Advances in few selected topics of perturbative LSS

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CERN

done in collaboration with:

- Martin White (Berkeley),
- Elisa Chisari (Oxford), Fabian Schmidt (MPA),
- Tomohiro Fujita (Kyoto, Geneva),
- Patrick McDonald (LBNL)

Agenda

What I would like to talk about:

- Redshift space distortions - categorization of few different approaches
- New look at the statistics of biased tracers - “Monkey bias“
- Clustering of shapes - applications to intrinsic alignments
- Lagrangian dynamics & stream crossing - statistical field theory attire

Early conclusions

Key messages:

- We have developed a new RSD framework based on the Fourier space cumulant expansion - highlight of differences and advantages.
- FoG are incorporated naturally - expressions are algebraic

- New and simplified bias framework - we rely only on symmetries & equiv. principle.

- We use decomposition into spherical tensors to construct the bias/EFT of shapes - applications to intrinsic alignments.
- General framework - worked out full sky correlators.

- Consistently included shell crossing into the perturbative Lagrangian scheme.
- Connection to EFT-PT via low-k expansion - EFT parameter matching.

Redshift space distortions - Fourier space cluster expansion

with: Martin White (Berkeley)

Real space:

Redshift space:

Object position in redshift-space:

$$\boldsymbol{s} = \boldsymbol{x} - f u_z(\boldsymbol{x})\hat{z}, \quad u_z \equiv -v_z/(f\mathcal{H})$$

Density in redshift-space:

$$\delta_s(\boldsymbol{k}) = \int_{\boldsymbol{x}} (1 + \delta(\boldsymbol{x}))e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \exp\left(-ifk_z u_z(\boldsymbol{x})\right)$$

I. Subjective overview of the RSD approaches

RSD power spectrum:

$$P_s(\boldsymbol{k}) = \int d^3r\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \mathcal{M}\big(k_{\|}\hat{z}, \boldsymbol{r}\big) = \int d^3r\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \mathcal{M}\big(\boldsymbol{J} = \boldsymbol{k}.\boldsymbol{R}, \boldsymbol{r}\big),$$

where $\boldsymbol{r} = \boldsymbol{x}_2 - \boldsymbol{x}_1$ and

$$1 + \mathcal{M}\big(\boldsymbol{J}, \boldsymbol{r}\big) = \Big\langle \big(1 + \delta(\boldsymbol{x})\big)\big(1 + \delta(\boldsymbol{x}')\big) e^{i\boldsymbol{J}\cdot\Delta\boldsymbol{u}} \Big\rangle\,,$$

is the pairwise velocity generating function, with $\Delta\boldsymbol{u} = \boldsymbol{u}_2 - \boldsymbol{u}_1$.
Approaches:

- Streaming model: configuration space cumulant expansion, $\mathcal{M} \to \ln \mathcal{M}$.
  [Peebles, White, ...]
- Velocity moment expansion: $\mathcal{M}$ expanded in moment correlators.
  [SPT, Seljak&McDonald, ...]
- Smoothing kernel: cumulant expansion or individual contributions in $\mathcal{M}$.
  [Scoccimarro, TNS ...]
- Direct Lagrangian approach: $\mathcal{M}$ is transformed into Lagrangian coordinates.
  [Matsubara, White ...]

I. Fifth strategy - Fourier space cluster expansion

- a note on cumulant expansion / cluster expansion / cluster decomposition in general

Config. space cumulant expansion: $\mathcal{Z}(\boldsymbol{J},\boldsymbol{r}) = \ln\left[1+\mathcal{M}\big(\boldsymbol{J},\boldsymbol{r}\big)\right]$ where

$$\mathcal{C}^{(n)}_{i_1\ldots i_n}(\boldsymbol{r}) = (-i)^n\, \partial\mathcal{Z}(\boldsymbol{J},\boldsymbol{r})/\partial J_{i_1}\ldots\partial J_{i_n}|_{\boldsymbol{J}=0}$$

are the cumulants of the (density weighted) velocities, Δu.

But we can work also in Fourier space $\widetilde{\mathcal{Z}}(\boldsymbol{J},\boldsymbol{k}) = \ln\left[1+\widetilde{\mathcal{M}}(\boldsymbol{J},\boldsymbol{k})\right]$, which gives the Fourier space cumulants $\widetilde{\mathcal{C}}^{(n)}_{i_1\ldots i_n}(\boldsymbol{k})$.

The nontrivial difference arises since $\ln x$ and FT do not commute!

Redshift space power spectrum:

$$\Delta^2_s(\boldsymbol{k}) = \left[1+\Delta^2(\boldsymbol{k})\right]\exp\left[\sum_{n=1}^{\infty}\frac{i^n}{n!}k_{i_1}\ldots k_{i_n}\widetilde{\mathcal{C}}^{(n)}_{i_1\ldots i_n}(\boldsymbol{k})\right]-1,$$

where $\Delta^2 = \frac{k^3}{2\pi^2}P$.

Ok... but how well does all this work?

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The nontrivial difference arises since $\ln x$ and FT do not commute!

Redshift space power spectrum, log for:

$$\ln\frac{1+\Delta^2_s(k,\nu)}{1+\Delta^2(k)} = i(\nu k)\widetilde{\mathcal{C}}^{(1)}_{\hat{n}}(k,\nu) - \frac{(\nu k)^2}{2}\widetilde{\mathcal{C}}^{(2)}_{\hat{n}}(k,\nu) + \cdots\,,$$

where $\Delta^2 = \frac{k^3}{2\pi^2}P$.

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I. Zeldovich approx. - as an controlled environment

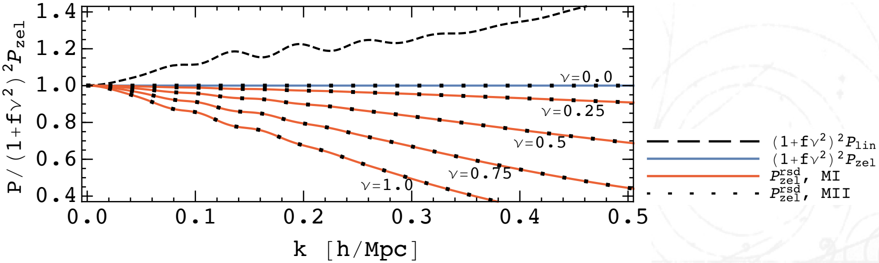
RSD in Zeldovich approx. : $\psi \rightarrow \psi_s = \psi + (\hat{n} \cdot \dot{\psi})\hat{n}/\mathcal{H}$

- a robust environment to test these different approaches
- more robust for range of scales than sims, we can go to arbitrary moments

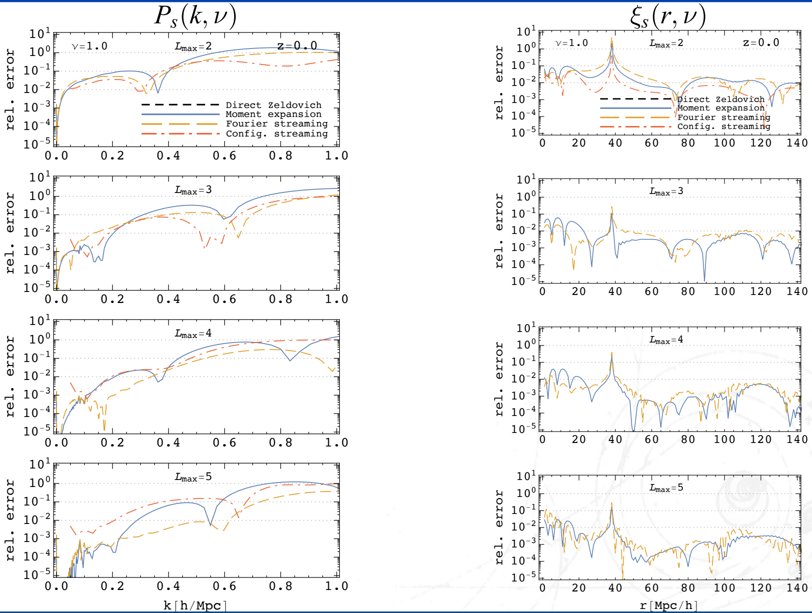
Zeldovich power spectrum (using $R_{ij} = \delta^K_{ij} + f\hat{n}_i\hat{n}_j$):

$$P_s(\boldsymbol{k}) = \int_{\boldsymbol{q}} e^{i\boldsymbol{k}\cdot\boldsymbol{q}} \exp\left(-\frac{1}{2}k_i k_j R_{in} R_{jm} \left\langle \Delta^L_n \Delta^L_m \right\rangle\right) .$$

[z.v.&White, '18]



I. Zeldovich approx. - convergence results



I. Bispectrum - Fourier cluster expansion framework

Velocity generating fnc. can be generalized to get RSD $n-$point functions.
For bispectrum:

$$\widetilde{\mathcal{M}}^{abc}(\boldsymbol{J}_1, \boldsymbol{J}_2; \boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{k_1^3 k_2^3}{4\pi^4} \int_{\boldsymbol{r}_1, \boldsymbol{r}_2} e^{i\boldsymbol{k}_1 \cdot \boldsymbol{r}_1 + i\boldsymbol{k}_2 \cdot \boldsymbol{r}_2} \left\langle (1+\delta_a)(1+\delta_b')(1+\delta_c'') e^{i\boldsymbol{J}_1 \cdot \Delta u_{ac} + i\boldsymbol{J}_2 \cdot \Delta u_{bc}} \right\rangle .$$

- RSD for higher $n-$pt. function is a quite difficult in the three canonical approaches
- In velocity moment expansion approach:

$$B_s^{abc}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \sum_{n,m=0}^{\infty} \frac{i^{n+m}}{n!m!} k_{1,i_1} \ldots k_{1,i_n} k_{2,j_1} \ldots k_{2,j_m} \widetilde{\Xi}_{i_1 \ldots i_n, j_1 \ldots j_m}(\boldsymbol{k}_1, \boldsymbol{k}_2)$$

In Fourier cluster expansion framework the algebraic structure of the velocity cumulants is preserved

$$1+\Delta^2_{s,abc} = \left[1+\Delta^2_{abc}\right] \exp\left[\sum_{n+m=1}^{\infty} \frac{i^{n+m}}{n!m!} k_{1,i_1} \ldots k_{1,i_n} k_{2,j_1} \ldots k_{2,j_m} \widetilde{\mathcal{C}}^{(n+m)}_{i_1 \ldots i_n, j_1 \ldots j_n}(\boldsymbol{k}_1, \boldsymbol{k}_2)\right].$$

Analogous structure valid for $n-$pt functions

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In Fourier cluster expansion framework the algebraic structure of the velocity cumulants is preserved

$$\ln \frac{1+\Delta^2_{s,abc}}{1+\Delta^2_{abc}} = \sum_{n+m=1}^{\infty} \frac{i^{n+m}}{n!m!} k_{1,i_1} \dots k_{1,i_n} k_{2,j_1} \dots k_{2,j_m} \widetilde{\mathcal{C}}^{(n+m)}_{i_1 \dots i_n, j_1 \dots j_n}(\boldsymbol{k}_1, \boldsymbol{k}_2).$$

Analogous structure valid for $n-$pt functions

I. Fourier framework - in application

We have developed an LSS analysis framework: [z.v. et. al., '15, '16, z.v.&White, '18]

- in application together with the Lagrangian EFT, bias

Codes that can be applied to real world data:

- power spectrum, correlation function
- link: `github.com/martinjameswhite/CLEFT_GSM`

Robust understanding of the BAO:

- did not have time to address here (see e.g. [Ding et. al., '17])
- application to reconstruction (in prep. [Chen et. al., '19])

Applied in the BOSS collaboration, as well as DESI in the future;

- something else?

“Monkey bias” - a new look at the statistics of biased tracers

with: Tomohiro Fujita (Kyoto, Geneva)

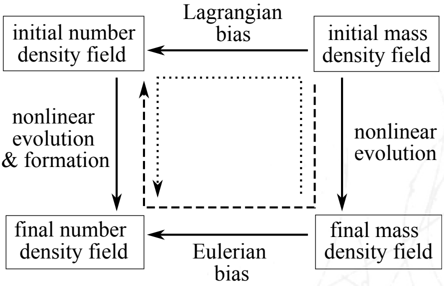
II. Non-linear dynamics and galaxy bias

- Eulerian bias: relates final d.m. density field and the final halo density field

$$\delta_{\rm g}(\boldsymbol{x}) = c^{\rm e}_{\delta}\delta(\boldsymbol{x}) + c^{\rm e}_{\delta^2}\delta^2(\boldsymbol{x}) + c^{\rm e}_{s^2}s^2(\boldsymbol{x}) + \ldots + c^{\rm e}_{\partial^2\delta}\frac{\partial_q^2}{k_L^2}\delta(\boldsymbol{x}) + \text{``stochastic''} + \ldots$$

- Lagrangian bias: relates initial d.m. density field and the proto-halo density field

$$\delta_{\rm g}(\boldsymbol{q}) = c^{\ell}_{\delta}\delta_L(\boldsymbol{q}) + c^{\ell}_{\delta^2}\delta_L^2(\boldsymbol{q}) + c^{\ell}_{s^2}s_L^2(\boldsymbol{q}) + \ldots + c^{\ell}_{\partial^2\delta}\frac{\partial_q^2}{k_*^2}\delta_L(\boldsymbol{q}) + \text{``stochastic''} + \ldots,$$



[Matsubara, '11]

II. Monkey (bias) business

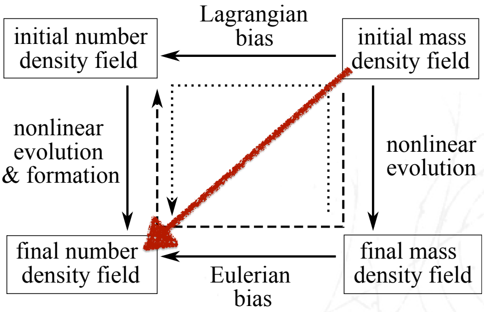
A new key idea:

(1) construct a bias of operators out of linear density - as a Monkey would,

(2) impose symmetries afterwords - application of consistency relations

$$\left\langle \delta^{\rm g}_{\boldsymbol{k}}(\eta)\delta^{\rm g}_{\boldsymbol{q}_1}(\eta_1)\dots\delta^{\rm g}_{\boldsymbol{q}_n}(\eta_n)\right\rangle' \sim P_{\rm g}(k,\eta)\sum_{\alpha} c_\alpha(\eta,\eta_\alpha)\frac{\boldsymbol{k}\cdot\boldsymbol{q}_\alpha}{k^2}\left\langle \delta^{\rm g}_{\boldsymbol{q}_1}(\eta_1)\dots\delta^{\rm g}_{\boldsymbol{q}_n}(\eta_n)\right\rangle', \quad \text{as } k\to 0.$$

[Peloso&Pietroni, '13, Kehagias&Riotto, '13,…]



Please check out Tomo's poster to hear the whole fascinating story!

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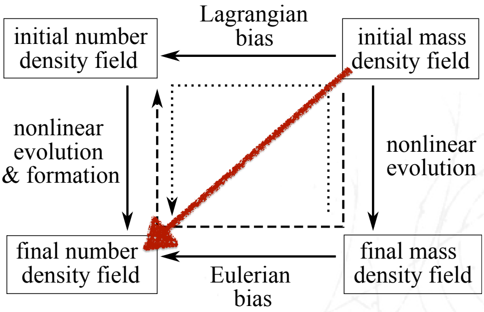

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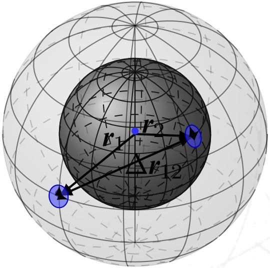
Clustering of shapes - applications to full-sky intrinsic alignment correlators

with: Elisa Chisari (Oxford), Fabian Schmidt (MPA)



Effective field theory of biasing

Alternatively we can be similarly expand density of tracers as

$$\delta_{\rm t}(\boldsymbol{x}) = \sum_{O} c_o O_t(\boldsymbol{x}),$$

where we list operators O_h:

[Desjacques et al, '16]

$$(1) \qquad \mathrm{tr}\left[\Pi^{[1]}\right],$$

$$(2) \qquad \mathrm{tr}\left[\left(\Pi^{[1]}\right)^2\right], \quad \left(\mathrm{tr}\left[\Pi^{[1]}\right]\right)^2,$$

$$(3) \qquad \mathrm{tr}\left[\left(\Pi^{[1]}\right)^3\right], \quad \mathrm{tr}\left[\left(\Pi^{[1]}\right)^2\right]\mathrm{tr}\left[\Pi^{[1]}\right], \quad \left(\mathrm{tr}\left[\Pi^{[1]}\right]\right)^3, \quad \mathrm{tr}\left[\Pi^{[1]}\Pi^{[2]}\right].$$

where $\Pi^{[1]}_{ij}(\boldsymbol{k}) = \frac{k_i k_j}{k^2}\delta_m(\boldsymbol{k})$, with derivative operators

$$R_*^2 \nabla^2 \mathrm{tr}\left[\Pi^{[1]}\right], \ldots .$$

– series allows one to estimate the higher order (theory) errors

– coefficients - physics from the R_* scale - degeneracies

III. Effective field theory of biasing

Expansion of the field of galaxy shapes:

$$g_{ij}(\boldsymbol{x}) = \sum_{O} b_o O_{ij}(\boldsymbol{x}).$$

where the list of operators (up to higher derivatives and stochastic contributions) is

$$\begin{aligned}
&(1) \qquad \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}, \\
&(2) \qquad \mathrm{TF}\left[\Pi^{[2]}\right]_{ij}, \;\; \mathrm{TF}\left[\left(\Pi^{[1]}\right)^2\right]_{ij}, \;\; \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}\mathrm{tr}\left[\Pi^{[1]}\right], \\
&(3) \qquad \mathrm{TF}\left[\Pi^{[3]}\right]_{ij}, \;\; \mathrm{TF}\left[\Pi^{[1]}\Pi^{[2]}\right]_{ij}, \;\; \mathrm{TF}\left[\Pi^{[2]}\right]_{ij}\mathrm{tr}\left[\Pi^{[1]}\right], \\
&\qquad\quad\;\; \mathrm{TF}\left[\left(\Pi^{[1]}\right)^3\right]_{ij}, \;\; \mathrm{TF}\left[\left(\Pi^{[1]}\right)^2\right]_{ij}\mathrm{tr}\left[\Pi^{[1]}\right], \;\; \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}\left(\mathrm{tr}\left[\Pi^{[1]}\right]\right)^2 \ldots
\end{aligned}$$

Derivative operators relevant for leading power spectrum corrections

$$R_*^2 \nabla^2 \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}.$$

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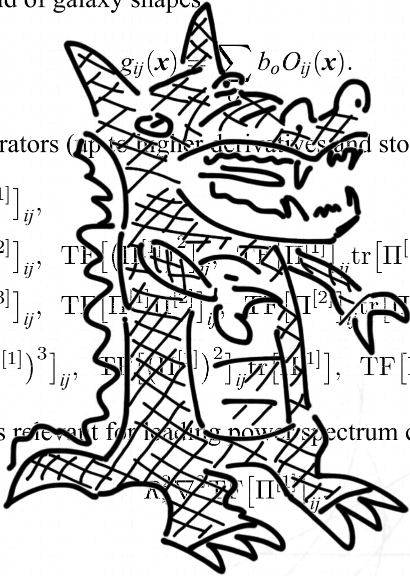


III. One-loop perturbation theory

Perturbative form of the shear tensor field

$$\Pi^{\rm t}_{ij}(\boldsymbol{k}) = \sum_{n=1}^{\infty} (2\pi)^3 \delta^D_{\boldsymbol{k}-\boldsymbol{q}_{1n}} \mathcal{K}^{(n)}_{ij,\rm bias}(\boldsymbol{q}_1 \ldots, \boldsymbol{q}_n) \delta_L(\boldsymbol{q}_1) \ldots \delta_L(\boldsymbol{q}_n),$$

where $\mathcal{K}^{(n)}_{\rm bias}$ are bias kernels (up to third order for one-loop).

PT results up to one-loop power spectrum

$$P^{\rm one-loop}_{ijlm} = P^{ab,\rm lin}_{ijlm} + P^{(22)}_{ijlm} + P^{(13)}_{ijlm} + P^{(31)}_{ijlm},$$

Linear, and loop (22), (13) contributions

$$P^{\rm lin}_{ijlm}(\boldsymbol{k}) = \frac{k_i k_j k_l k_m}{k^4} c^2_{\Pi^{[1]}} P_{\rm lin}(k),$$

$$P^{(22)}_{ijlm}(\boldsymbol{k}) = 2\,\mathcal{K}^{(2)}_{ij}(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}) \mathcal{K}^{(2)}_{lm}(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}) P_{\rm lin}(q) P_{\rm lin}(|\boldsymbol{k}-\boldsymbol{q}|),$$

$$P^{(13)}_{ijlm}(\boldsymbol{k}) = 3 c_{\Pi^{[1]}} \frac{k_i k_j}{k^2} P_{\rm lin}(k)\, \mathcal{K}^{(3)}_{lm,b}(\boldsymbol{k}, \boldsymbol{q}, -\boldsymbol{q}) P_{\rm lin}(q).$$

Similar for bispectrum...

Symmetries and Spherical Tensors

Separation: "symmetries" + "dynamics"

$$\text{Spherical tensors}: \quad \boldsymbol{Y}^{(\ell)m}(\hat{k}') = \sum_{q=-\ell}^{\ell} \left(\mathcal{D}^{(\ell)}\right)^{m}{}_{q} \boldsymbol{Y}^{(\ell)q}(\hat{k})$$

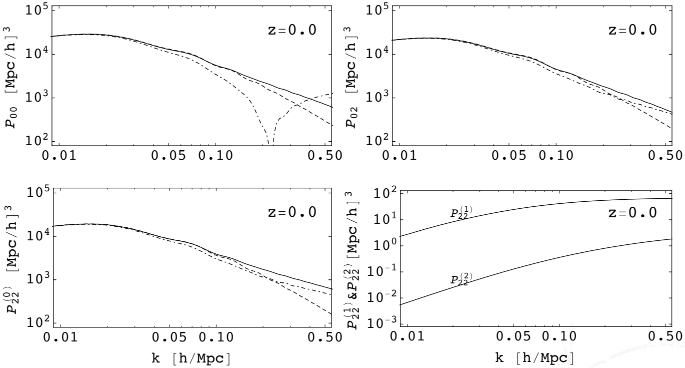
Rank $0, 1, 2$ form the orthogonal basis constructed from reps. of so(3):

$$\begin{aligned}
&\text{scalar} && \boldsymbol{Y}^{(0)} = 1, \\
&\text{vector} && \boldsymbol{Y}_i^{(0)} = \hat{k}_i, \quad \boldsymbol{Y}_i^{(\pm 1)} = e_i^{\pm}, \\
&\text{tensor} && \boldsymbol{Y}_{ij}^{(0)} = \hat{k}_i\hat{k}_j - \tfrac{1}{3}\delta^K_{ij}, \ \boldsymbol{Y}_{ij}^{(\pm 1)} = \hat{k}_j e_i^{\pm} + \hat{k}_i e_j^{\pm}, \ \boldsymbol{Y}_{ij}^{(\pm 2)} = e_i^{\pm} e_j^{\pm},
\end{aligned}$$

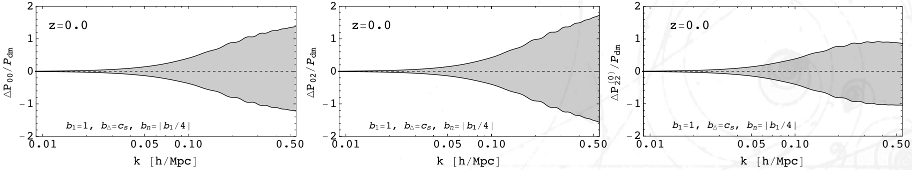
This gives the expansion

$$\Pi_{ij}(\boldsymbol{k}) = \frac{1}{3}\Pi_0^{(0)}(\boldsymbol{k})\delta^K_{ij} + \sum_{m=-2}^{2} \Pi_2^{(m)}(\boldsymbol{k})\boldsymbol{Y}_{ij}^{(m)}$$

This equivalent to the usual cosmological SVT decomposition.



Power spectra $P_{00}^{(0)}$, $P_{02}^{(0)}$, $P_{22}^{(0)}$, $P_{22}^{(1)}$ and $P_{22}^{(2)}$.



III. Projections onto the sky: flat sky approximation

3D shape of galaxies get projected onto the onto the sky:

$$\gamma_{I,ij}(\boldsymbol{r},z) = \left(\mathcal{P}_{ik}(\hat{n})\mathcal{P}_{jl}(\hat{n}) - \frac{1}{2}\mathcal{P}_{ij}(\hat{n})\mathcal{P}_{kl}(\hat{n})\right) g_{kl}(\boldsymbol{r},z),$$

where $\mathcal{P}_{ij}(\hat{r}) \equiv \delta^K_{ij} - \hat{r}_i\hat{r}_j$.

Integrating along the line of sight for photometric survey

$$\hat{\gamma}_{I,ij}(\boldsymbol{\theta}) = \int d\chi\ W(\chi)\gamma_{I,ij}\big(\chi\hat{n}, \chi\boldsymbol{\theta}\big),$$

These rotation of the basis leads to the following spectra

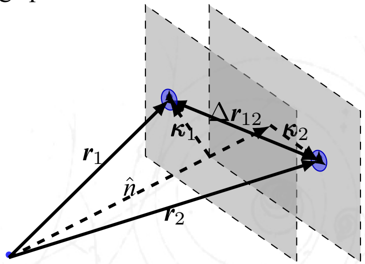
$$C_{\delta E}(\ell) = \int d\chi \frac{W^2(\chi)}{\chi^2} P^{(0)}_{02}(\ell/\chi),$$

$$C_{\delta B}(\ell) = 0,$$

$$C_{EE}(\ell) = \int d\chi \frac{W^2(\chi)}{\chi^2} \left(2P^{(0)}_{22}(\ell/\chi) + P^{(2)}_{22}(\ell/\chi)\right)$$

$$C_{BB}(\ell) = \int d\chi \frac{W^2(\chi)}{\chi^2} P^{(1)}_{22}(\ell/\chi)$$

$$C_{EB}(\ell) = 0.$$



III. Projections onto the sky: full sky

Note that configuration space basis vectors are eigenfunctions of the projection operator $\mathcal{P}.m^{\pm} = m^{\pm}$ Thus projections are simple!

$$\hat{\gamma}_{\pm 2}(\hat{r}) = \boldsymbol{M}_{ij}^{(\pm 2)*}\hat{\gamma}_{I,ij}(\hat{r}) = \int d\chi\, W(\chi)\gamma_{\pm 2}\left(\chi\hat{r}\right) = \int d\chi\, W(\chi) g_{\pm 2}(\chi\hat{r}).$$

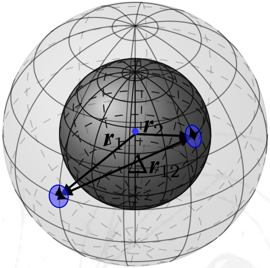
that can be decomposed into spin weighted spherical harmonics

$$\hat{\gamma}_{\pm 2} = \sum_{lm} {}_{\pm}\hat{\gamma}_{lm\pm 2} Y_l^{(m)}$$

Full angle power spectrum is than given by

$$\langle X_{lm}^{*} X_{lm}' \rangle = (2\pi)^2 \delta_{ll'}^{K} \delta_{mm'}^{K} C_l^{XX'}.$$

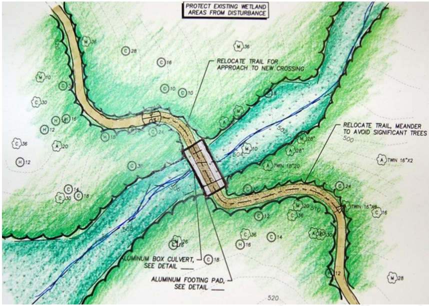
Leads to familiar E & B mode full sky form:



$$\langle {}_{s}\hat{\gamma}_{\ell m}^{*} | {}_{s'}\hat{\gamma}_{\ell' m'} \rangle = \delta_{\ell,\ell'}^{K} \delta_{m,m'}^{K} 4\pi \sum_{q=0}^{2} \int_k P_{22}^{(q)}(k) \left[\int_{\chi_1} W_1 \mathcal{J}_{\ell,2}^{s,\{q}(\chi_1 k) \right] \left[\int_{\chi_2} W_2 \mathcal{J}_{\ell,2}^{s',-q\}}(\chi_2 k) \right].$$

Lagrangian dynamics & stream crossing
- statistical field theory attire

with: Patrick McDonald (LBL)



IV. Path integrals and going beyond shell crossing

- as we saw the Lagrangian framework includes shell crossing
- Lagrangian dynamics can be compactly written using

$$\boldsymbol{L}_0\phi + \boldsymbol{\Delta}_0(\phi) = \boldsymbol{\epsilon},$$

where:

$$\phi \equiv (\psi, \upsilon)\,, \quad [\boldsymbol{L}_0]_{i_2 i_1} = \begin{pmatrix} \frac{\partial}{\partial \eta_2} & -1 \\ -\frac{3}{2} & \frac{\partial}{\partial \eta_2} + \frac{1}{2} \end{pmatrix}, \quad \boldsymbol{\Delta}_0(\phi) = \frac{3}{2}\left(0, \partial_{\boldsymbol{x}}\partial_{\boldsymbol{x}}^{-2}\delta + \psi\right).$$

Statistics of interest given by generating function

$$Z(\boldsymbol{j}) \equiv \int d\boldsymbol{\epsilon}\; e^{-\frac{1}{2}\boldsymbol{\epsilon}\boldsymbol{N}^{-1}\boldsymbol{\epsilon} + \boldsymbol{j}\phi[\boldsymbol{\epsilon}]} \;\text{ and }\; \langle \phi_{i_1}\phi_{i_2}\rangle = \frac{\partial^2}{\partial j_{i_1}\partial j_{i_2}} Z(\boldsymbol{j})\Big|_{\boldsymbol{j}=0},$$

which after the variable change becomes

$$Z(\boldsymbol{j}) \equiv \int d\phi\; e^{-S(\phi) + \boldsymbol{j}\phi},$$

with $S(\phi) = 1/2\,[\boldsymbol{L}_0\phi + \boldsymbol{\Delta}_0(\phi)]\,\boldsymbol{N}^{-1}\,[\boldsymbol{L}_0\phi + \boldsymbol{\Delta}_0(\phi)]\,.$ [McDonald&Vlah, '17]

IV. Path integrals and going beyond shell crossing

We can organize our perturbation theory as:

$$S = S_g + S_p, \quad \text{where then we do} \quad \exp(-S) = \exp(-S_g)(1 - S_p + S_p^2/2 + ...)$$

where we can choose what the "Gaussian part" will be, i.e.

$$S_g \equiv 1/2\chi \boldsymbol{N}\chi + i\chi[W^{-1}\boldsymbol{L}_0]\phi \equiv 1/2\chi\boldsymbol{N}\chi + i\chi\boldsymbol{L}\phi$$

and

$$S_p \equiv i\chi\boldsymbol{\Delta}_0(\phi) + i\chi[(1 - W^{-1})\boldsymbol{L}_0]\phi \equiv i\chi\boldsymbol{\Delta}(\phi),$$

where χ is the auxiliary field from the Hubbard-Stratonovich transformation.

Perturbation theory result : $Z_0(\boldsymbol{j}) = Z_0(\boldsymbol{j}) + Z_1(\boldsymbol{j}) + \ldots$

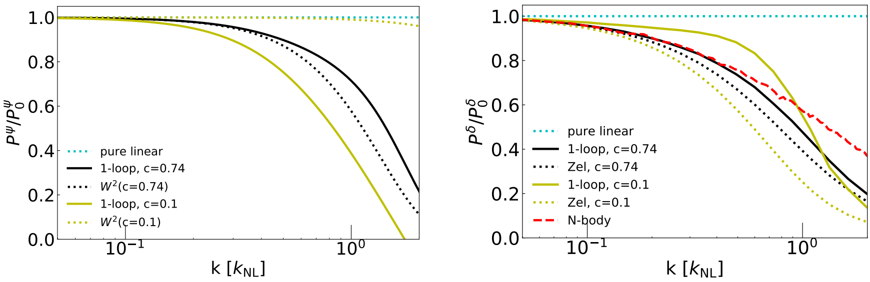
Leading order result: truncate Zel'dovich dynamics!!!

$$Z_0 = e^{\frac{1}{2}\boldsymbol{j}.\boldsymbol{C}.\boldsymbol{j}} \quad \text{and} \quad P(k) = \int d^3q \; e^{i\boldsymbol{q}\cdot\boldsymbol{k}} e^{-\frac{1}{2}k_ik_jA^W_{ij}}$$

higher orders more complicated, build in renormalization! [McDonald&Vlah, '17]

IV. Path integrals and going beyond shell crossing

$n = 0.5$

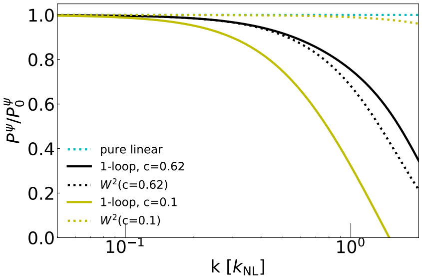


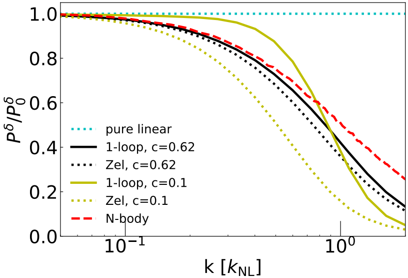
Significance and connection EFT formalism:

- no need of EFT free parameters, i.e. counter terms are predicted
- CMB lensing: direct information on baryonic and neutrinos physics
- reduction of degeneracy in galaxy bias coefficients
- possible connection to the EFT formalism by matching the $k \to 0$ limit

IV. Path integrals and going beyond shell crossing

$n = 1.0$





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Concluding thoughts

To recapitulate:

- We have developed a new RSD framework based on the Fourier space cumulant expansion - highlight of differences and advantages.
- FoG are incorporated naturally - expressions are algebraic

- New and simplified bias framework - we rely only on symmetries & equiv. principle.

- We use decomposition into spherical tensors to construct the bias/EFT of shapes - applications to intrinsic alignments.
- General framework - worked out full sky correlators.

- Consistently included shell crossing into the perturbative Lagrangian scheme.
- Connection to EFT-PT via low-k expansion - EFT parameter matching.