Responses in and on Large Scale Structure

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with Elisabeth Krause & Fabian Schmidt

PTChat Kyoto – April 2019

<u>R(k)</u>

In this talk ...

1) Response Approach to Perturbation Theory

Barreira, Schmidt , 1703.09212 Barreira, Schmidt , 1705.01092

2) Covariance applications

Barreira, Krause, Schmidt, 1711.07467 Barreira, Krause, Schmidt, 1807.04266 Barreira, 1901.01243

3) Baryonic effects on higher-order N-point functions

Barreira et al 1904.02070

Response Approach to PT

Barreira, Schmidt , 1703.09212 Barreira, Schmidt , 1705.01092

What are responses?

Responses describe how the power spectrum responds to the presence of large-scale perturbations.

$$\mathcal{R}_{n} \equiv \frac{1}{n!P(k)} \frac{\mathrm{d}^{n} P(\boldsymbol{k}, \delta_{1} \cdots \delta_{n})}{\mathrm{d}\delta_{1} \cdots \mathrm{d}\delta_{n}} \bigg|_{\delta_{a}=0}$$



What are responses?

What are they good for? ctrum responds to the presence urbations.

To "resum" squeezed PT kernels



$$-\frac{k}{P_{\rm L}(k)} \stackrel{k}{=} = P_L(k,t)$$

• The linear matter power spectrum.

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• Interaction of P(k) with a mode **p**



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Response interaction

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The n-th order response vertex:

the interaction of the power spectrum with N soft modes.



Barreira, Schmidt , 1703.09212

The n-th order response vertex:

the interaction of the power spectrum with N soft modes.



Predictive for:

- Quasi-linear values of the soft modes, p_n
- Nonlinear values of the hard modes, k,k'

Barreira, Schmidt , 1703.09212

- 1. Take any N-point function and write all SPT terms.
- 2. Identify hard-to-soft coupling terms.
- 3. Replace these SPT kernels by the responses.

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= Hard-to-soft terms + Remainder
Evaluate with responses. Evaluate with SPT kernels













$$\lim_{p \to 0} \left\langle \delta(\boldsymbol{k}) \delta(\boldsymbol{k}') \delta(\boldsymbol{p}) \right\rangle_c$$



 $\lim_{p \to 0} \left< \frac{\delta(\boldsymbol{k})\delta(\boldsymbol{k}')\delta(\boldsymbol{p})}{\delta(\boldsymbol{p})} \right>_c$



Example 2: SSC

$$\operatorname{Cov}\left[\begin{array}{c} P_m(\boldsymbol{k}_1), P_m(\boldsymbol{k}_2) \\ hard hard \end{array} \right]$$

"Internal" response

interactions

Approach not limited to

squeezed N-pt functions



 $= \mathcal{R}_1(k_1, -\mu_{\boldsymbol{p}, \boldsymbol{k}_1}) \mathcal{R}_1(k_2, \mu_{\boldsymbol{p}, \boldsymbol{k}_2}) P_m(k_1) P_m(k_2) P_L(p)$

Two 1st order responses



The response approach requires simulation measurements of



Power spectrum "Normal" simulations.



Responses Separate Universe Simulations

Write the response in terms of all possible local gravitational observables

$$\mathcal{R}_n = \sum_{\mathcal{O}} R_{\mathcal{O}}(k) \ \mathcal{K}_{\mathcal{O},\text{large scale}}^{(n)}$$

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All possible configurations of large-scale density/tidal fields;

Given by perturbation theory.

Write the response in terms of all possible local gravitational observables

$$\mathcal{R}_n = \sum_{\mathcal{O}} \frac{R_{\mathcal{O}}(k)}{\mathcal{O}(k)} \begin{array}{c} \mathcal{K}_{\mathcal{O},\text{large scale}}^{(n)} \\ \downarrow \end{array}$$

Measure the response to each specific large-scale configuration;

What we will get from simulations.

All possible configurations of large-scale density/tidal fields;

Given by perturbation theory.



Nitty-gritty: Li et al (1401.0385) ; Wagner et al (1409.6294); Baldauf et al (1511.01465) Schmidt et al (1803.03274);

$$\mathcal{R}_n = \sum_{k} R_{\mathcal{O}}(k) \mathcal{K}_{\mathcal{O},\text{large scale}}^{(n)}$$

⑦ Response to specific perturbations All possible configurations of large-scale density/tidal fields;

Nitty-gritty: Li et al (1401.0385) ; Wagner et al (1409.6294); Baldauf et al (1511.01465) Schmidt et al (1803.03274);



$$P_m(\boldsymbol{k}, \boldsymbol{x}) = P_m(k) \left[1 + \frac{R_1(k)}{\delta(\boldsymbol{x})} + \frac{R_K(k)}{k^i \hat{k}^j K_{ij}(\boldsymbol{x})} \right]$$







Lensing covariances

Barreira, Krause, Schmidt, 1711.07467 Barreira, Krause, Schmidt, 1807.04266 Barreira, 1901.01243

The covariance decomposition

- <u>Windowed lensing convergence</u> $\kappa_{\mathcal{W}}(\boldsymbol{\theta}) = \mathcal{W}(\boldsymbol{\theta})\kappa(\boldsymbol{\theta})$
- <u>Estimator of its power spectrum</u>

$$\hat{C}_{\kappa}(\boldsymbol{\ell}) = \frac{\tilde{\kappa}_{\mathcal{W}}(\boldsymbol{\ell})\tilde{\kappa}_{\mathcal{W}}(-\boldsymbol{\ell})}{\Omega_{\mathcal{W}}}$$



e.g. Takada&Hu (1302.6994)

<u>Covariance matrix of the estimator</u>

$$\operatorname{Cov}_{\kappa}\left(\hat{C}_{\kappa}(\ell_{1}),\hat{C}_{\kappa}(\ell_{2})\right) = \left\langle \hat{C}_{\kappa}(\ell_{1})\hat{C}_{\kappa}(\ell_{2})\right\rangle - \left\langle \hat{C}_{\kappa}(\ell_{1})\right\rangle \left\langle \hat{C}_{\kappa}(\ell_{2})\right\rangle$$
$$= \underbrace{\operatorname{Cov}_{\kappa}^{G}(\ell_{1},\ell_{2})}_{\operatorname{Gaussian}} + \underbrace{\operatorname{Cov}_{\kappa}^{cNG}(\ell_{1},\ell_{2})}_{\operatorname{Connected}} + \underbrace{\operatorname{Cov}_{\kappa}^{SSC}(\ell_{1},\ell_{2})}_{\operatorname{Super-sample}}$$

Connected non-Gaussian term : cNG

 Describes the <u>coupling of different Fourier modes due to nonlinear</u> <u>structure formation</u> (given by the parallelogram matter trispectrum).

$$\operatorname{Cov}_{\kappa}^{cNG}(\boldsymbol{\ell}_{1},\boldsymbol{\ell}_{2}) = \frac{1}{\Omega_{\mathcal{W}}} \int \mathrm{d}\chi \frac{\left[g(\chi)\right]^{4}}{\chi^{6}} T_{m}(\boldsymbol{k}_{\ell_{1}},-\boldsymbol{k}_{\ell_{1}},\boldsymbol{k}_{\ell_{2}},-\boldsymbol{k}_{\ell_{2}})$$
$$\boldsymbol{k}_{\ell} = \frac{\boldsymbol{\ell}+1/2}{k_{\ell_{1}}}$$

 χ

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$$k_{\ell} = \frac{\ell + 1/2}{\chi}$$

$$LOS$$

$$projection$$

$$Trispectrum$$

Evaluate it with the response approach.

cNG : response vs simulations



cNG : response vs simulations



The super-sample term : SSC

 Describes the coupling of modes inside the survey with <u>unobserved modes</u> outside the survey.
 Hamilton et al (0511416)

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$$\operatorname{Cov}_{\kappa}^{SSC}(\boldsymbol{\ell}_{1},\boldsymbol{\ell}_{2}) = \frac{1}{\Omega_{\mathcal{W}}^{2}} \int \mathrm{d}\chi \frac{[g(\chi)]^{4}}{\chi^{6}} \int \frac{\mathrm{d}^{2}\boldsymbol{\ell}}{(2\pi)^{2}} |\tilde{\mathcal{W}}(\boldsymbol{\ell})|^{2} \mathcal{R}_{1}(\boldsymbol{k}_{\boldsymbol{\ell}_{1}},\mu_{\boldsymbol{\ell}_{1},\boldsymbol{\ell}}) \mathcal{R}_{1}(\boldsymbol{k}_{\boldsymbol{\ell}_{2}},\mu_{\boldsymbol{\ell}_{2},\boldsymbol{\ell}}) \times P_{m}(\boldsymbol{k}_{\ell_{1}}) P_{m}(\boldsymbol{k}_{\ell_{2}}) P_{L}(\boldsymbol{k}_{\ell})$$

<u>Note</u>: This expression assumes Limber for the super-survey modes, which is only accurate up to 10% for Euclid/LSST. Beyond Limber SSC expressions exist however (Barreira, Krause, Schmidt, arXiv:1711.07467).

The super-sample term : SSC

• Describes the coupling of modes inside the survey with <u>unobserved modes</u> <u>outside the survey</u>. Hamilton et al (0511416)

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$$\frac{LOS}{projection} \frac{Window}{function} \frac{1st order responses}{||\mathbf{x}_{1}||^{2}}$$
$$Cov_{\kappa}^{SSC}(\boldsymbol{\ell}_{1},\boldsymbol{\ell}_{2}) = \frac{1}{\Omega_{W}^{2}} \int d\chi \frac{[g(\chi)]^{4}}{\chi^{6}} \int \frac{d^{2}\boldsymbol{\ell}}{(2\pi)^{2}} ||\tilde{\mathcal{W}}(\boldsymbol{\ell})|^{2} \mathcal{R}_{1}(\boldsymbol{k}_{\boldsymbol{\ell}_{1}},\mu_{\boldsymbol{\ell}_{1},\boldsymbol{\ell}})\mathcal{R}_{1}(\boldsymbol{k}_{\boldsymbol{\ell}_{2}},\mu_{\boldsymbol{\ell}_{2},\boldsymbol{\ell}})$$
$$\times P_{m}(\boldsymbol{k}_{\ell_{1}})P_{m}(\boldsymbol{k}_{\ell_{2}})P_{L}(\boldsymbol{k}_{\ell})$$

Responses fully specify the SSC term

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Lensing covariance summary

$\operatorname{Cov}_{\kappa}(\boldsymbol{\ell}_{1},\boldsymbol{\ell}_{2}) \quad = \quad \operatorname{Cov}_{\kappa}^{G} \quad + \quad \operatorname{Cov}_{\kappa}^{cNG} \quad + \quad \operatorname{Cov}_{\kappa}^{SSC}$

Lensing covariance summary



Lensing covariance summary



The unimportance of the cNG term for future surveys

Euclid-like lensing setup

- Tomographic convergence power spectrum 10 tomographic bins
- 20 ell bins in [20, 5000]
- Mask: spherical cap 15000 deg²
- Source density: 30 / arcmin^2

Barreira, Krause, Schmidt 1807.04266

$$\mathcal{L}(w_0) \propto \exp\left[-\frac{1}{2}\left(\overrightarrow{Theory} - \overrightarrow{Data}\right)\mathbf{Cov}^{-1}\left(\overrightarrow{Theory} - \overrightarrow{Data}\right)\right]$$

What is the impact of different matrices on parameter constraints ?

The unimportance of the cNG term for future surveys

Euclid-like lensing setup



The unimportance of the cNG term for future surveys

Euclid-like lensing setup

 Tomographic convergence power spectrum 10 tomographic bins

Barreira, Krause, Schmidt 1807.04266

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Assumes no systematics,

The cNG is the hardest-to-evaluate covariance term, but it is also the smallest ...

... accuracy of analytical methods is likely sufficient for lensing.

increases error by only 6%.

cNG has also a small impact on parameter biases.



Squeezed bispectrum covariance

<u>Squeezed bispectrum</u>

Barreira (arXiv:1901.01243)

 $B_i = B_m(k_i, k'_i, s_i), \quad s_i \ll k_i, k'_i$

<u>Covariance decomposition</u>

Negligible

$$\operatorname{Cov}(B_1, B_2) = \operatorname{Cov}^{PPP} + \operatorname{Cov}^{BB} + \operatorname{Cov}^{TP} + \operatorname{Cov}^{SSC} + \frac{\operatorname{Cov}^{cNG}}{\operatorname{Fully given by P(k) and its responses}}$$

Squeezed bispectrum covariance

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 $B_i = B_m(k_i, k'_i, s_i), \quad s_i \ll k_i, k'_i$

<u>Covariance decomposition</u>

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Analytical calculation of the sq. bispectrum covariance.

The SSC term is negligible ! (Okay to skip from f_nl forecasts)

Baryons and higher-order statistics

Barreira et al 1904.02070

Baryonic effects on N-point functions

Separate Universe Simulations with IllustrisTNG

with the TNG collaboration



Baryonic effects on N-point functions

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Baryonic effects on N-point functions

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Baryons actually lower the covariance matrix !

Ignoring baryons is conservative on errors bars!



Summary

