Perturbation theory challenge for cosmological parameters estimation

PTchat@Kyoto
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Yukawa Institute of Theoretical Physics

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In collaboration with
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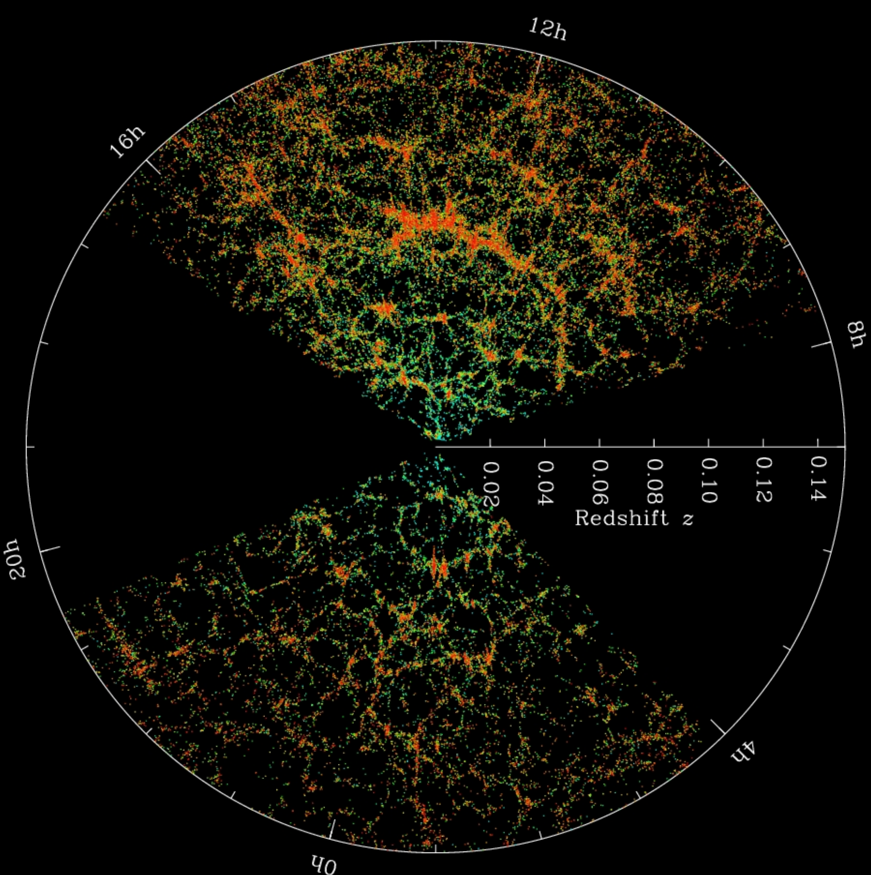
and Atushi Taruya (YITP)

Based on PRD 99, 063530 (2019)

Cosmology with Galaxy Clustering

✦ The matter distribution, ***large-scale structures***, reflects underlying physics (***modified gravity***, ***massive neutrinos***…).

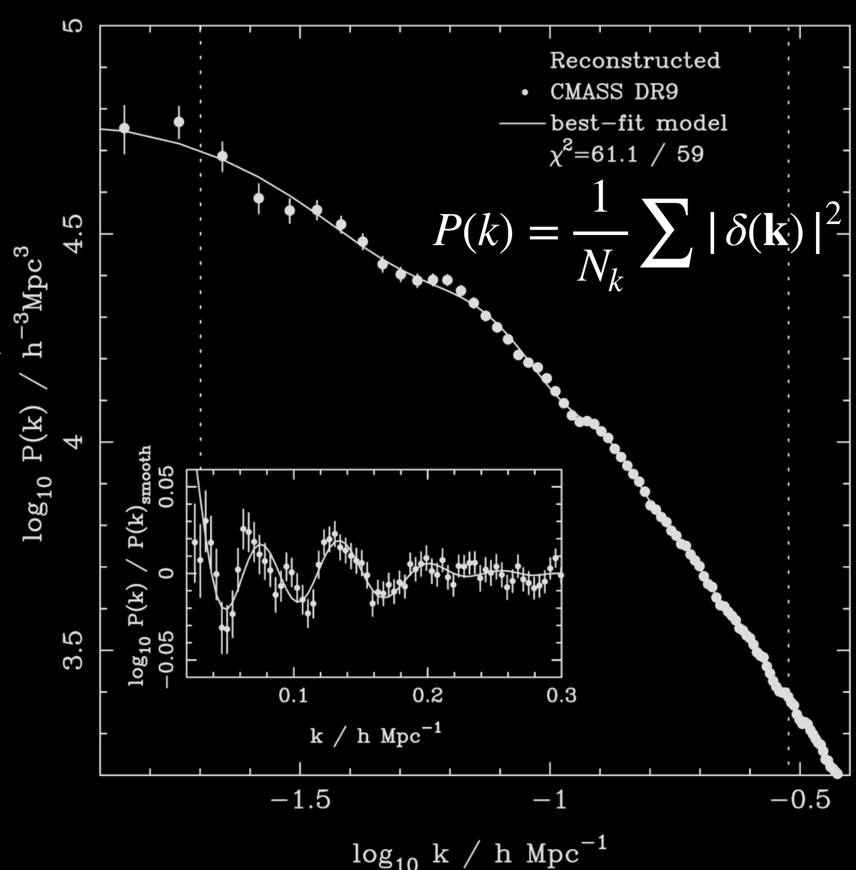
➡ This information is imprinted on ***power spectrum*** and it can be measured from spectroscopic surveys of galaxy distribution.



SDSS Galaxy Map

Statistics

Power spectrum



Anderson+ (2012)

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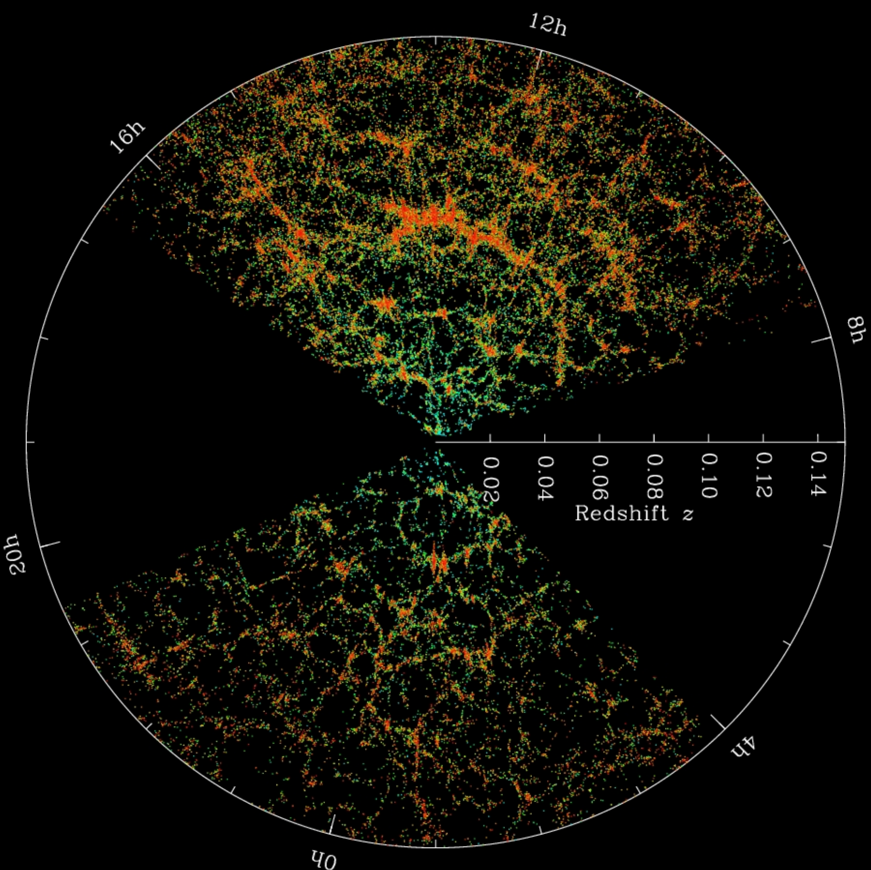

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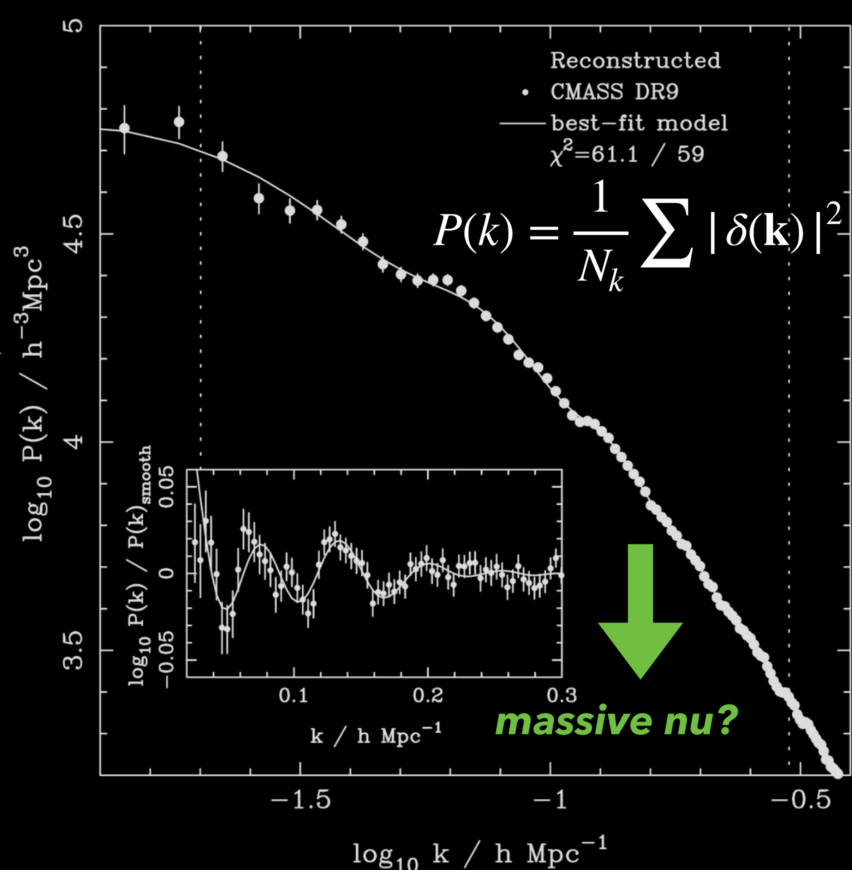

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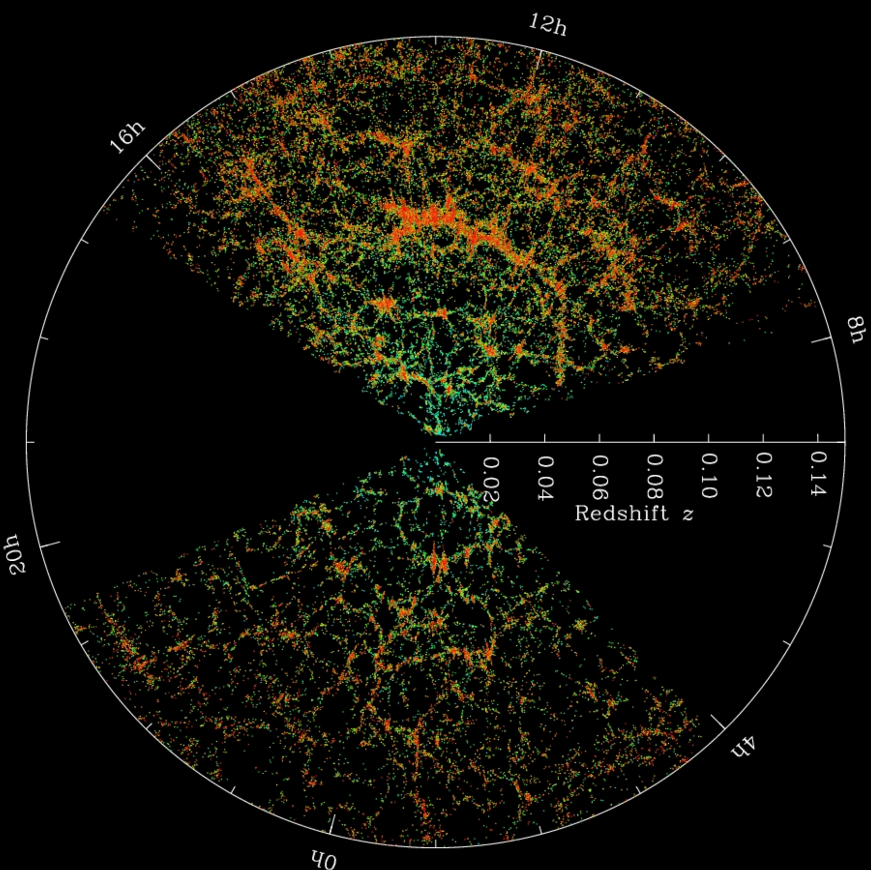

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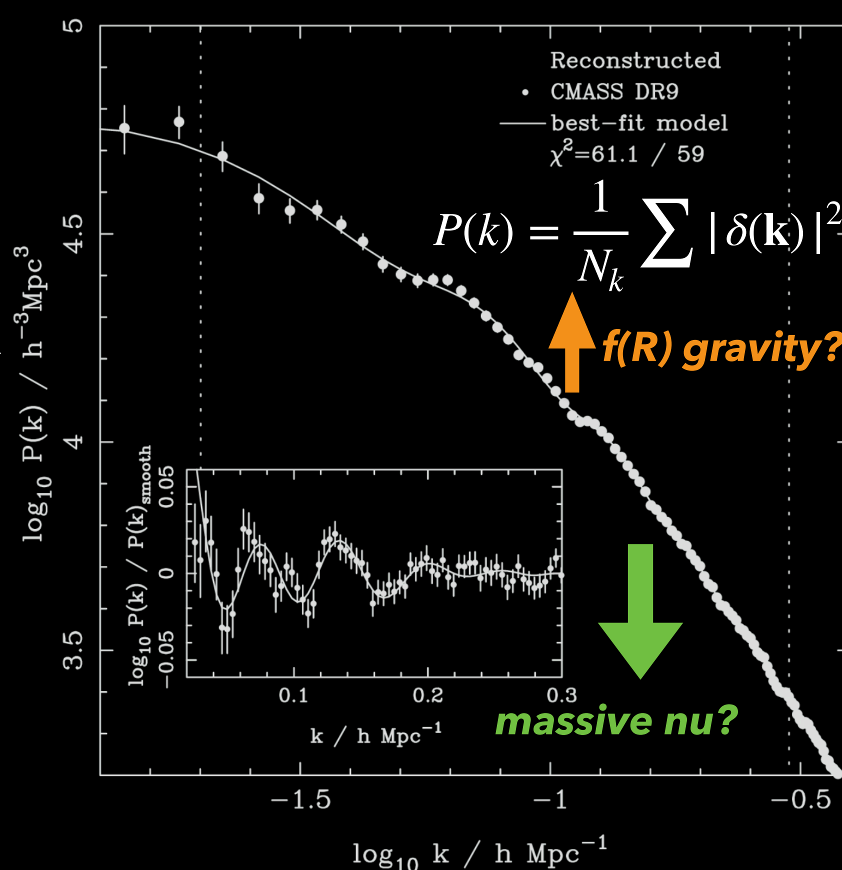

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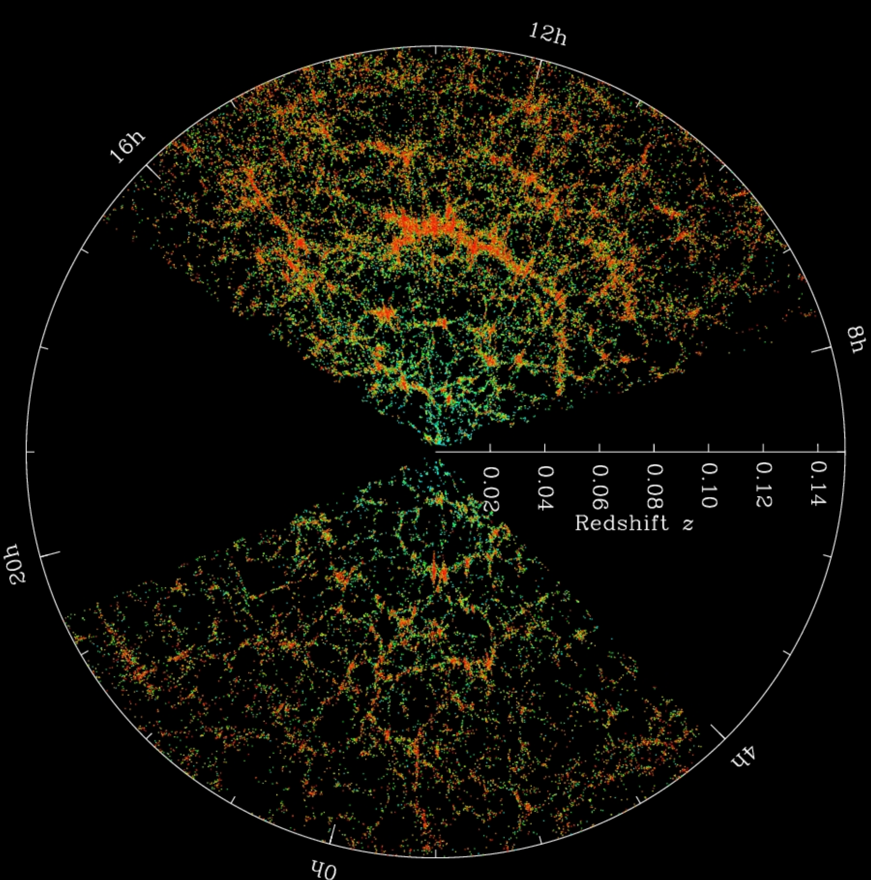

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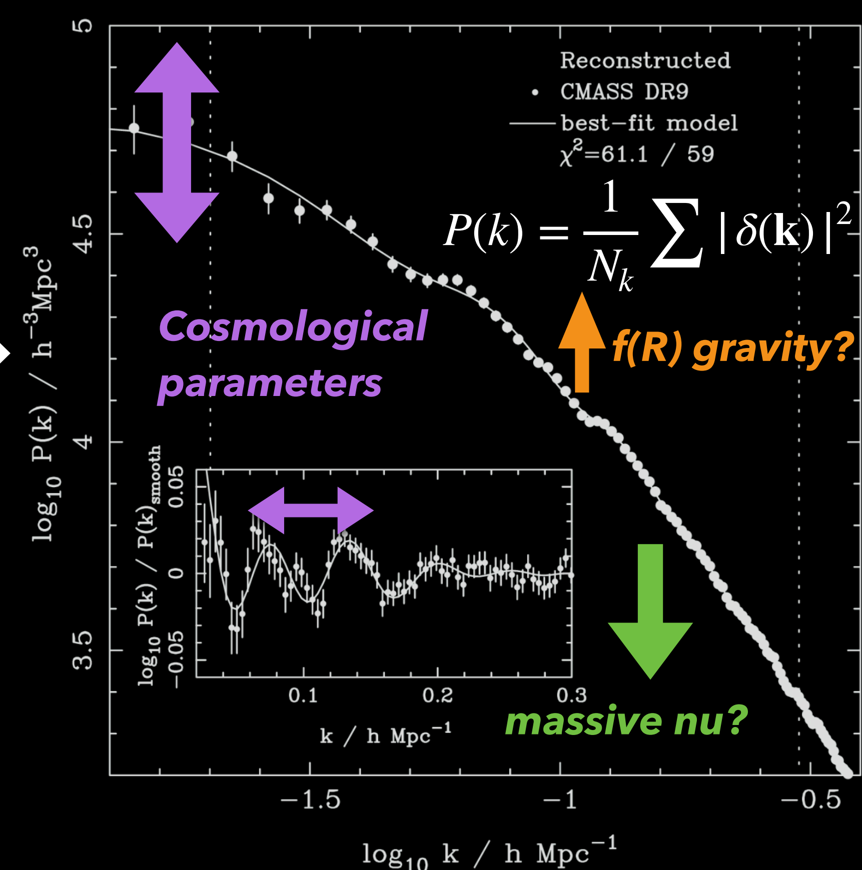


Anderson+ (2012)

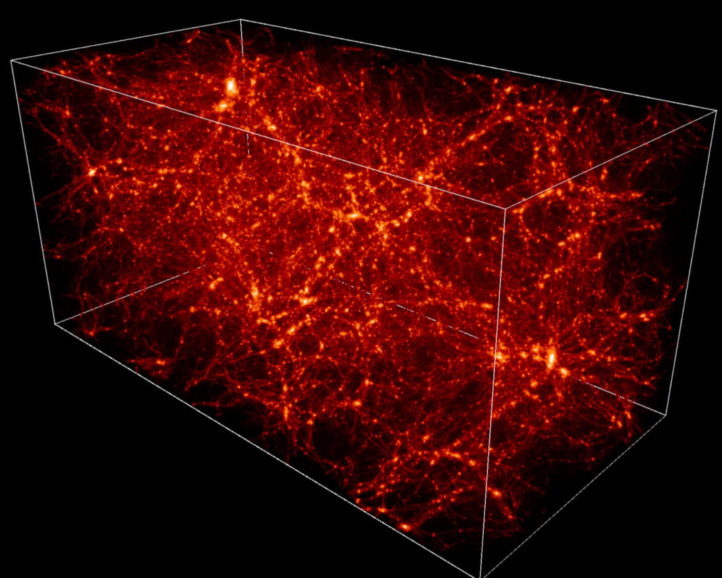
Modeling of LSS at nonlinear regimes

N-body simulations

Matter distribution is discretized as particles. The evolution is governed by Newtonian gravity.

Pros: Accurate down to resolution

Cons: Computationally expensive
Not suitable for MCMC



Perturbation theory (PT)

Based on single-stream approx., fluid equations are expanded with respect to density contrast.

Pros: Analytical and fast

Cons: The applicable range is limited to mildly nonlinear regime.

$$\delta(\mathbf{k}) = \sum_{n=1} D_+^n \delta^{(n)}(\mathbf{k})$$

$$\delta^{(n)}(\mathbf{k}) = \int \frac{d^3q_1 \cdots d^3q_n}{(2\pi)^{3(n-1)}} \delta_D(\mathbf{k} - \mathbf{q}_1 - \cdots - \mathbf{q}_n) \times F_{\rm sym}^{(n)}(\mathbf{q}_1, \ldots, \mathbf{q}_n) \delta_0(\mathbf{q}_1) \cdots \delta_0(\mathbf{q}_n)$$

Bernardeau+ (2002)

PT Models

Standard Perturbation Theory (SPT) :

$$\delta(\mathbf{k}) = \sum_{n=1} D_+^n \delta^{(n)}(\mathbf{k})$$

Widely used standard way to expand fluid eqs. 2-loop level expansion gives reasonable predictions.

$$\delta^{(n)}(\mathbf{k}) = \int \frac{d^3q_1\cdots d^3q_n}{(2\pi)^{3(n-1)}}\delta_{\rm D}(\mathbf{k}-\mathbf{q}_1-\cdots-\mathbf{q}_n) \times F^{(n)}_{\rm sym}(\mathbf{q}_1,\ldots,\mathbf{q}_n)\delta_0(\mathbf{q}_1)\cdots\delta_0(\mathbf{q}_n)$$

Regularized Perturbation Theory (RegPT) : *Bernardeau+ (2008), Taruya+ (2012)*

Reorganizing SPT expansion with ***propagator*** (Γ-expansion)
At high-k regime, the power spectrum damps as $e^{-\sigma_{\rm d}^2 k^2/2}$.
The r.m.s. displacement can be a free parameter. → **RegPT+**

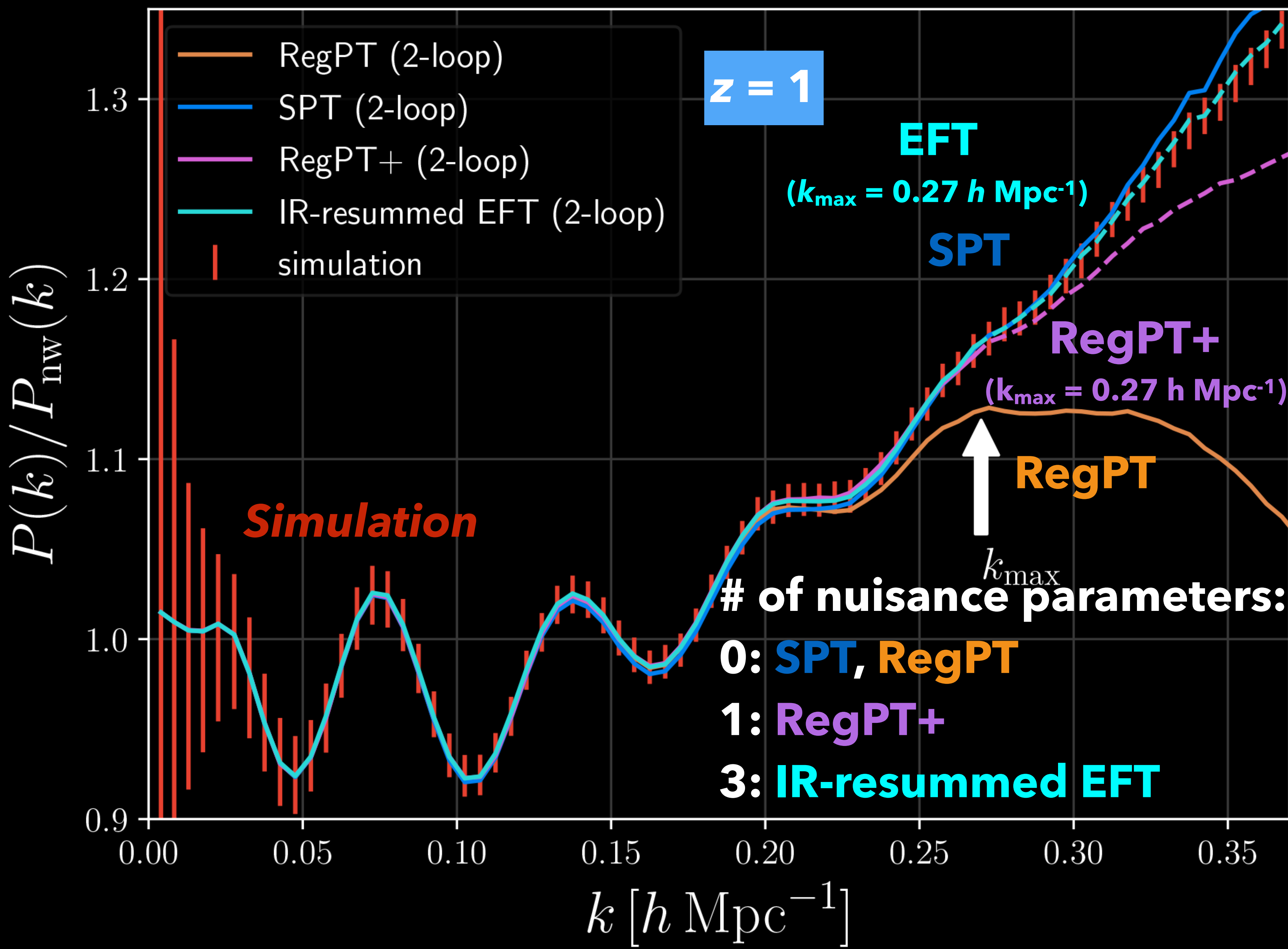
(IR-resummed) Effective Field Theory (EFT) : *Baumann+ (2012), Carrasco+ (2012)*

Introducing counter terms to describe small-scale behavior characterized by effective sound speed.
The sound speed is treated as free parameter.

$$-2(2\pi)c_{s(1)}^2\left(\frac{k}{k_{\rm NL}}\right)^2 P_{\rm L}(k)$$

LPT, closure, Time-RG, RPT, and more…

Predictions of Power Spectra



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- **Is it applicable to smaller scales?**
- **How accurate in cosmological parameter estimation?**
- **Some models contain nuisance parameters but is it fair to compare just the smallest applicable scales?**

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This study: Systematic tests of various 2-loop level PT schemes with *N*-body simulations in the context of cosmological parameter estimation with P(k)

Strategy

- ***End-to-end test in cosmological parameter inference***

1. Generate initial condition

h Hubble parameter

$\Omega_{\rm m}$ Matter density

$A_{\rm s}$ Amplitude of scalar perturbation

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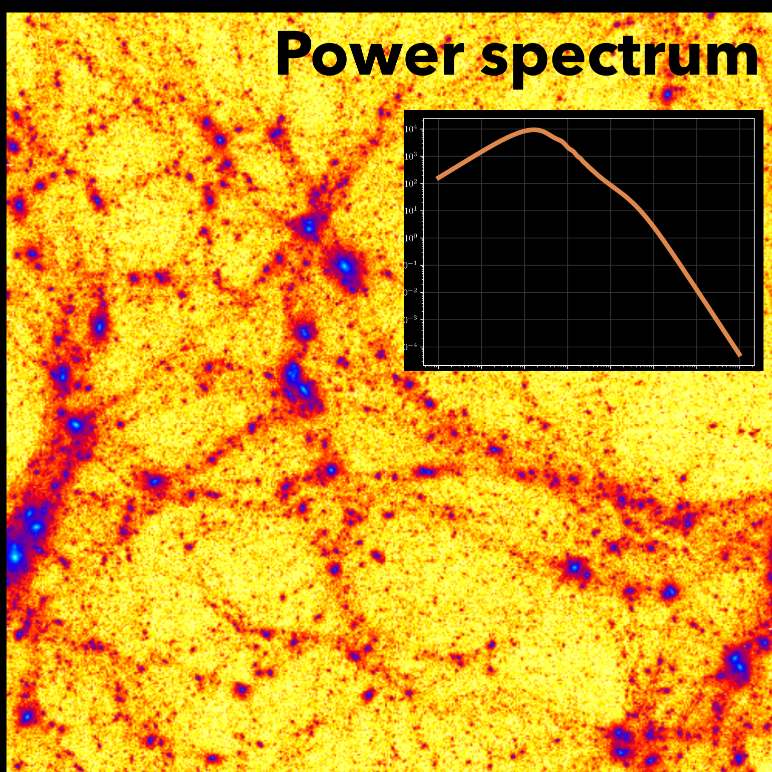

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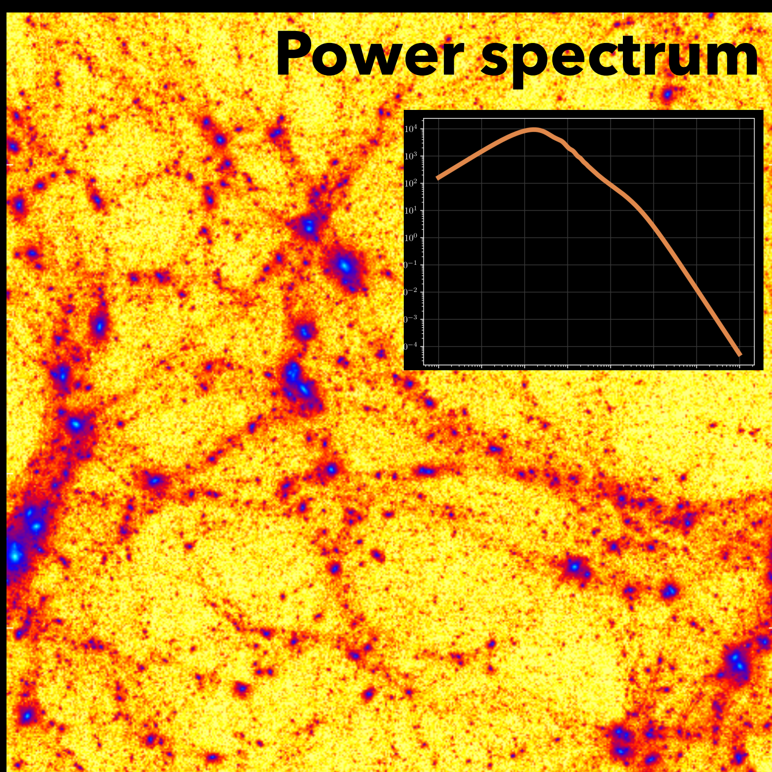


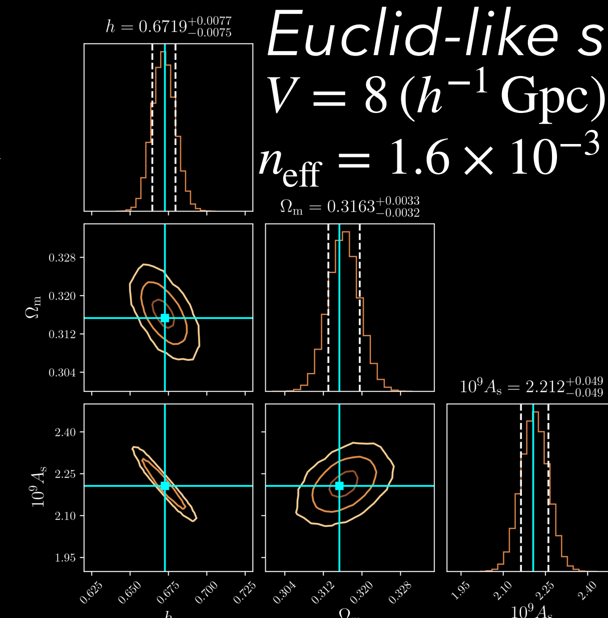
3. Infer cosmological params. with MCMC

2-loop level PT schemes of SPT, RegPT, RegPT+, EFT

Euclid-like survey

$V = 8\,(h^{-1}\,\mathrm{Gpc})^3$

$n_{\rm eff} = 1.6 \times 10^{-3}\,(h^{-1}\,\mathrm{Mpc})^{-3}$



Strategy

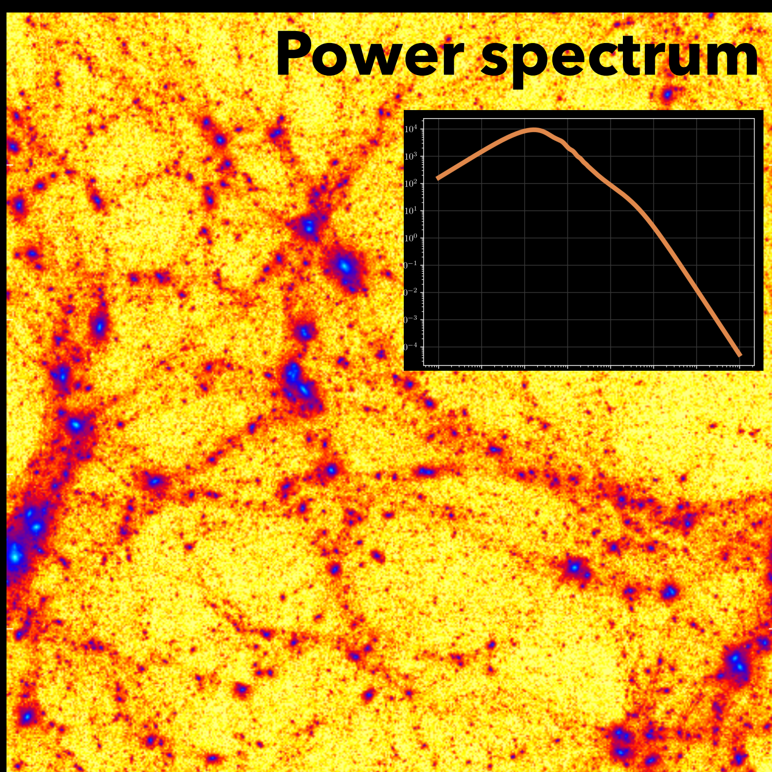
✦ ***End-to-end test in cosmological parameter inference***

1. Generate initial condition

Can PT reproduce?

h	Hubble parameter
Ω_m	Matter density
A_s	Amplitude of scalar perturbation

2. Run *N*-body sim. & measure power spectrum @ *z* = 1



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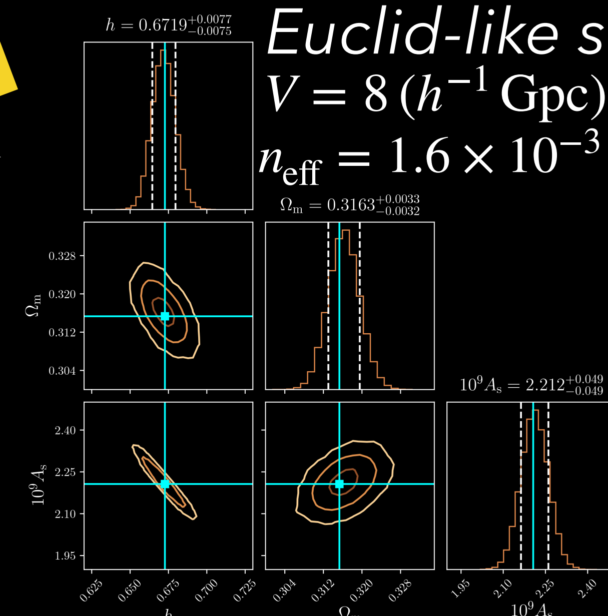
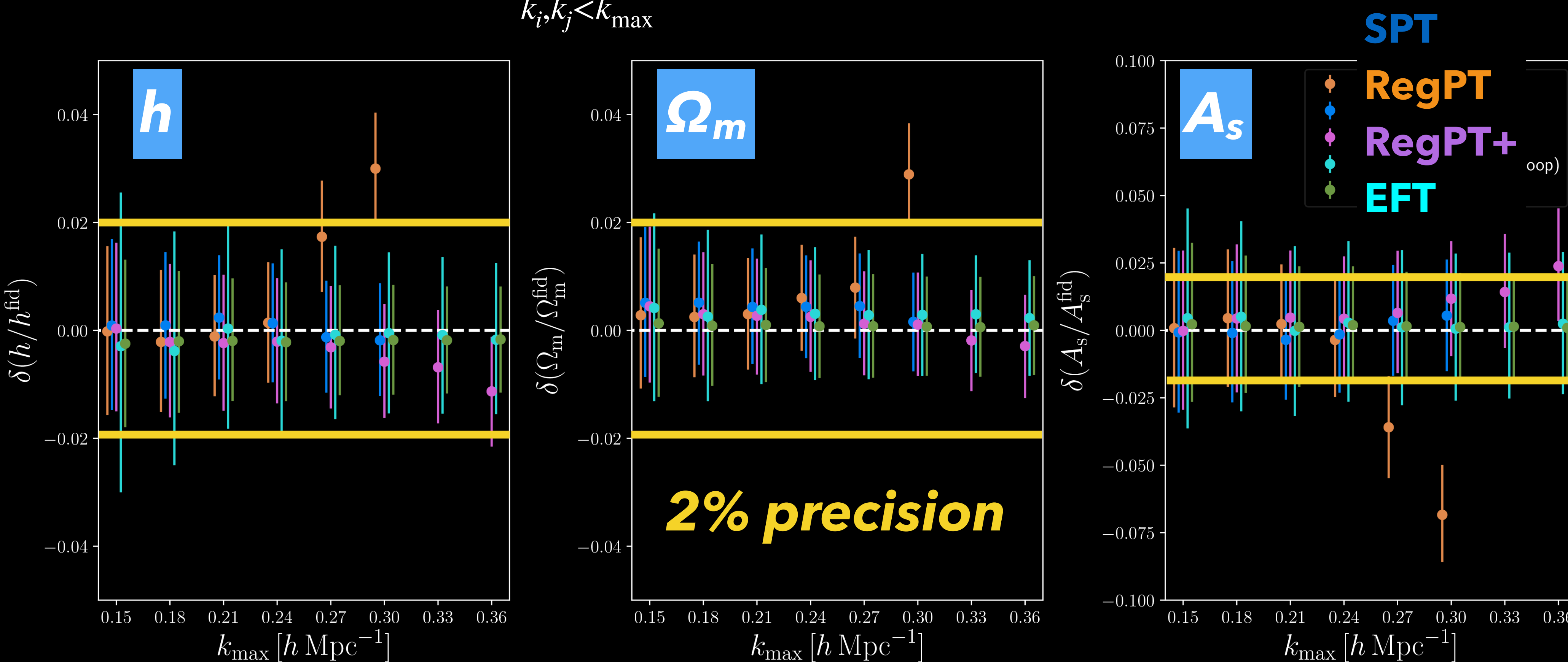


Results: Estimated Parameters

- ✦ ***Inferred parameters normalized by fiducial values for three cosmological parameters as a function of k_{max}.***

$$\log L(\theta\,|\,\hat{P}) = -\frac{1}{2} \sum_{k_i,k_j<k_{\rm max}} (\hat{P}(k_i) - P(k_i;\theta))(C^{-1})_{ij}(\hat{P}(k_j) - P(k_j;\theta))$$



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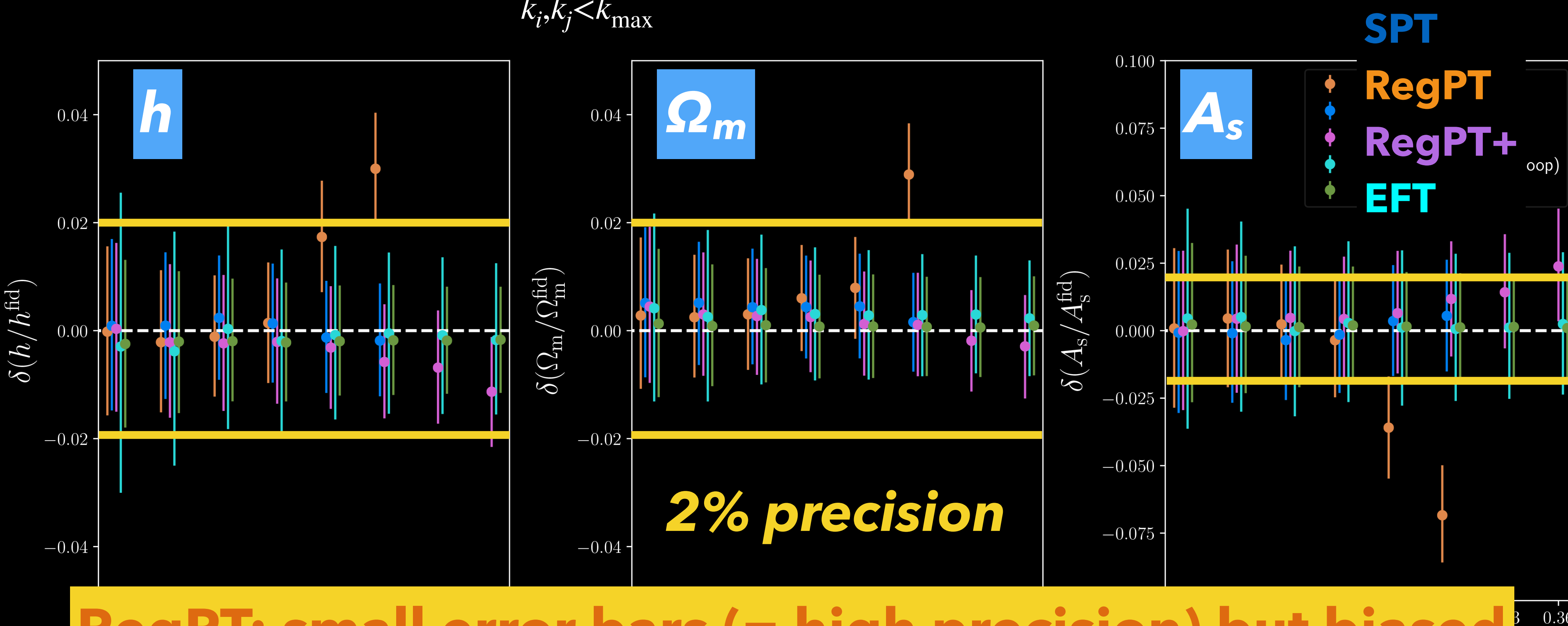


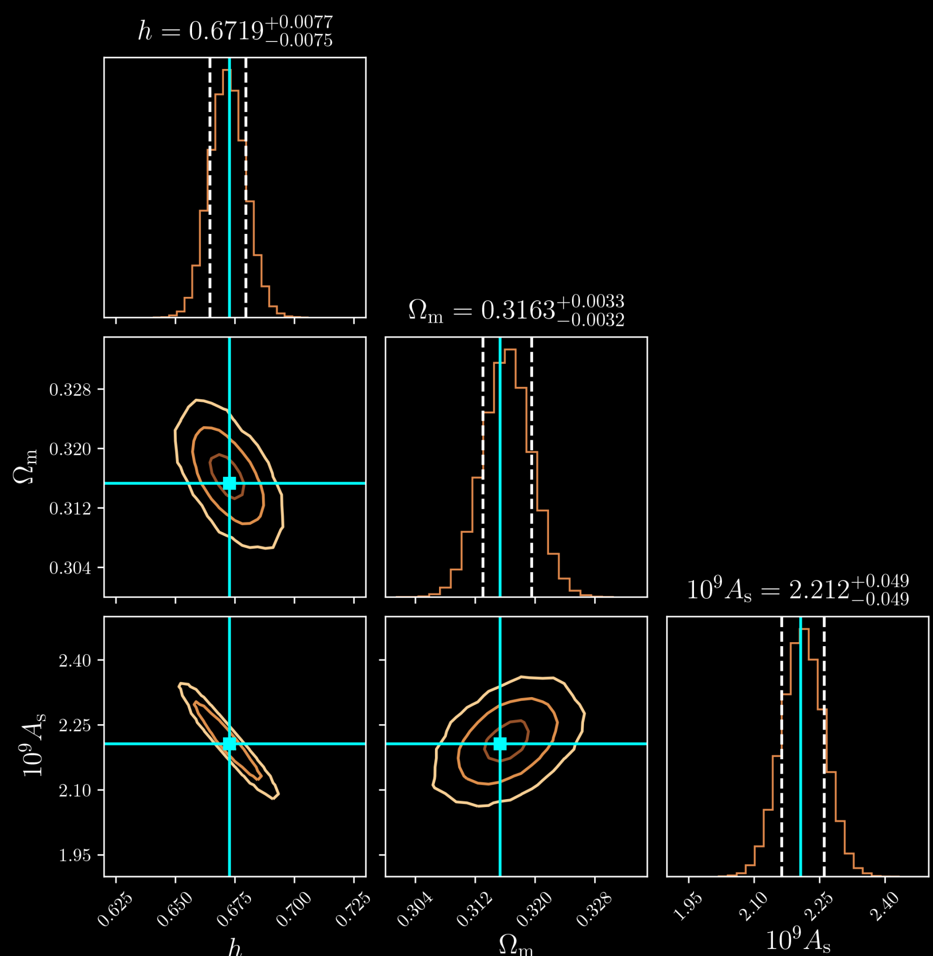
RegPT: small error bars (= high precision) but biased

EFT: unbiased even for high k_{max} but large error bars

Results: 2D Contours

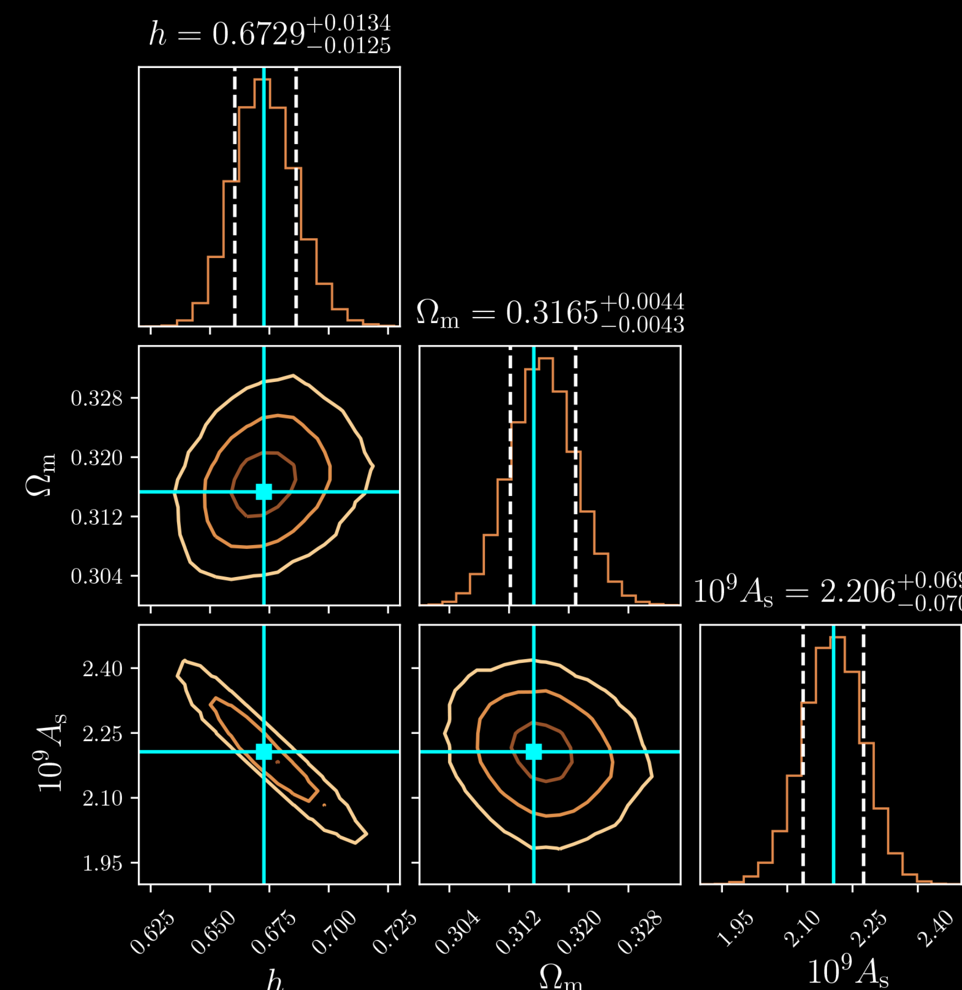
RegPT

(No nuisance param.)



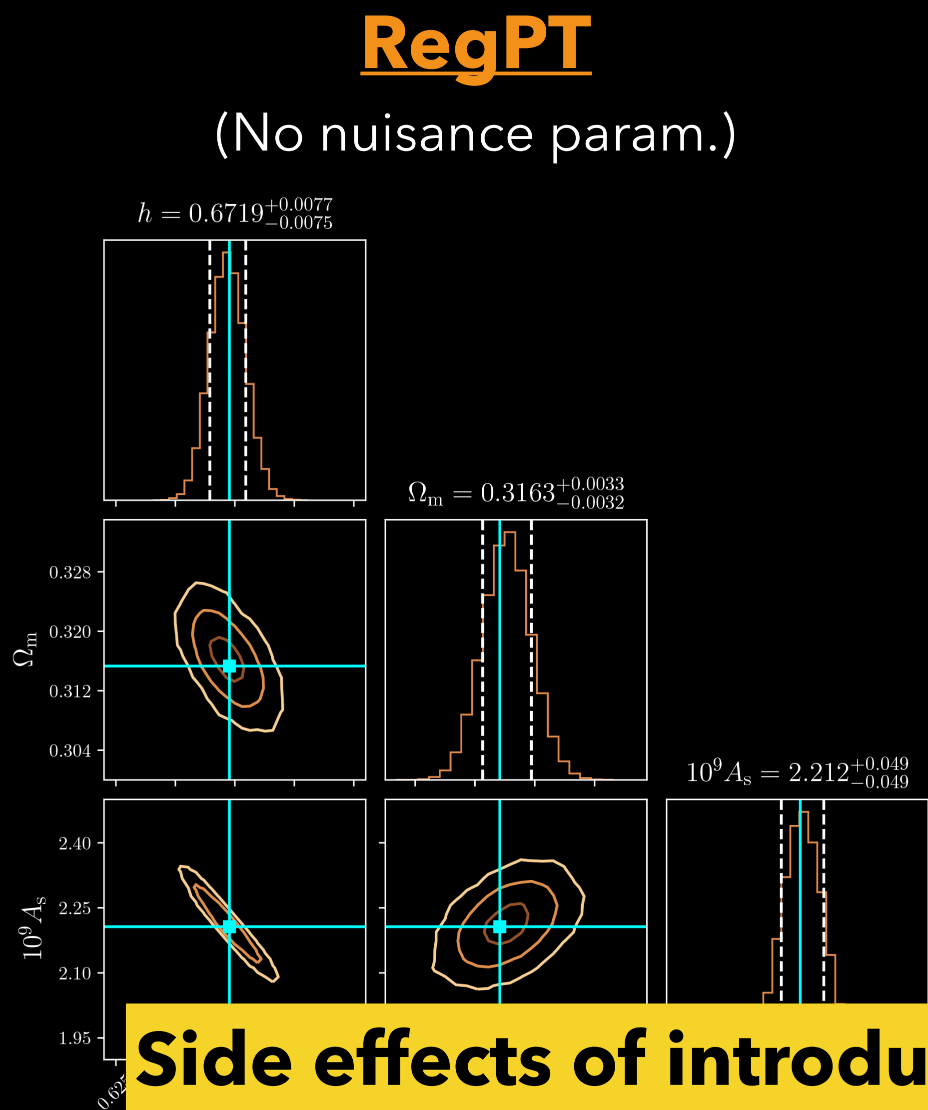
IR-resummed EFT

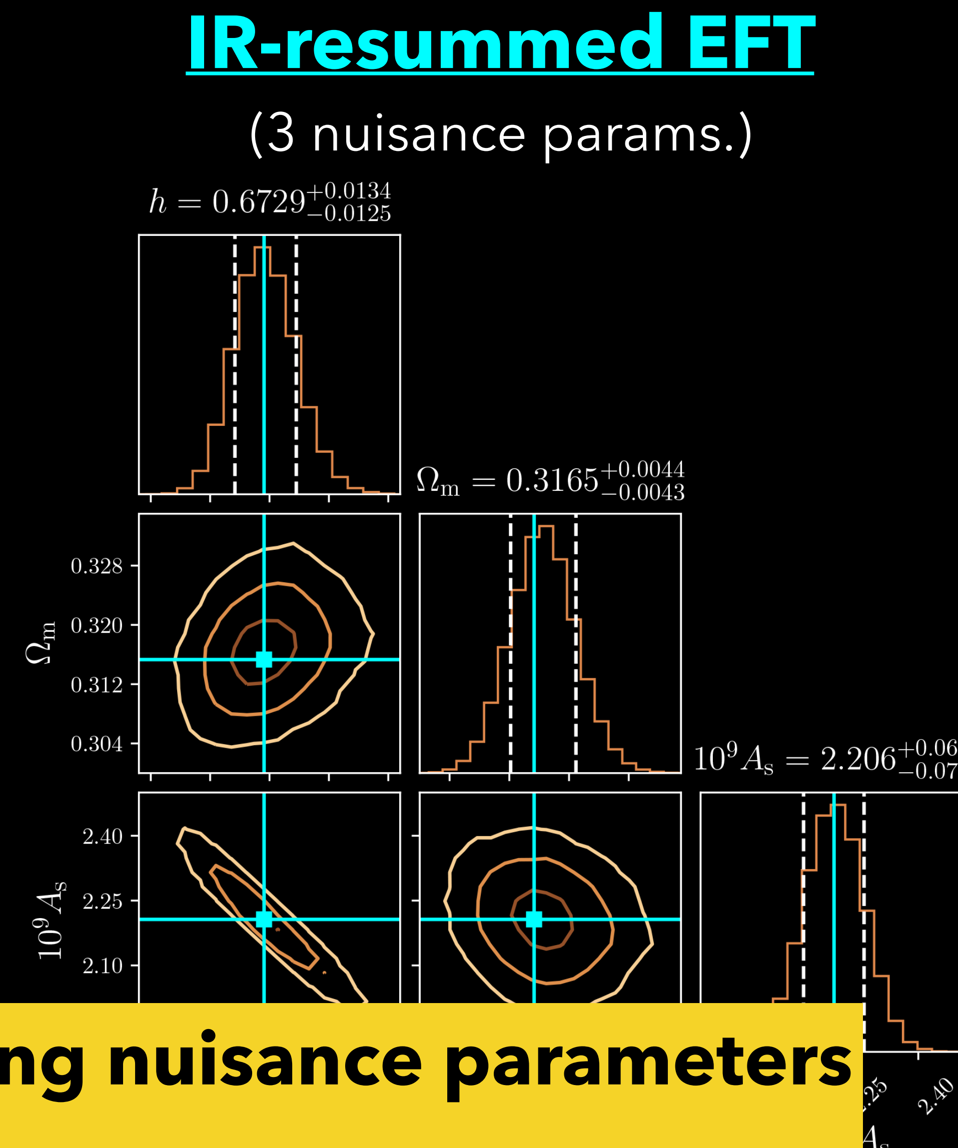
(3 nuisance params.)



$z = 1.0$, $k_{max} = 0.21\ h$ Mpc^{-1}

Results: 2D Contours





Side effects of introducing nuisance parameters

1. Degrades constraints

2. Distorts parameter degeneracy

FoM & FoB

Introduce goodness of fit and parameter bias as summary quantities

Correlation matrix: estimated from MCMC chains

$$S_{\alpha\beta} = \frac{1}{N-1}\sum_{k}^{N}(\theta_\alpha^k - \bar{\theta}_\alpha)(\theta_\beta^k - \bar{\theta}_\beta)$$

Parameter vector

$$\theta = (h, \Omega_\mathrm{m}, A_\mathrm{s}, \ldots)$$

Figure of Merit (FoM): goodness of fit

$$\mathrm{FoM} = \left[\det \tilde{S}\right]^{-\frac{1}{2}}$$ ~ (Volume of 1-σ C.L.)$^{-1}$

marginalized for nuisance params.

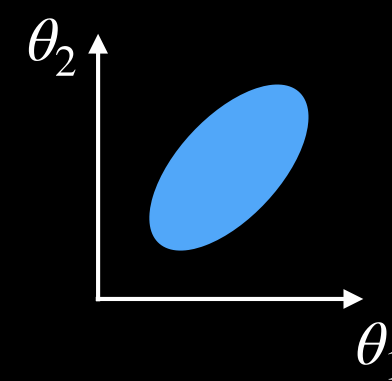
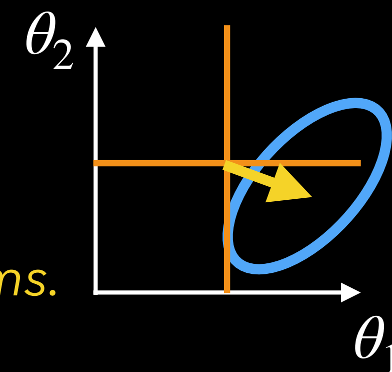


Figure of Bias (FoB): parameter bias

$$\mathrm{FoB} = \left[(\bar{\theta}_\alpha - \theta_\alpha^{\mathrm{fid.}})\tilde{S}_{\alpha\beta}(\bar{\theta}_\beta - \theta_\beta^{\mathrm{fid.}})\right]^{-\frac{1}{2}}$$

Distance between true and estimated params. normalized by variances.



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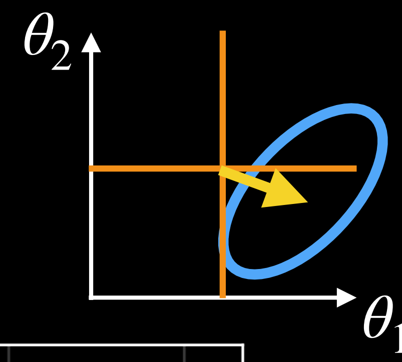


Applicable scale without biased

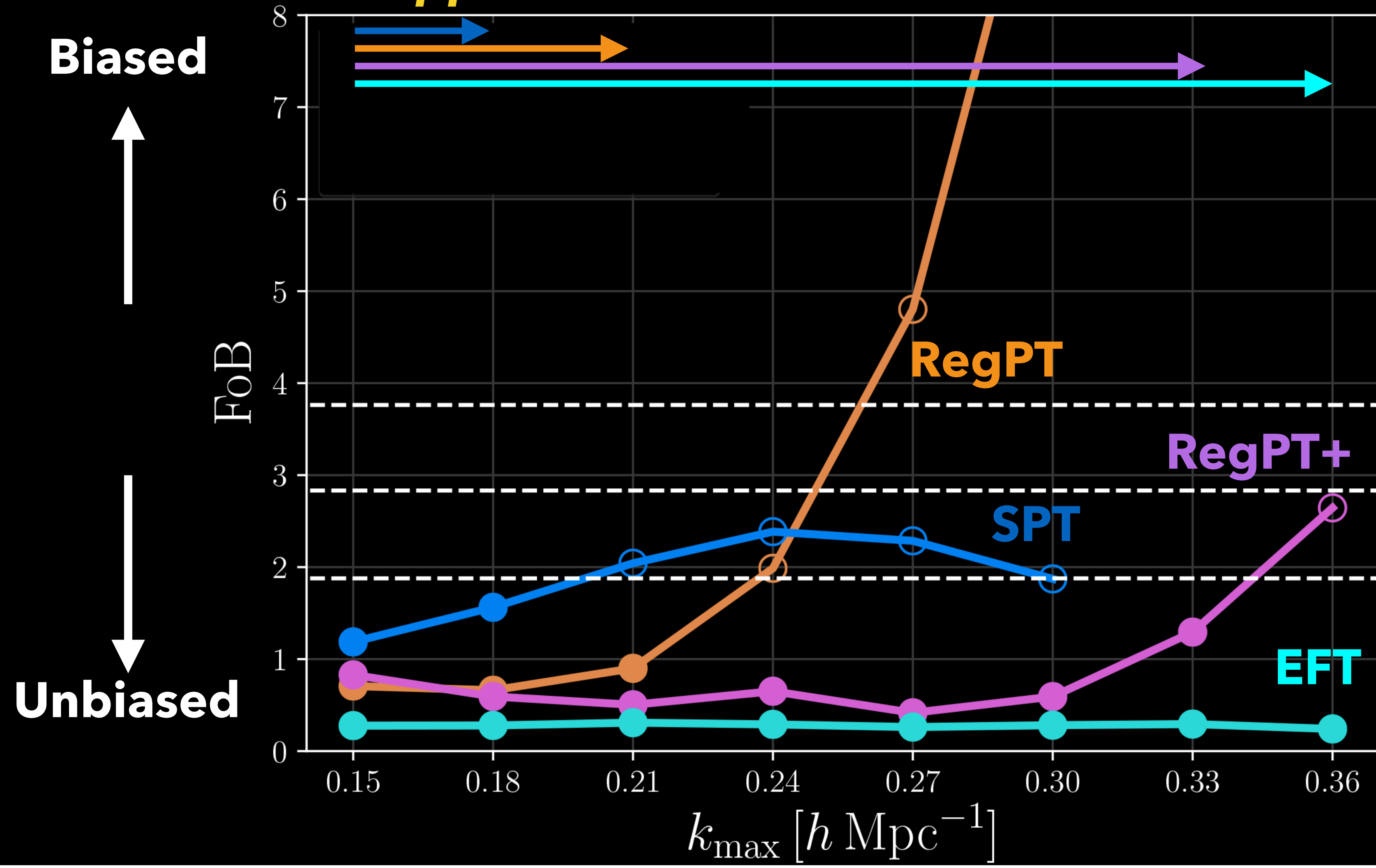
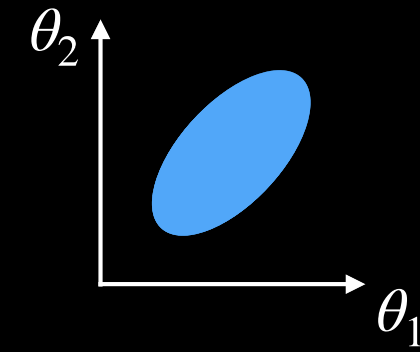
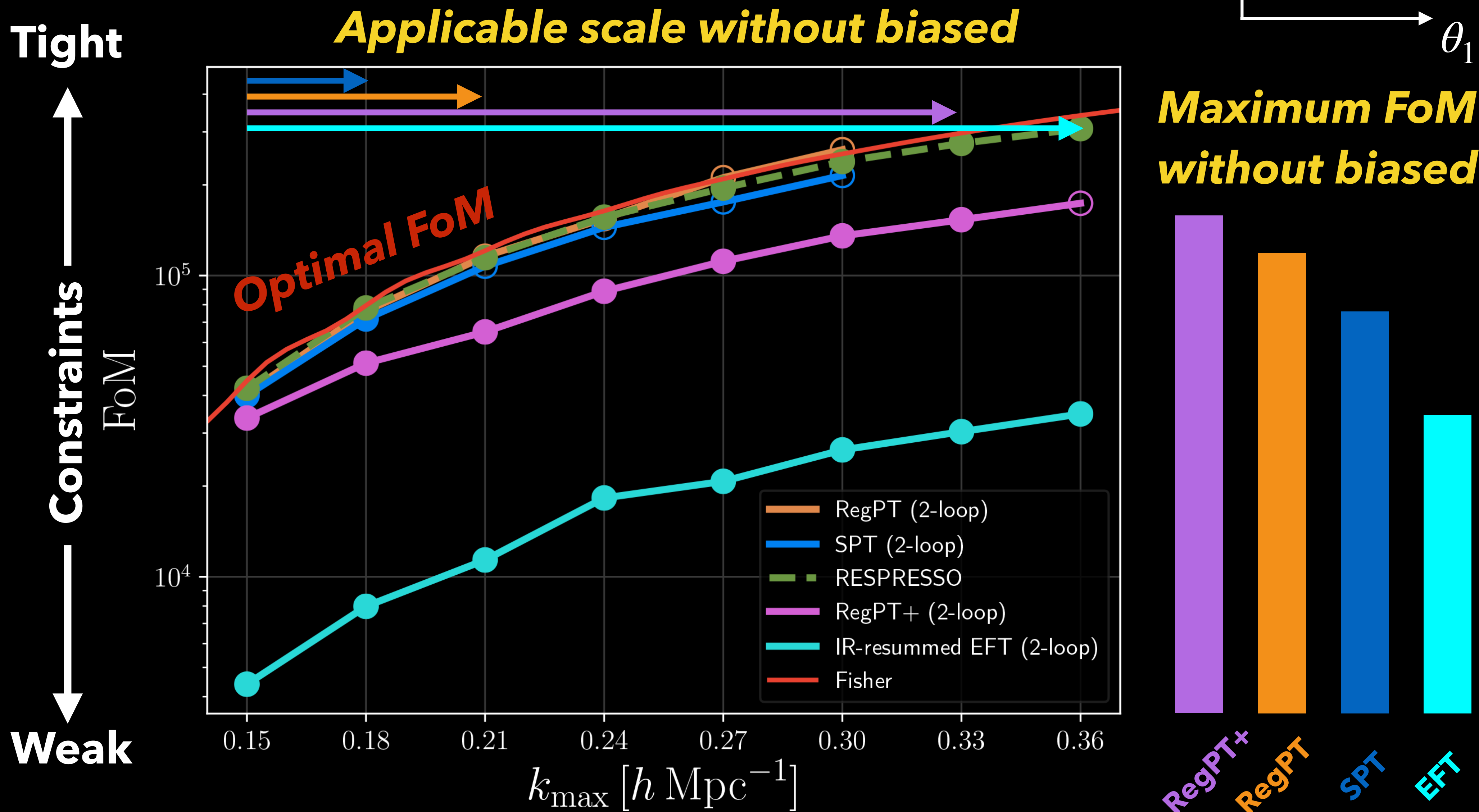


Figure of Merit

Figure of Merit (FoM): goodness of fit

$$\mathrm{FoM} = \left[\det \tilde{S}\right]^{-\frac{1}{2}}$$ ~ (Volume of 1-σ C.L.)⁻¹





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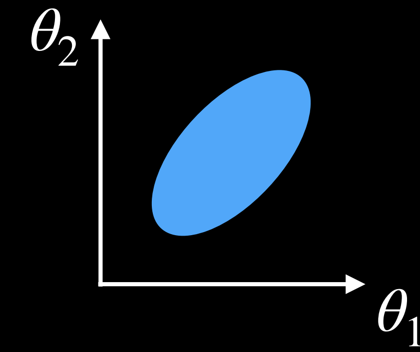
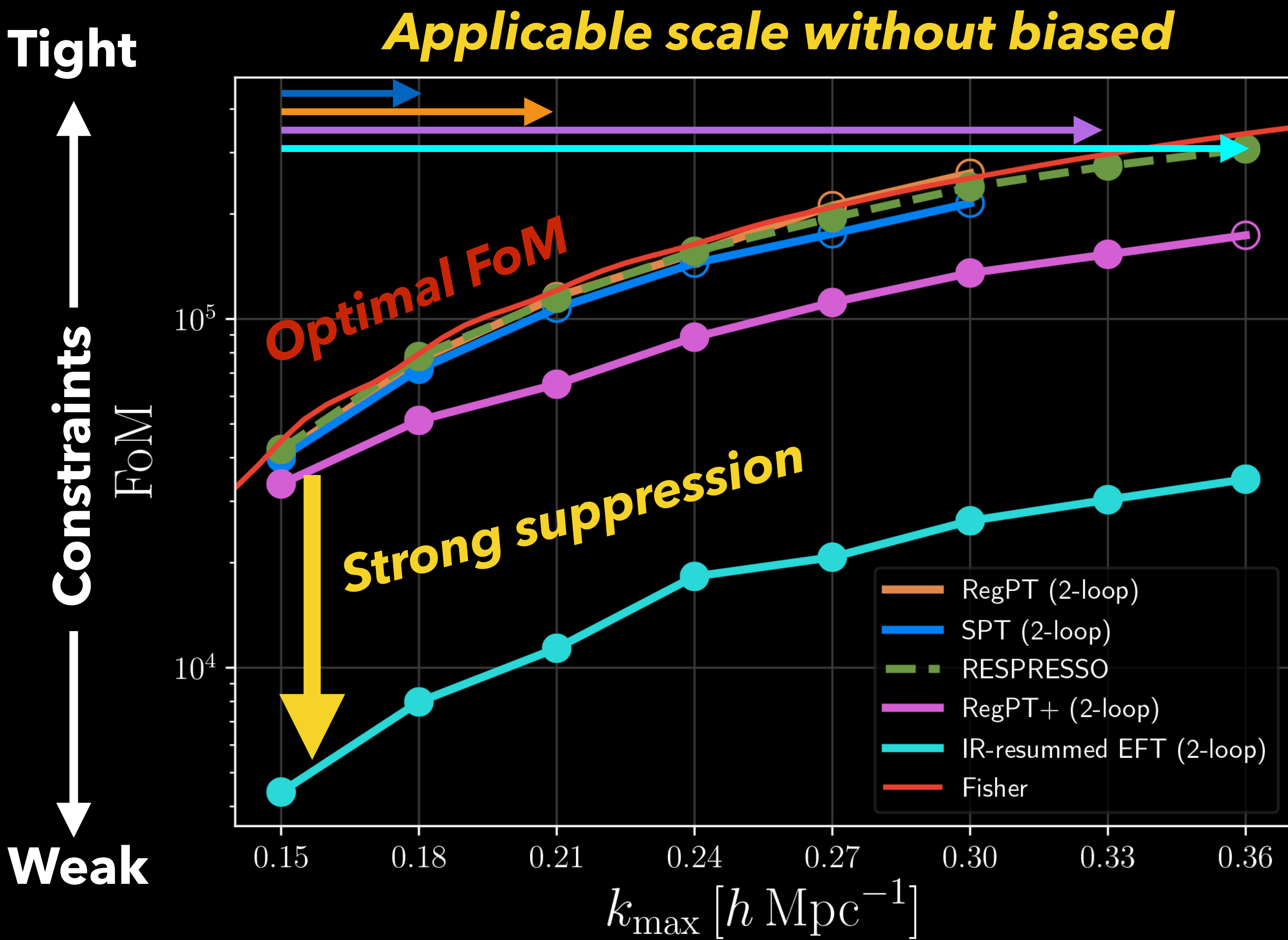
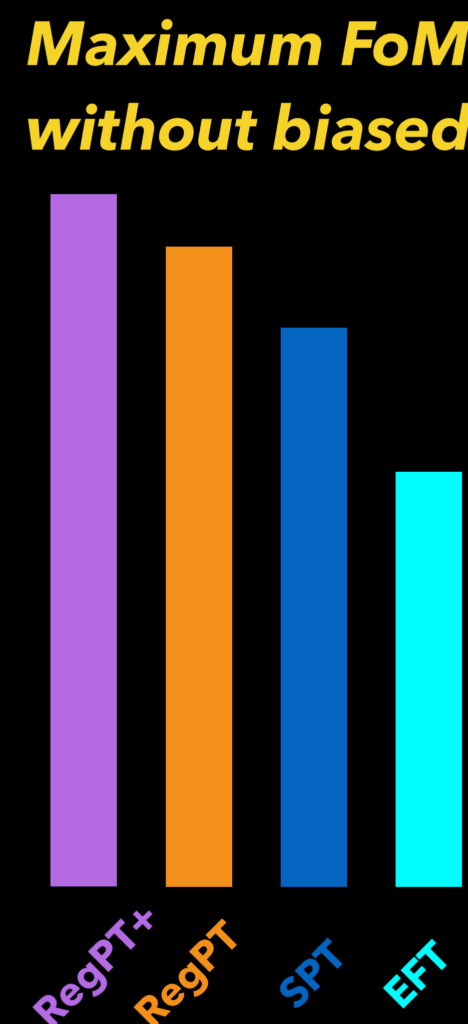
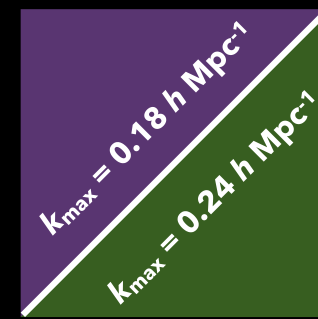






Correlation Coefficient

Correlation coefficient: $R_{\alpha\beta} = \dfrac{S_{\alpha\beta}}{\sqrt{S_{\alpha\alpha}S_{\beta\beta}}}$



$R_{\alpha\beta} = \pm 1$ **Positive/Negative correlation**

$R_{\alpha\beta} = 0$ **No correlation**

RegPT+

	h	Ω_m	A_s	σ_d
σ_d	0.31	0.14	−0.30	
A_s	−0.99	0.39		−0.27
Ω_m	−0.49		0.34	0.35
h		−0.48	−0.98	0.25

IR-resummed EFT

	h	Ω_m	A_s	α_1	α_2	Σ
Σ	−0.26	−0.09	0.21	0.36	−0.52	
α_2	0.55	0.46	−0.39	−0.91		−0.28
α_1	−0.71	−0.63	0.55		−0.94	0.19
A_s	−0.97	−0.15		0.60	−0.48	0.12
Ω_m	0.24		−0.12	−0.54	0.40	−0.04
h		0.12	−0.98	−0.71	0.59	−0.14

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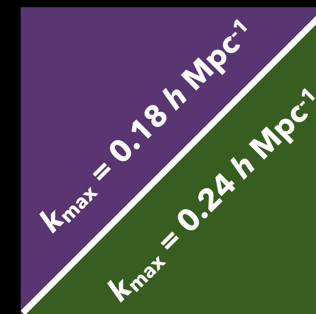

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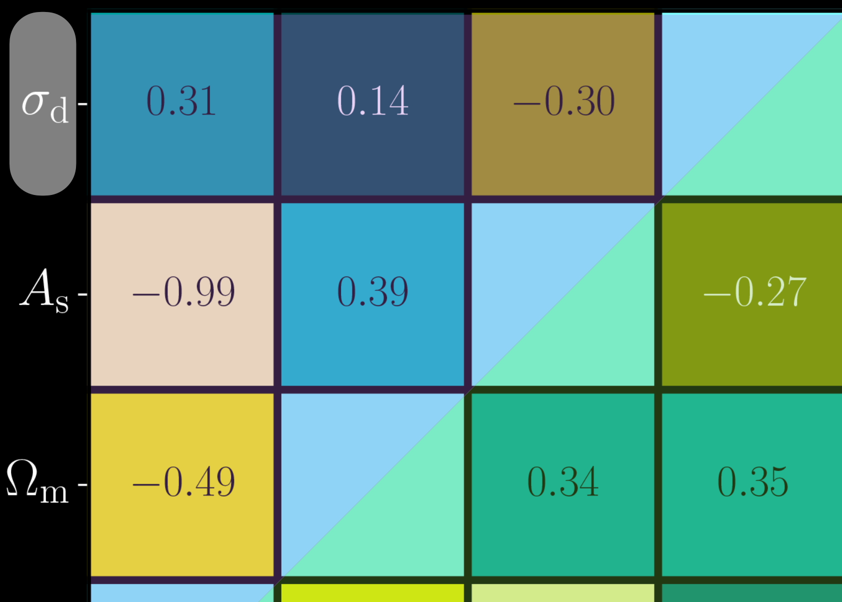

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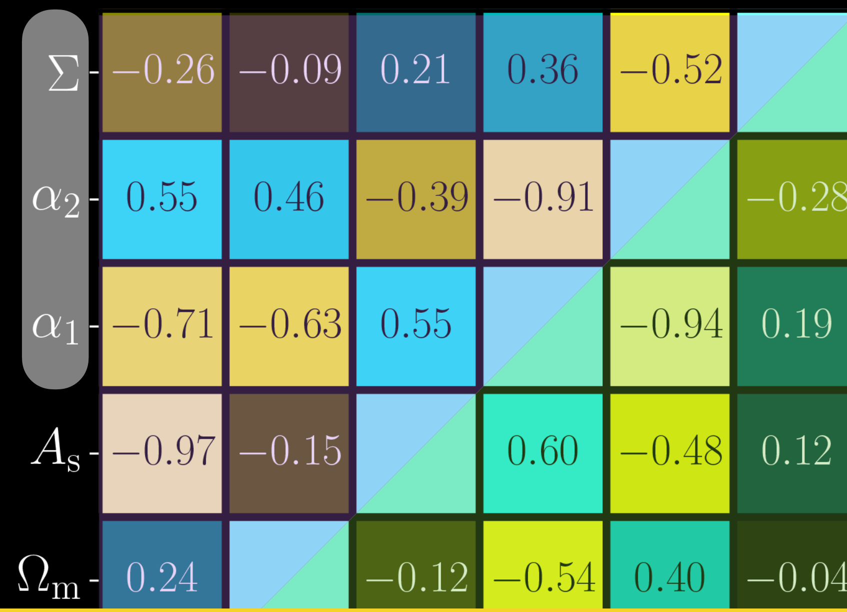

Strong correlation with cosmological parameters degrades constraints!

Summary

End-to-end test of PT schemes (SPT, RegPT, RegPT+, EFT) in the analysis of cosmological parameter inference with real-space matter power spectrum.

- **Best FoB model : IR-resummed EFT** (3 nuisance params.)
  but weak constraining power
- **Best FoM model : SPT / RegPT** (no nuisance param.)
  but biased parameter estimation
- **Reasonable choice : RegPT+** (1 nuisance param.)
  works well up to even $k_{max} \sim 0.33\ h\ \mathrm{Mpc}^{-1}$

Prospects: Incorporating RSD and galaxy bias for more realistic modeling of power spectrum and fast calculation suitable for MCMC analysis