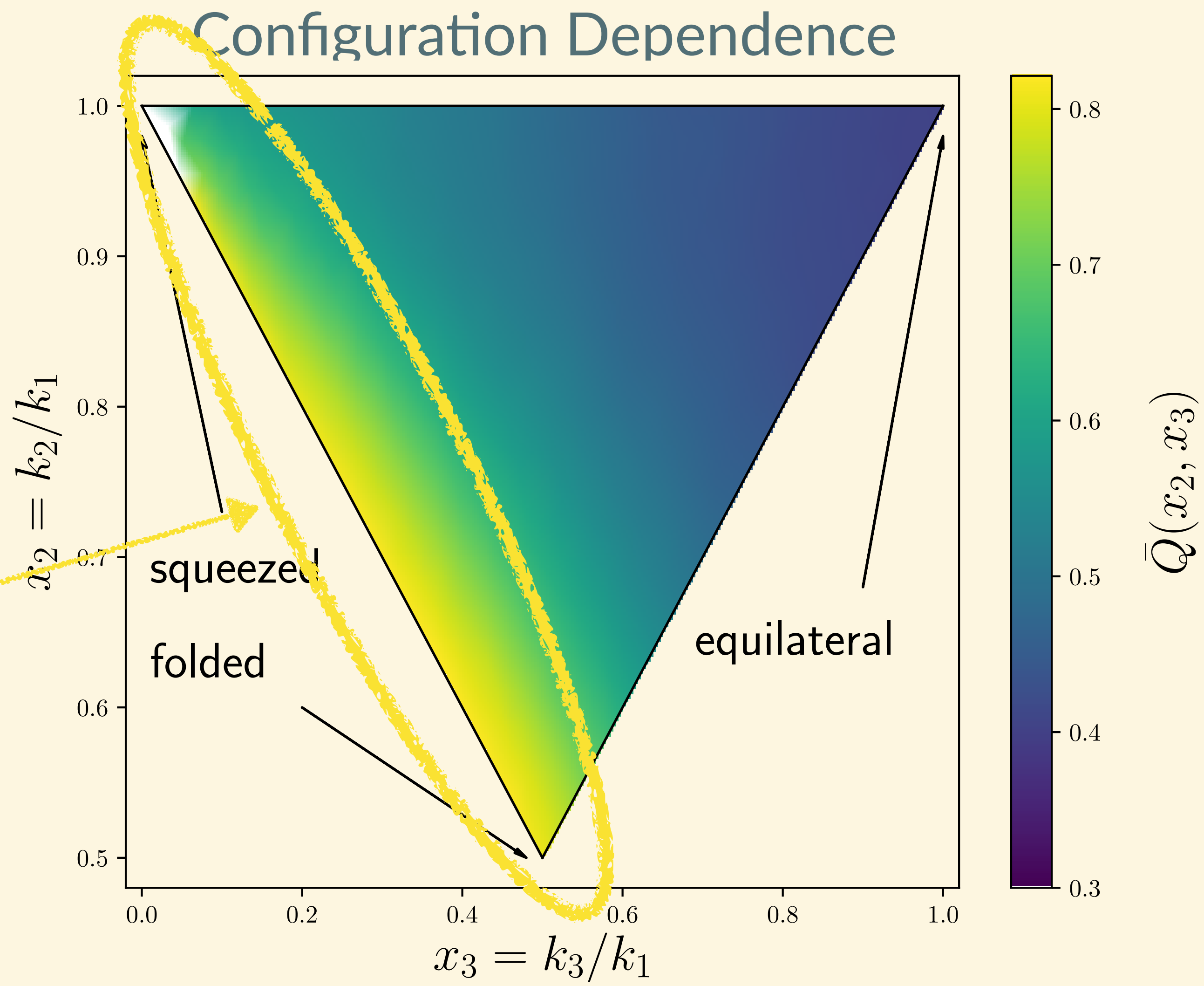

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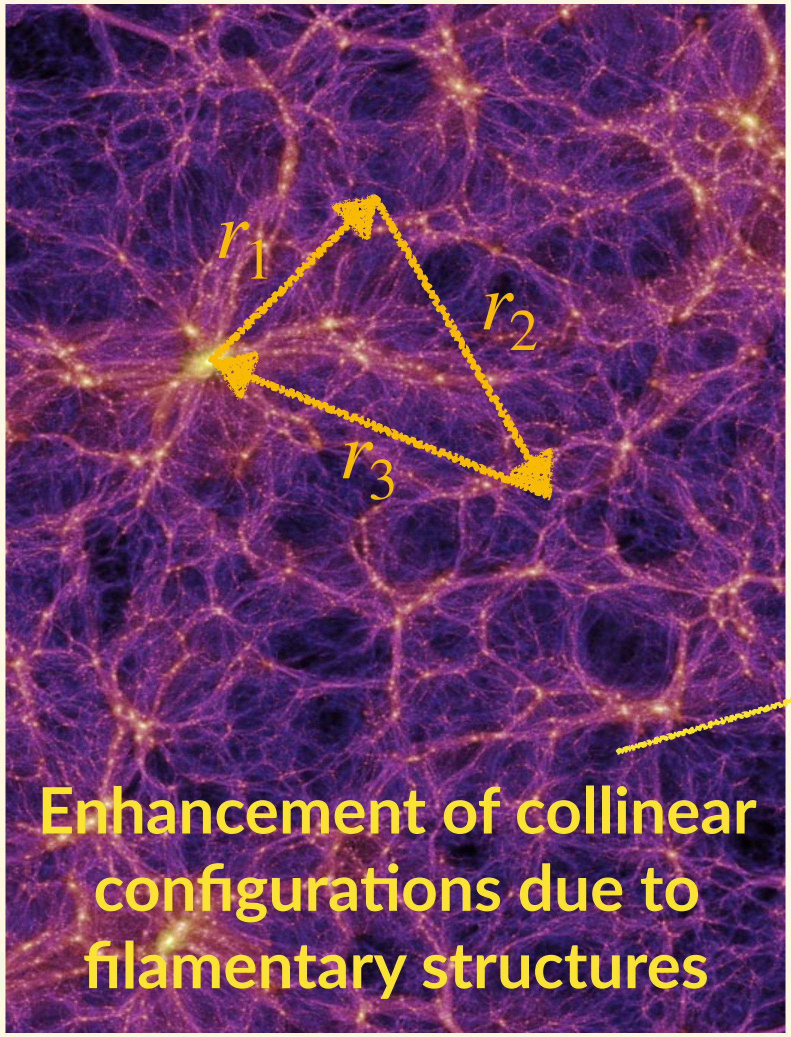

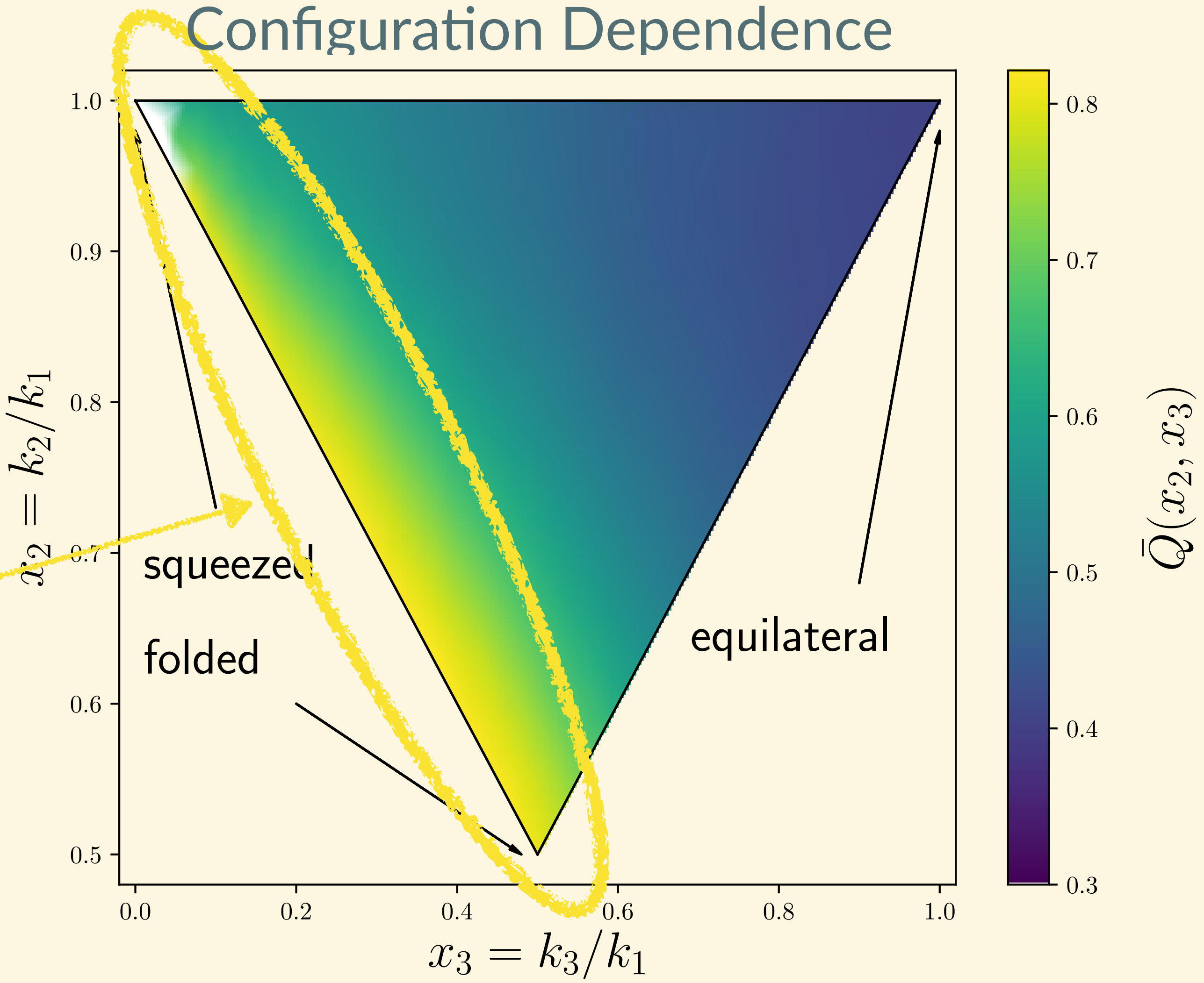

Why bother?

- Three-point statistics probe shapes in cosmic web ⟶ extra information
- Combination of two- and three-point statistics **breaks degeneracies**

$$\begin{cases} P \sim (f\sigma_8)^2 \\ B \sim \left[f^3 + \ldots\right] \sigma_8^4 \end{cases}$$

Improvements of 2x to 3x for DESI LRGs in redshift range 0.6 < z < 0.7

Gagrani & Samushia 17

Model Requirements

Robust interpretation of the data requires accurate control of:

- ✓ Non-linear evolution of matter density
- ✓ Redshift-space distortions
- ✓ Galaxy bias

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$$B(k_1,k_2,k_3) = \underbrace{B_{\rm tree}(k_1,k_2,k_3)}_{\langle \delta^{(2)}\delta^{(1)}\delta^{(1)}\rangle} + \underbrace{B_{1-\rm loop}(k_1,k_2,k_3)}_{\langle \delta^{(4)}\delta^{(1)}\delta^{(1)}\rangle,\ \langle \delta^{(3)}\delta^{(2)}\delta^{(1)}\rangle,\ \langle \delta^{(2)}\delta^{(2)}\delta^{(2)}\rangle} + \ \dots$$

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e.g. Gil-Marin+ 16

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up to **fourth order** in linear matter density

(assuming Gaussian initial conditions)

Galaxy Bias Expansion

How to account for galaxy bias in perturbative models?

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Kaiser 84

free bias parameter

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$$\Rightarrow \quad \delta_g(\boldsymbol{x}) = b_1\delta(\boldsymbol{x}) + \frac{b_2}{2}\delta^2(\boldsymbol{x}) + \frac{b_3}{3!}\delta^3(\boldsymbol{x}) + \ldots$$

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More generally, **all gravitational effects** that can influence galaxy formation should appear in bias relation McDonald & Roy 09, Chan+ 12, Mirbabayi+ 15, Desjacques+ 18

Second-order non-linear matter density

$$= \left[\frac{17}{21} + \frac{\mu}{2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right) + \frac{2}{7}\left(\mu^2 - \frac{1}{3}\right)\right] \delta_L(\boldsymbol{k}_1)\,\delta_L(\boldsymbol{k}_2)$$

Sph. collapse $\rightarrow \delta^2$

Bulk flow (velocity bias)

Tidal streams $\rightarrow \mathcal{G}_2$

Linear density

$$\mu \equiv \hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2$$

Galaxy Bias Expansion

Expand galaxy density up to fourth order: Assassi+ 14, Desjacques+ 18, Eggemeier+ 18

$$\delta_g(\boldsymbol{x}) = b_1\delta(\boldsymbol{x}) + \frac{b_2}{2}\delta^2(\boldsymbol{x}) + \gamma_2\mathcal{G}_2(\boldsymbol{x})$$

1st & 2nd order

$$+ \frac{b_3}{3!}\delta^3(\boldsymbol{x}) + \gamma_2^{\times}\delta(\boldsymbol{x})\mathcal{G}_2(\boldsymbol{x}) + \gamma_3\mathcal{G}_3(\boldsymbol{x}) + \gamma_{21}\mathcal{G}_2(\varphi_2, \varphi_1\,|\,\boldsymbol{x})$$

3rd order

$$+ \left[8 \text{ fourth order terms}\right] + \ldots \quad + \text{stochastic terms}$$

Spherical collapse

Tidal effects

Tidal effects can be expressed through **Galilean operators** $\mathcal{G}_2, \mathcal{G}_3$ and **LPT potentials** $\varphi_1, \varphi_2, \ \ldots$

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Complication: computing observables $(P_g, B_g, \ \ldots)$ leads to divergencies $\propto \sigma^2 \equiv \left\langle \delta^2(\boldsymbol{x}) \right\rangle$

➡ Absorb σ^2's by **renormalization** of bias parameters
Parameters in bias expansion above are **NOT** measurable quantities

Multi-Point Propagator Formalism

Renormalization of bias parameters = Resumming reducible diagrams

$$\Rightarrow \quad \left\langle \frac{\partial^n \delta_g(\boldsymbol{k})}{\partial \delta_L(\boldsymbol{k}_1) \,\cdots\, \partial \delta_L(\boldsymbol{k}_n)} \right\rangle = (2\pi)^3 \, \Gamma_g^{(n)}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n) \, \delta_D(\boldsymbol{k} - \boldsymbol{k}_1 - \ldots - \boldsymbol{k}_n)$$

correspond to **observable** bias parameters

Multi-point propagators

Crocce & Scoccimarro 06, Bernardeau+ 08, Matsubara 11, ...

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⟶ New galaxy bias expansion: $\delta_g = \left\langle \frac{\partial \delta_g}{\partial \delta_L} \right\rangle \delta_L + \frac{1}{2} \left\langle \frac{\partial^2 \delta_g}{\partial \delta_L^2} \right\rangle \left(\delta_L^2 - \langle \delta_L^2 \rangle \right) + \ldots$

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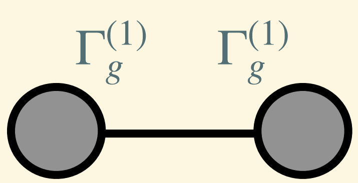
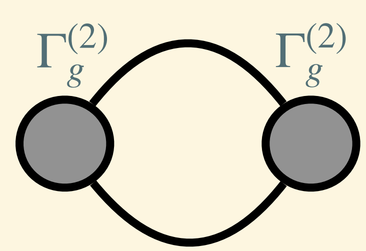
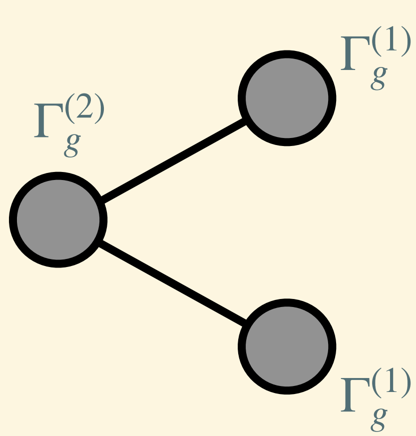
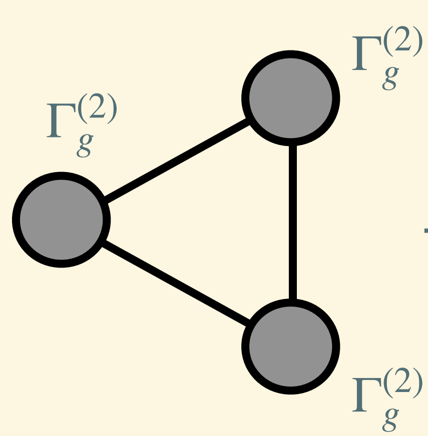
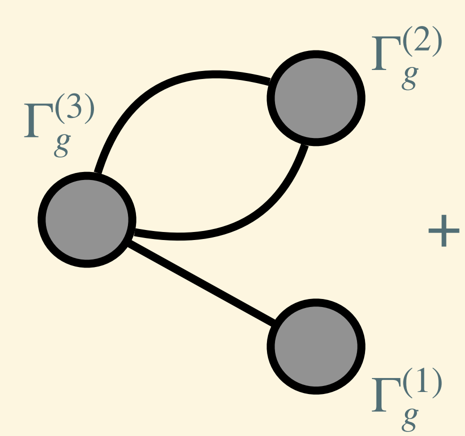
P_g = 

+ 

+ ... + $P_{\rm noise}$

B_g = 

+ 

+ 

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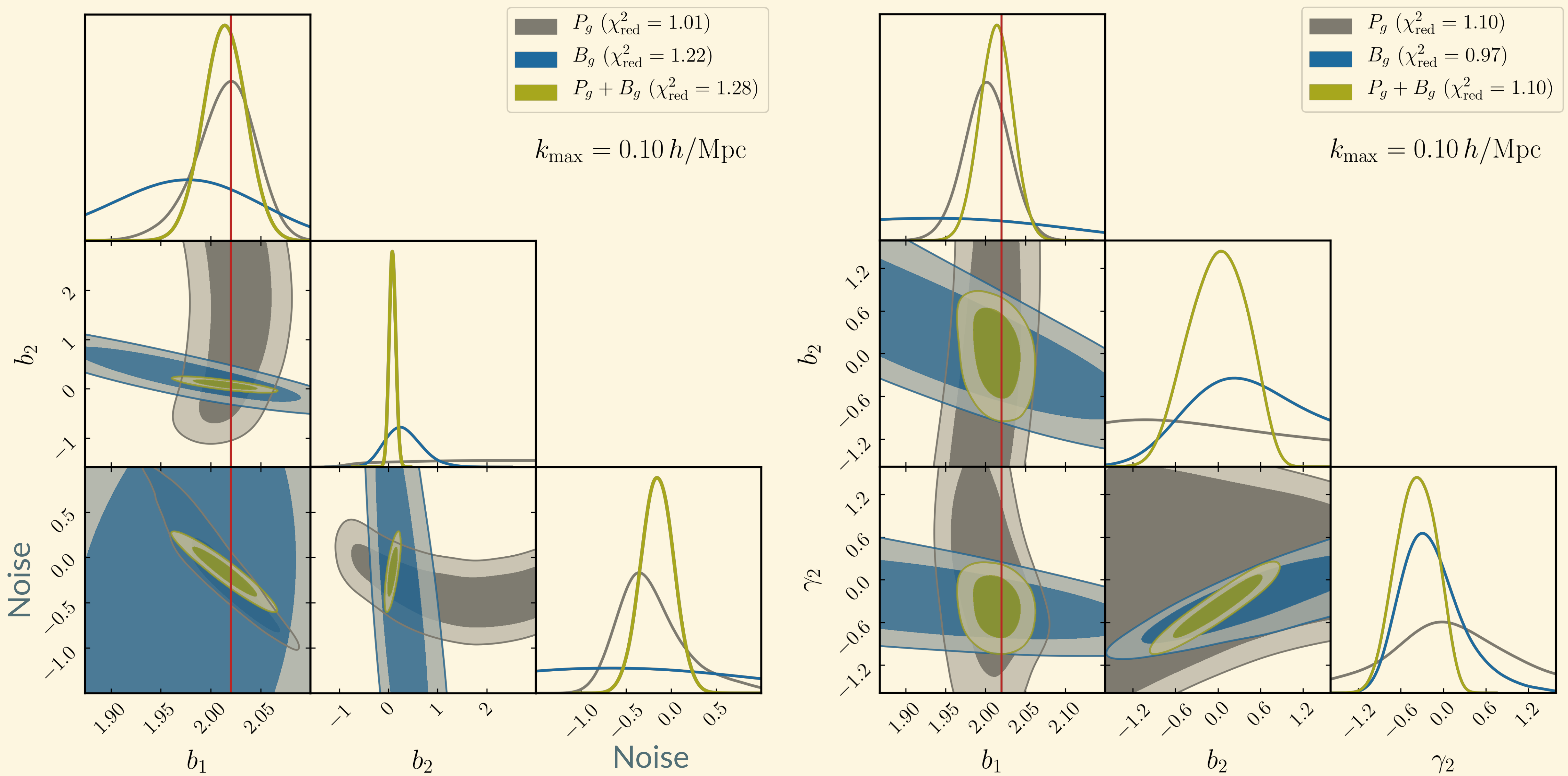
Combined Constraints

Redshift = 0.57



Gil-Marin+ 16 (BOSS) model
(3 parameters: 2 bias + 1 noise)

One-loop model
(10 parameters: 7 bias + 3 noise)

Mock CMASS galaxies, $V = (1.5\,{\rm Gpc}/h)^3$, fitting up to **50** P_g and **1400** B_g bins

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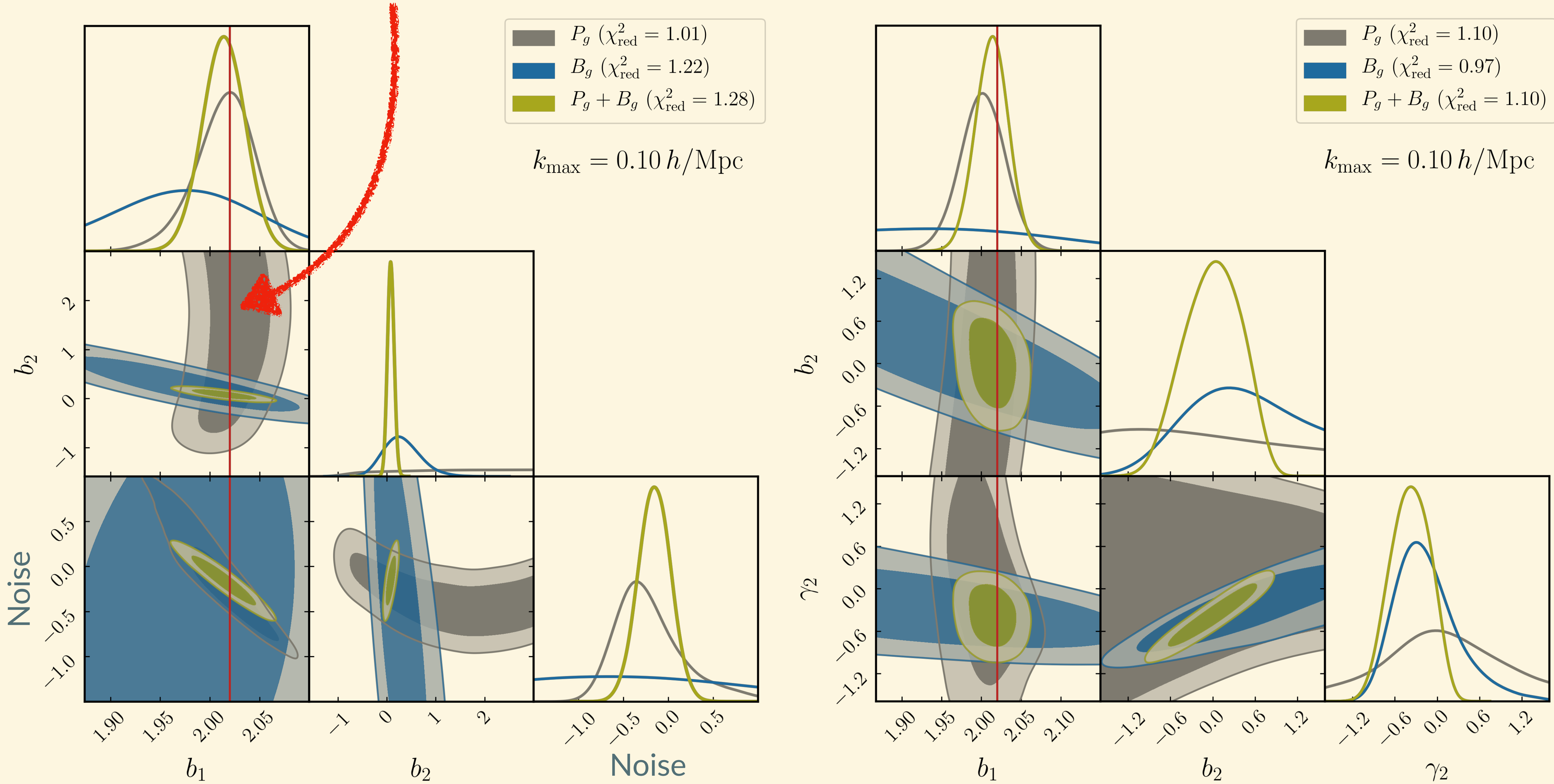
"true" linear bias from galaxy — matter cross-correlation



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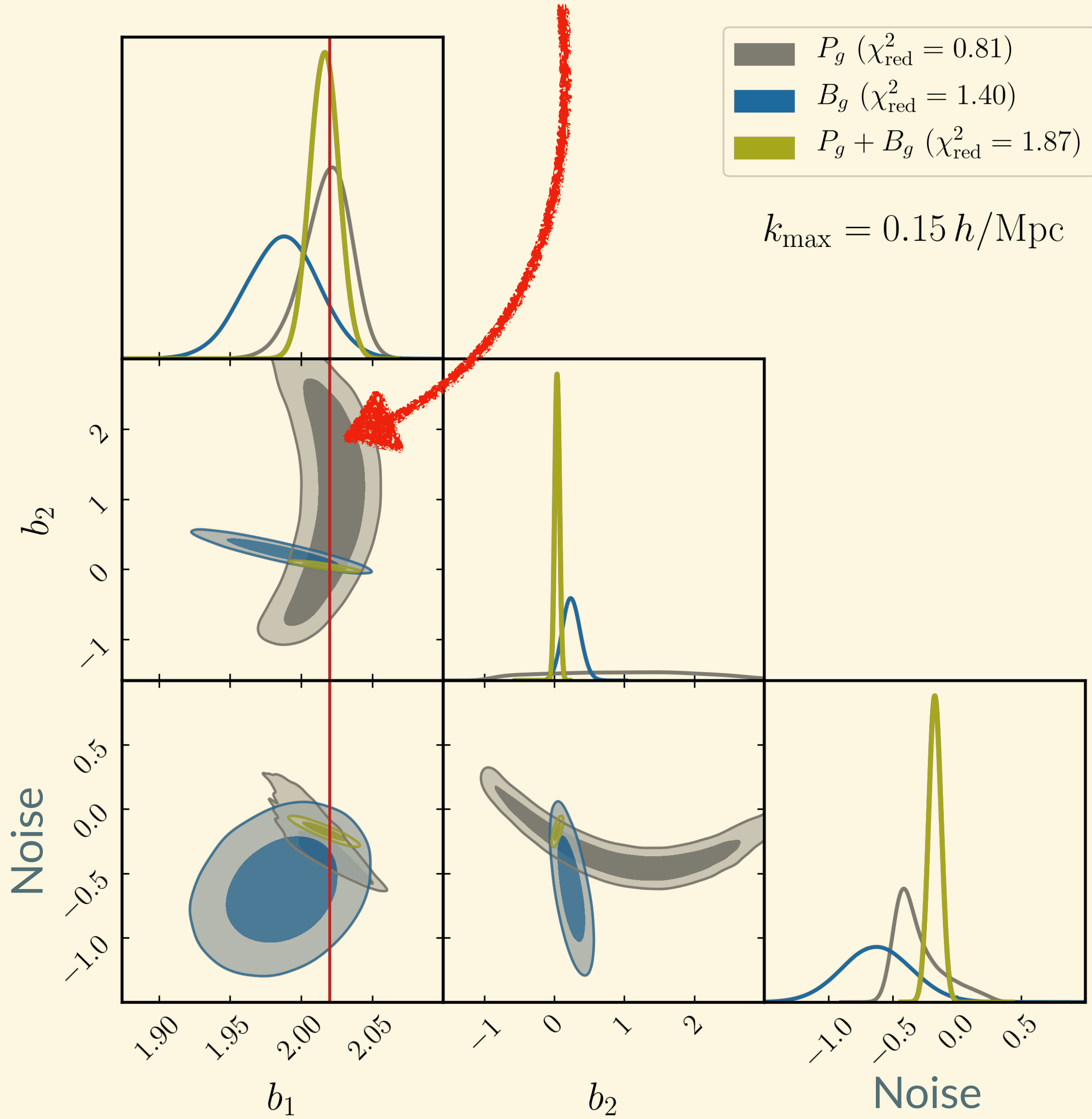

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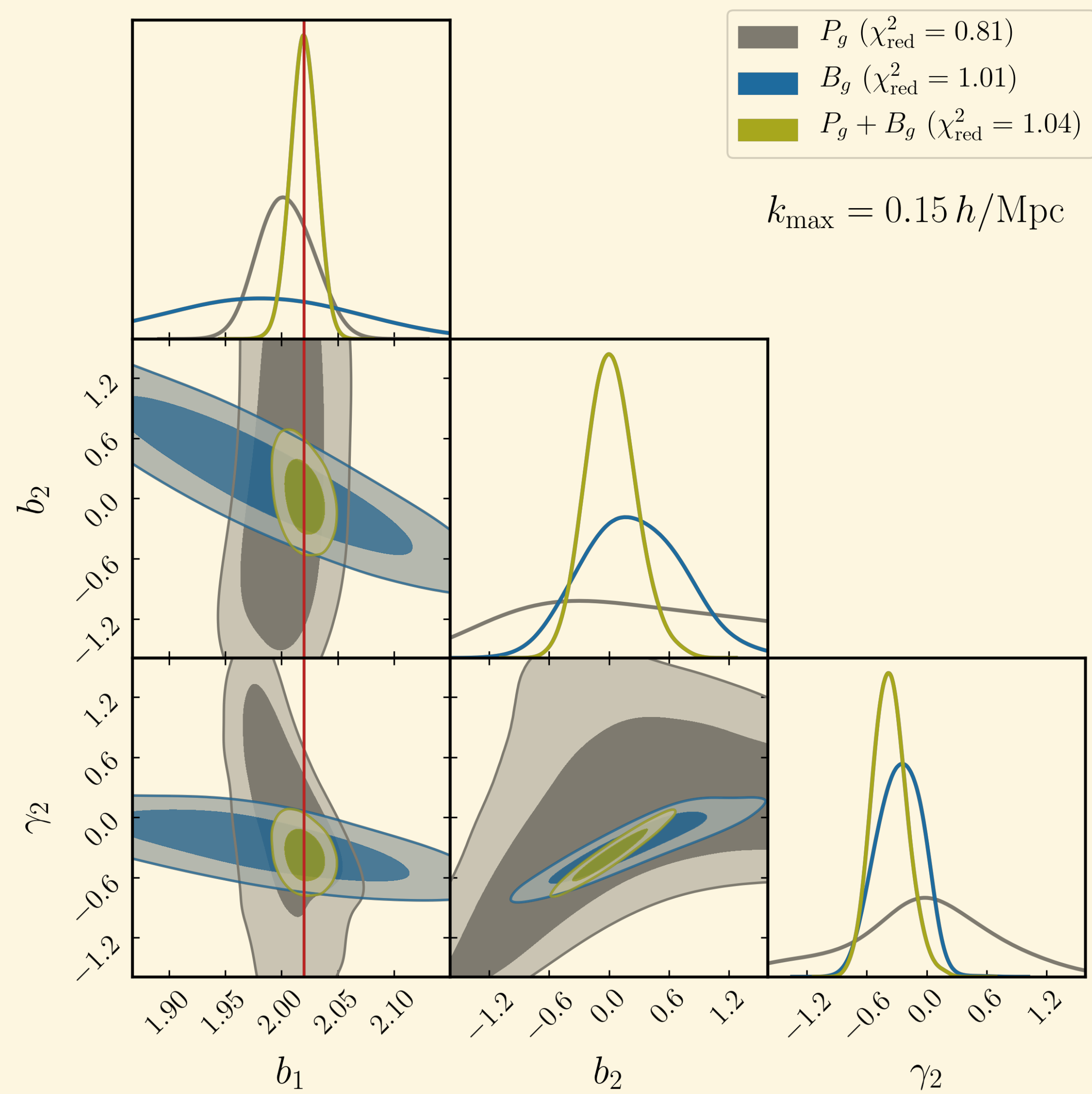

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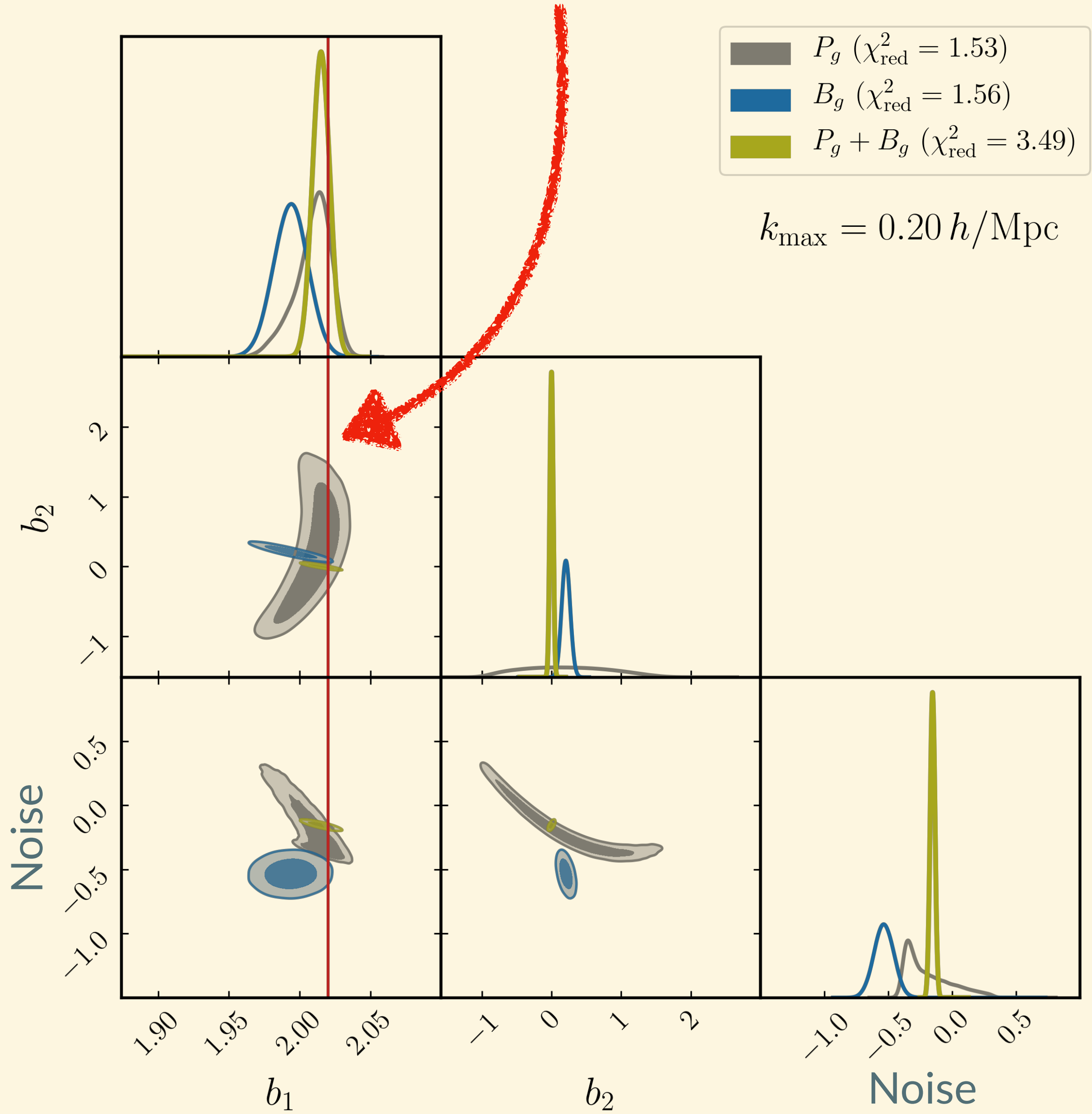

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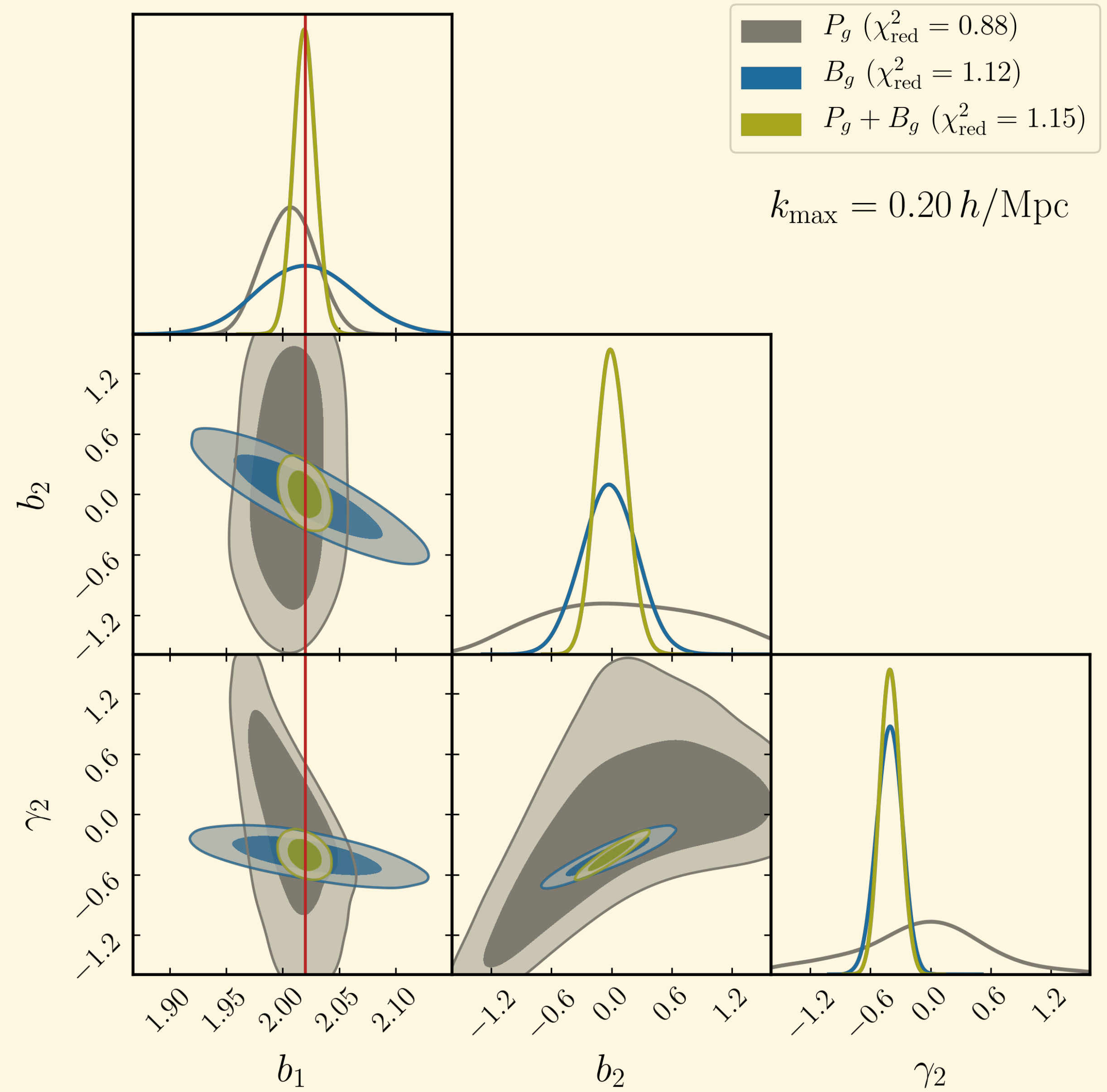

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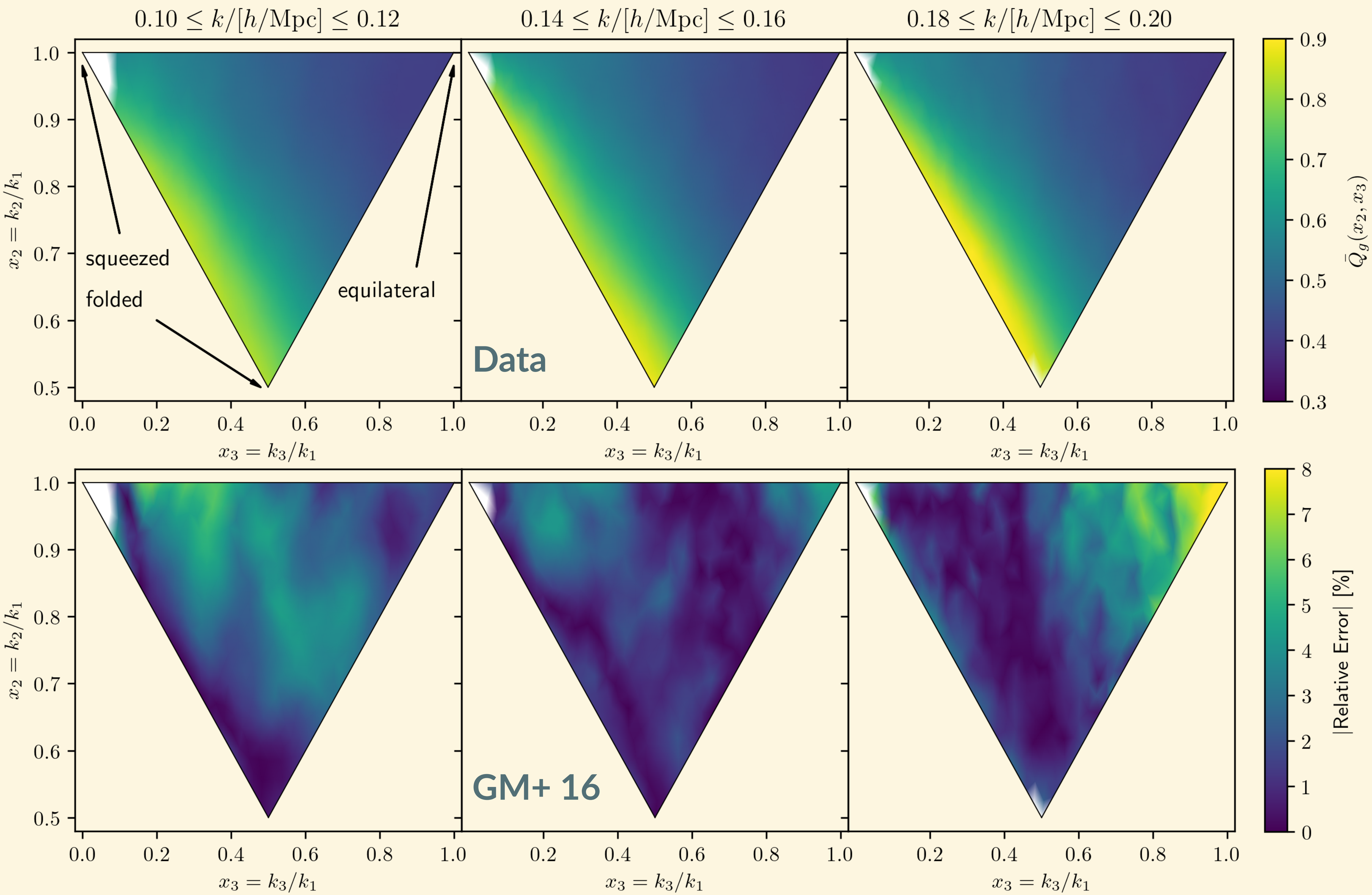

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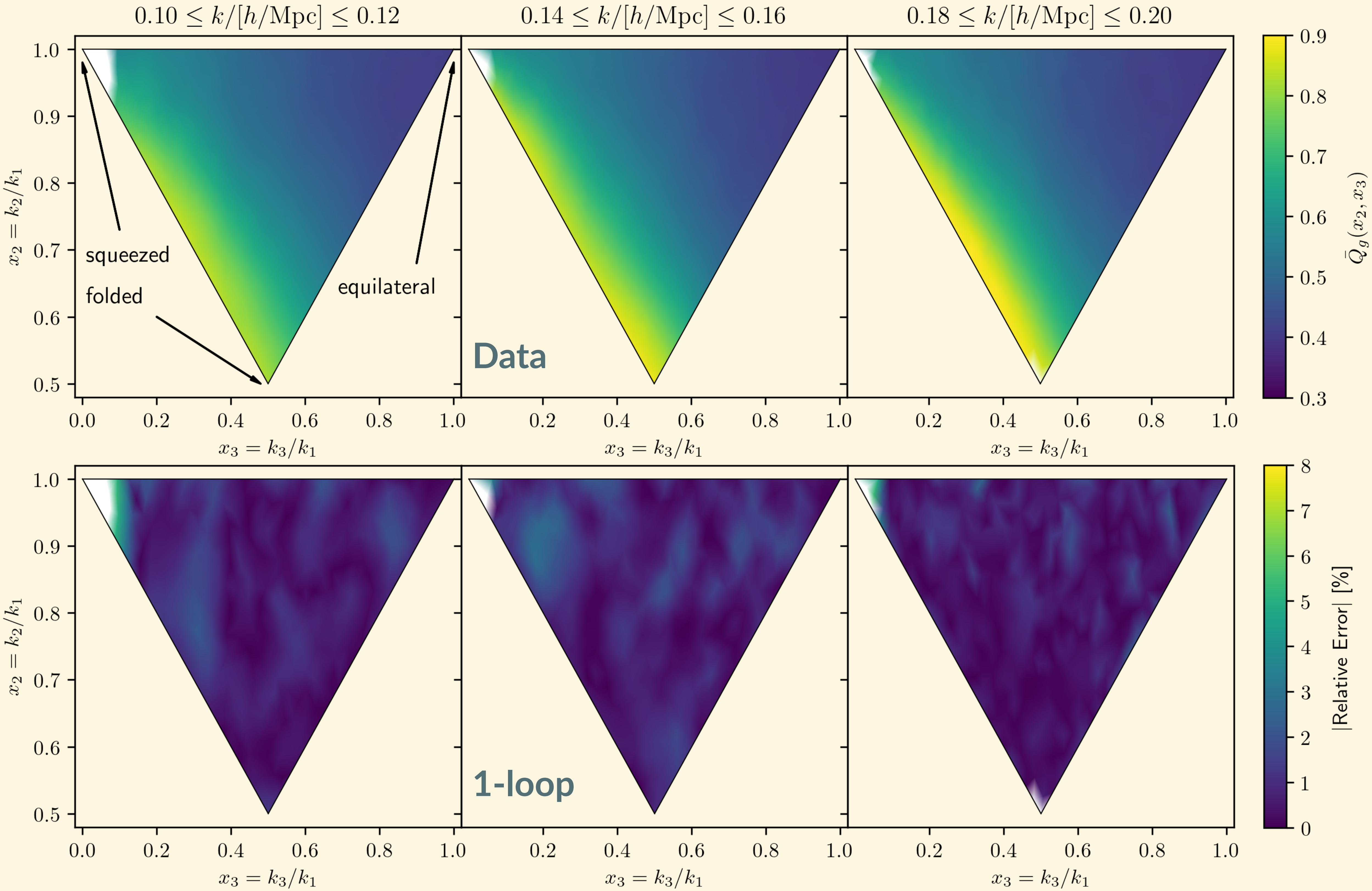

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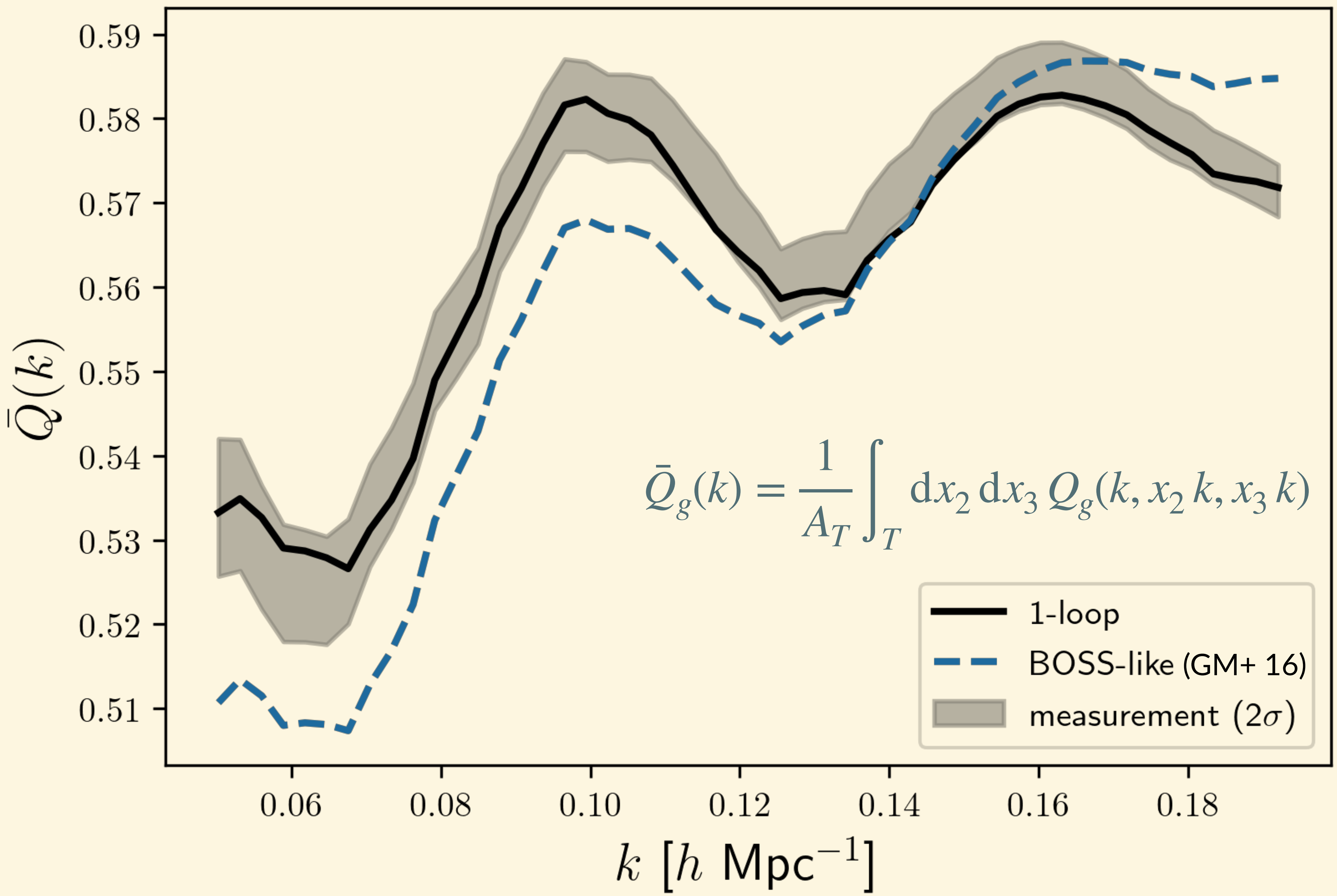

Towards smaller scales →

Reducing the Parameter Space

Time evolution produces non-zero contributions to late-time bias parameters

Lagrangian (initial) bias parameters

2nd order

$$\gamma_2 = -\frac{2}{7}\left(b_1 - 1\right) + \gamma_{2,\mathscr{L}}$$

3rd order

$$\gamma_3 = -\frac{1}{9}\left(b_1 - 1 + 9\gamma_2\right) + \gamma_{3,\mathscr{L}}$$

$$\gamma_{21} = \frac{2}{21}\left(b_1 - 1 + 9\gamma_2\right) + \gamma_{21,\mathscr{L}}$$

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Chan+ 12, Eggemeier+ 18

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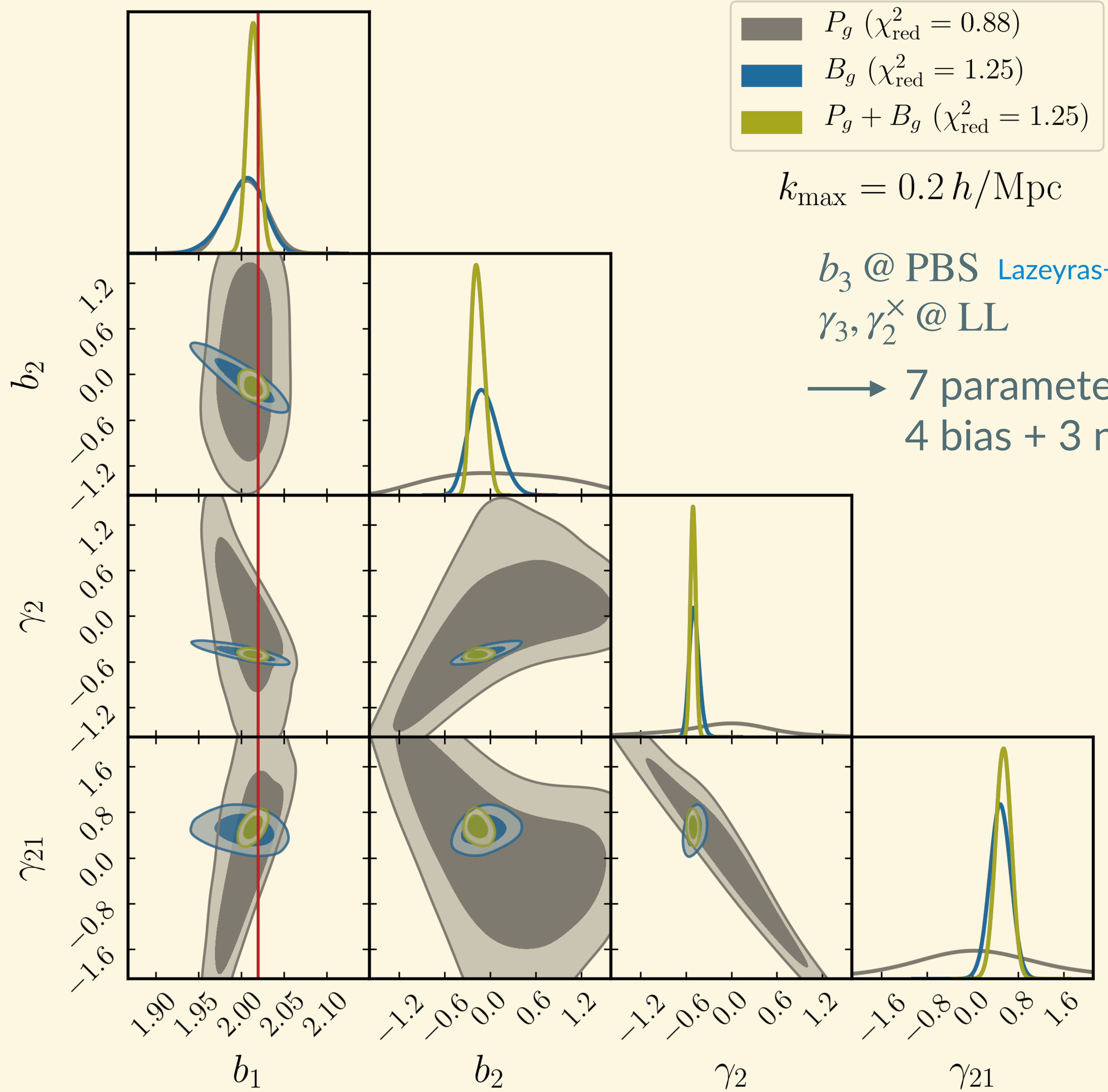
Local Lagrangian relation

Chan+ 12, Eggemeier+ 18

➡ Can we use it to fix a subset of 3rd order bias parameters?

Reducing the Parameter Space



$k_{\rm max} = 0.2\, h/{\rm Mpc}$

b_3 @ PBS Lazeyras+ 16

$\gamma_3, \gamma_2^\times$ @ LL

⟶ 7 parameters:
4 bias + 3 noise

Varying the Amplitude of Fluctuations

Combine power spectrum and bispectrum to break $b_1 - A_S$ degeneracy

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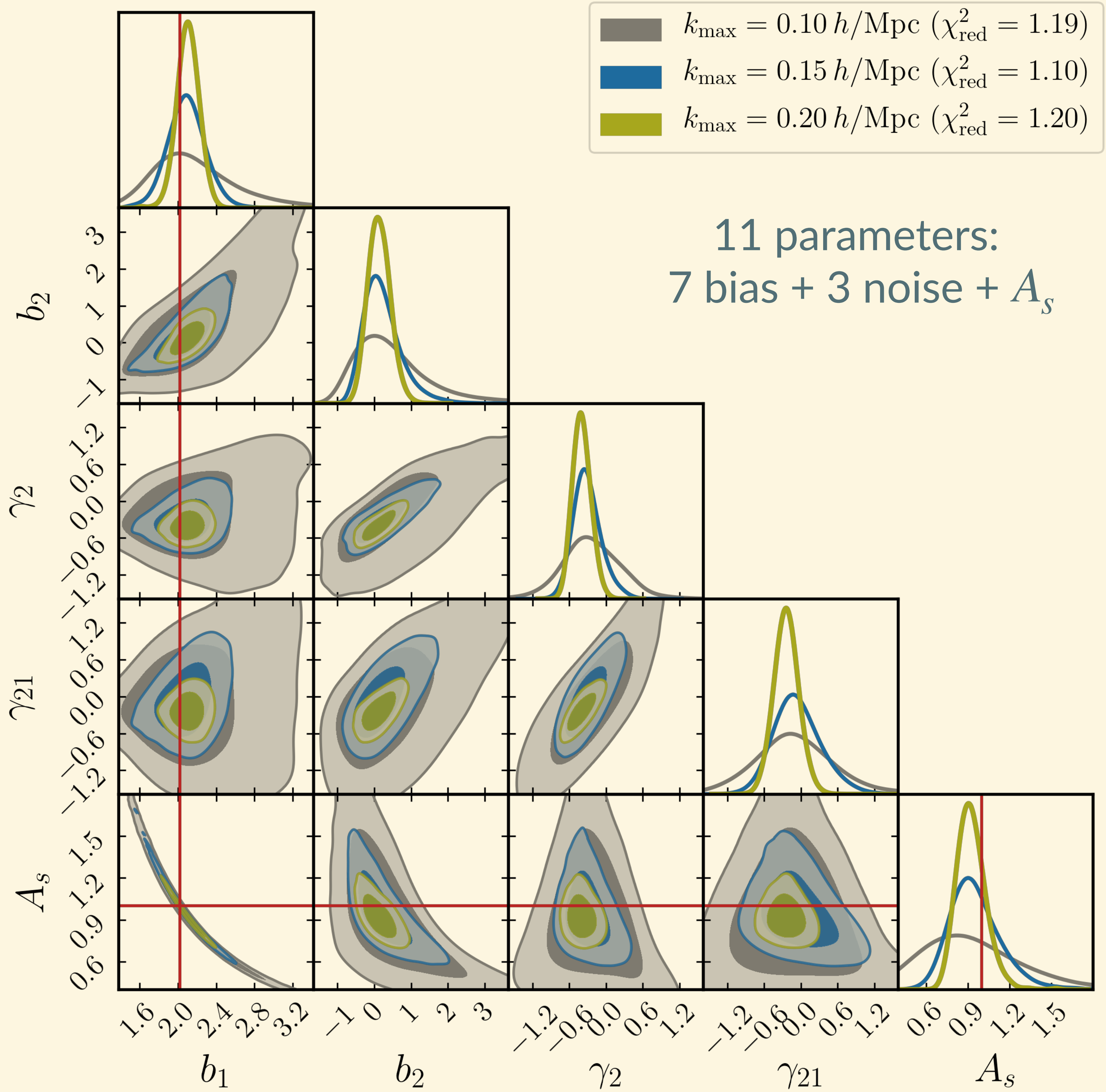
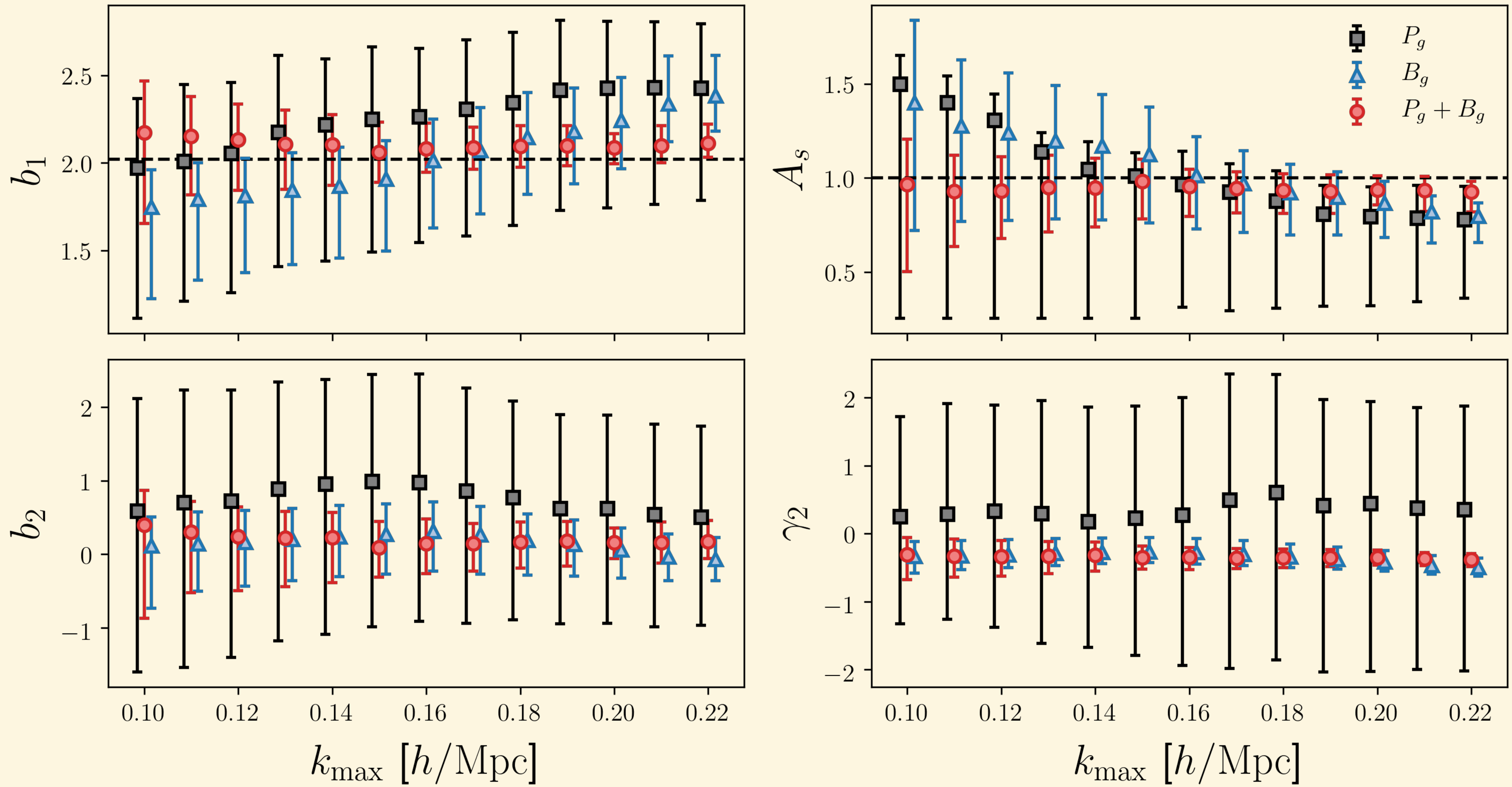


11 parameters:
7 bias + 3 noise + A_s

Varying the Amplitude of Fluctuations



Improvement on amplitude of fluctuations, A_s, **up to ~4x** over power spectrum!

Take-Away

- ★ Beyond $k_{\max} \sim 0.1\, h/\mathrm{Mpc}$ loops in the galaxy bias expansion become important

- ★ **Consistent treatment** of power spectrum and bispectrum necessary for unbiased recovery of b_1 and A_s

- ★ Time evolution of bias can help reduce quickly inflating parameter space (→ 7 parameter model for P + B works up to $k_{\max} \sim 0.2\, h/\mathrm{Mpc}$)

THANK YOU!