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PTchat@Kyoto
YITP

CGP
Center for Gravitational Physics
Yukawa Institute for Theoretical Physics

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YUKAWA INSTITUTE FOR
THEORETICAL PHYSICS

Wide-angle redshift-space distortions at quasi-linear scales

~ *modeling relativistic dipole* ~

Atsushi Taruya

(Yukawa Institute for Theoretical Physics)

Plan of talk

What we did/are doing

- Modeling wide-angle redshift-space distortions at quasi-linear scales
- Predicting halo cross-correlation functions with relativistic effect

  → comparison with N-body simulation

- Introduction & motivation
- Modeling relativistic dipole
- Results
- Summary

Collaborators

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Introduction

Observed large-scale structure *generally* appears distorted

In galaxy redshift surveys

Line-of-sight position $\not\longleftrightarrow$ Actual position

(Inferred from redshift measurements)

Redshift-space distortions (RSD)
(Clustering anisotropies)

Major source

(classical)

Doppler effect induced by peculiar velocity of galaxy

Observed galaxy position (comoving)

$$\boldsymbol{s} = \boldsymbol{x} + \frac{1}{a\,H}(\mathbf{v}\cdot\hat{x})\boxed{\hat{x}}$$

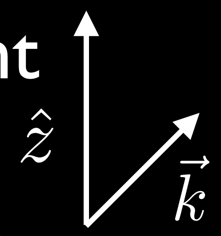
observer's line-of-sight

Actual position

peculiar velocity of galaxy

Kaiser formula

(Kaiser ’87)



Observed density field (Fourier space)

$$\delta^{(\mathrm{S})}(\boldsymbol{k}) = (1 + \boxed{f}\,\mu_k^2)\,\delta(\boldsymbol{k})$$

‘Real’ density field

$$\mu_k = \frac{\vec{k}\cdot\hat{z}}{|\vec{k}|}$$

This parameter tells us

how the nature of gravity affects the growth of structure:

$$f \equiv \frac{d\ln D_+}{d\ln a}$$

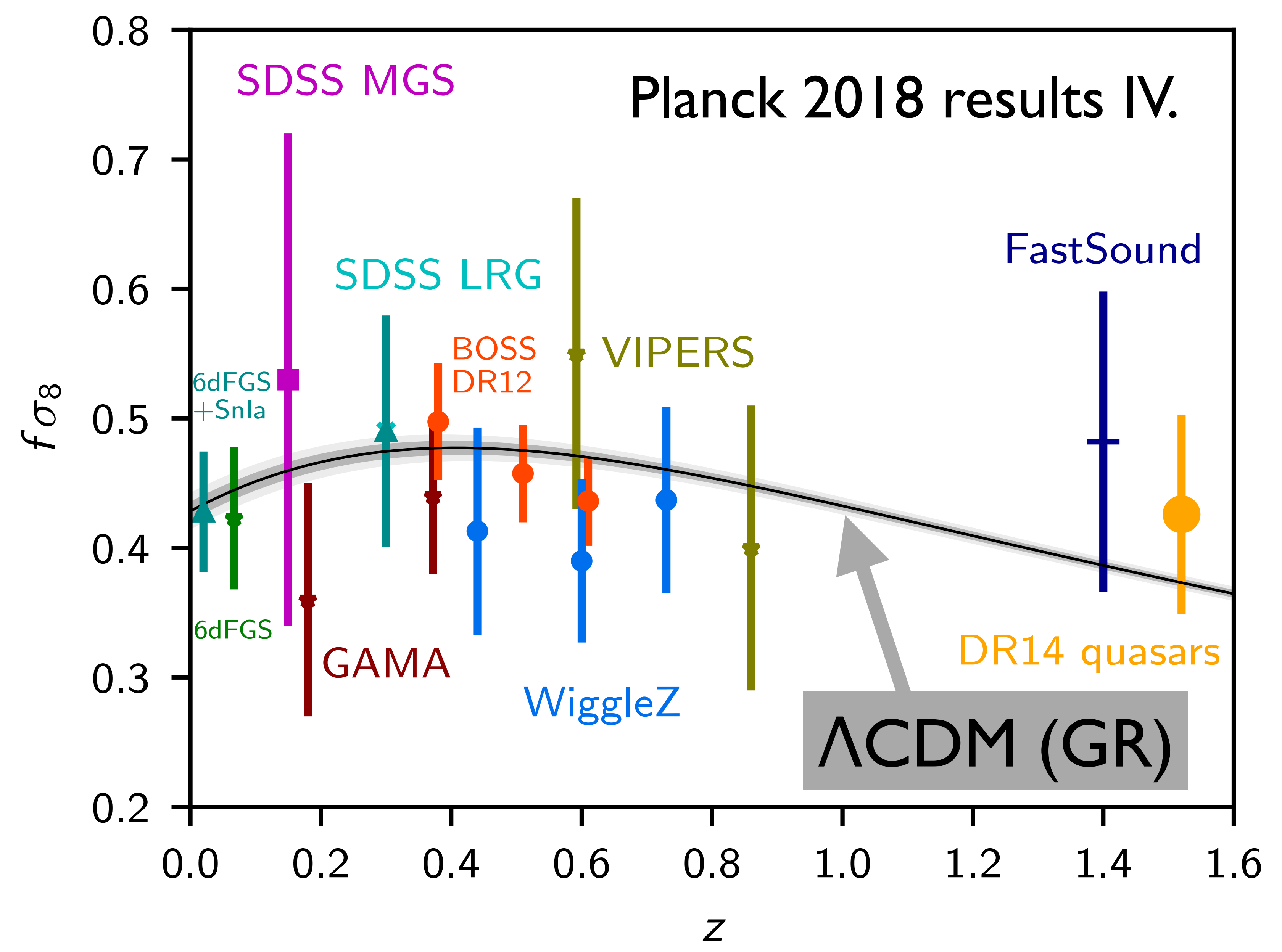
Growth of structure induced by gravity

Scale factor

This formula holds irrespective of gravity theory

probe of gravity (general relativity) on cosmological scales

Cosmological test of gravity



So far,

Consistent with ΛCDM (GR)

Euclid

DESI

SuMIRe-PFS

Dramatic improvement is expected in future RSD measurements, which will also open up a possibility to detect something new !

Generalized

Redshift-space distortions

Redshift we actually measure involves not only Doppler effect but also several relativistic contributions

Yoo et al. ('09), Yoo ('10), Challinor & Lewis ('11), Bonvin & Durrer ('11)

Actual position

Observed galaxy position (comoving)

$$\vec{s} = \vec{x} + \hat{x}\left\{\frac{c}{H}\delta z - \frac{1}{c^2}\int_0^{\chi(z_{\mathrm{obs}})} d\chi'(\psi-\phi)\right\} - \chi(z_{\mathrm{obs}})\,\vec{\alpha}$$

Shapiro time-delay

gravitational lensing

For rest-frame observer

$$\delta z = (1+z_{\mathrm{obs}})\left\{\frac{\vec{\mathrm{v}}_{\mathrm{s}}\cdot\hat{x}}{c} - \frac{\psi_{\mathrm{s}}}{c^2} + \frac{1}{2}\frac{\mathrm{v}_{\mathrm{s}}^2}{c^2} - \frac{1}{c^2}\int_{t_{\mathrm{s}}}^{t_{\mathrm{o}}} dt'(\dot{\psi}-\dot{\phi})\right\}$$

standard RSD (Doppler)

gravitational redshift

transverse Doppler

integrated Sachs-Wolfe

Detection of these relativistic contributions would be an important target in future RSD measurements

Signature of relativistic effects

Relativistic contributions generate *dipole asymmetry* when cross-correlating two galaxy/halo populations

e.g., McDonald ('09), Bonvin et al. ('14)

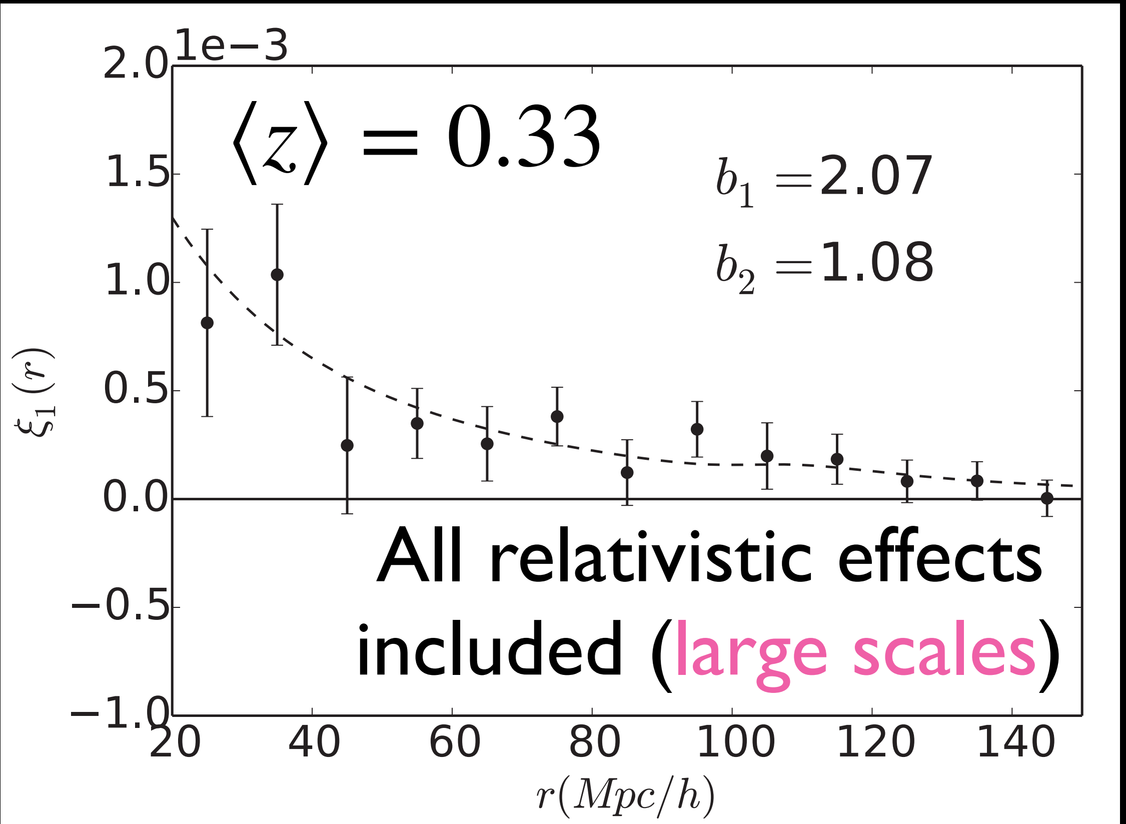
Full-sky light-cone simulation + light-ray propagation

$(2{,}625\,h^{-1}\mathrm{Mpc})^3$

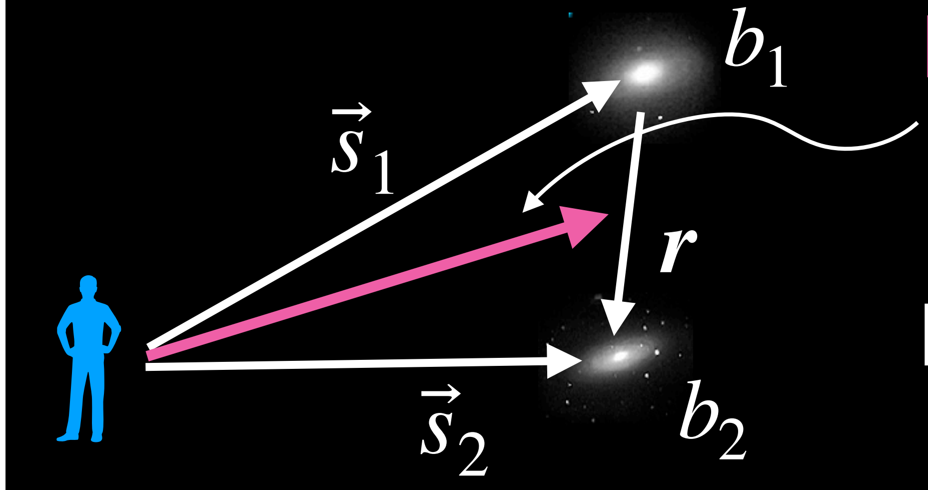
$N_{\mathrm{DM}} = 4{,}096^3$

Halo catalog with observational relativistic effects

Breton, Rasera, AT, Lacombe & Saga ('19)



Dipole cross correlation
$$\xi_1(r) \equiv \frac{3}{2}\int_{-1}^{1} d\mu\, \xi^{(\mathrm{S})}(\boldsymbol{s}_1, \boldsymbol{s}_2)$$



Line-of-sight
$$\vec{d} \equiv (\vec{s}_1 + \vec{s}_2)/2$$

Directional cosine
$$\mu \equiv \hat{d} \cdot \hat{r}$$

Signature of relativistic effects

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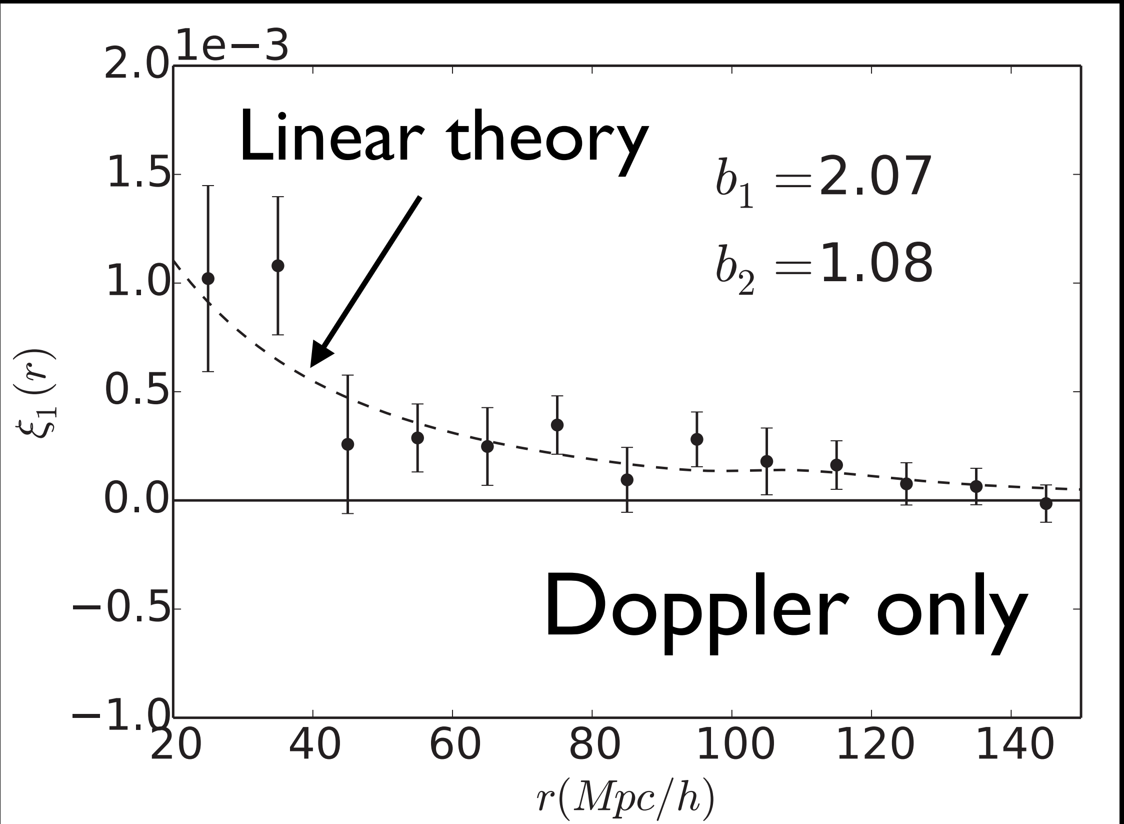

Doppler effect also produces dipole (*wide-angle effects* → Paolo's talk)

⟶ Major contribution at large scales

Signature of relativistic effects

Relativistic contributions generate *dipole asymmetry*
when cross-correlating two galaxy/halo populations

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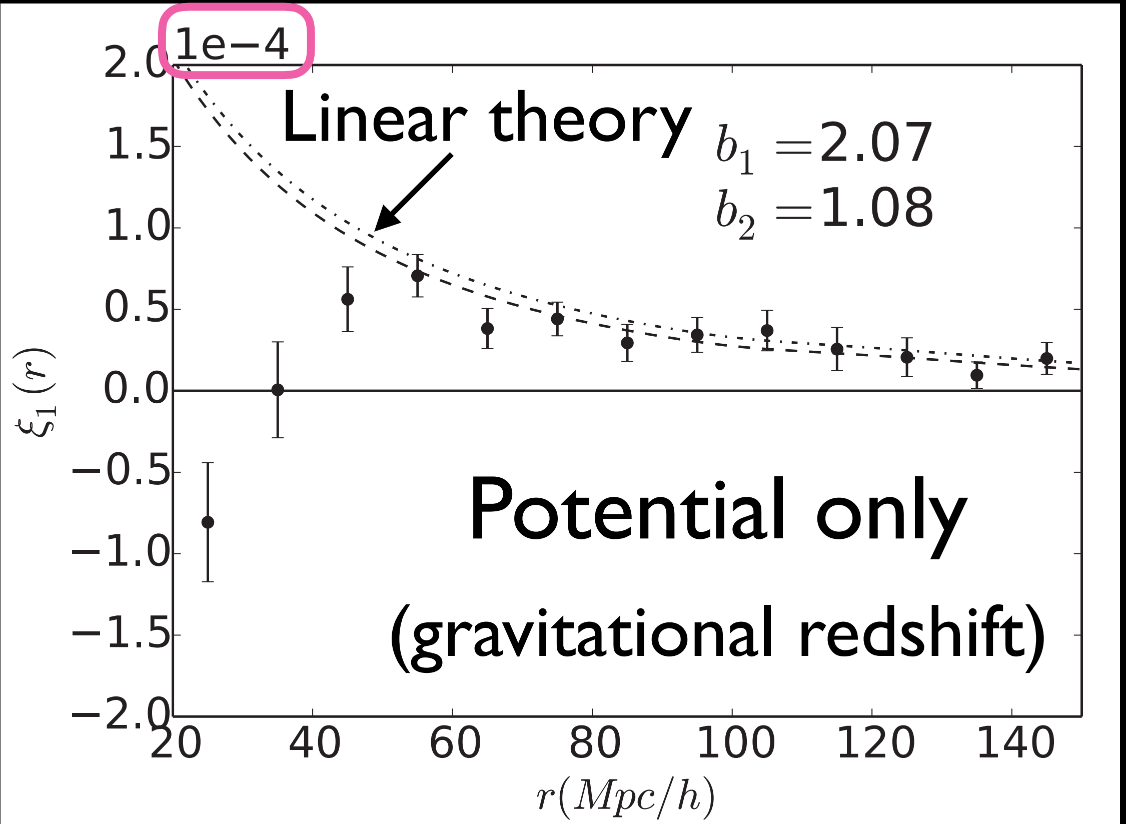

Gravitational redshift is the largest
among relativistic contributions

Still, subdominant at large scales

however, *at small scales*, …

Signature of relativistic effects

Relativistic contributions generate *dipole asymmetry* when cross-correlating two galaxy/halo populations

e.g., McDonald ('09), Bonvin et al. ('14)

Full-sky light-cone simulation + light-ray propagation

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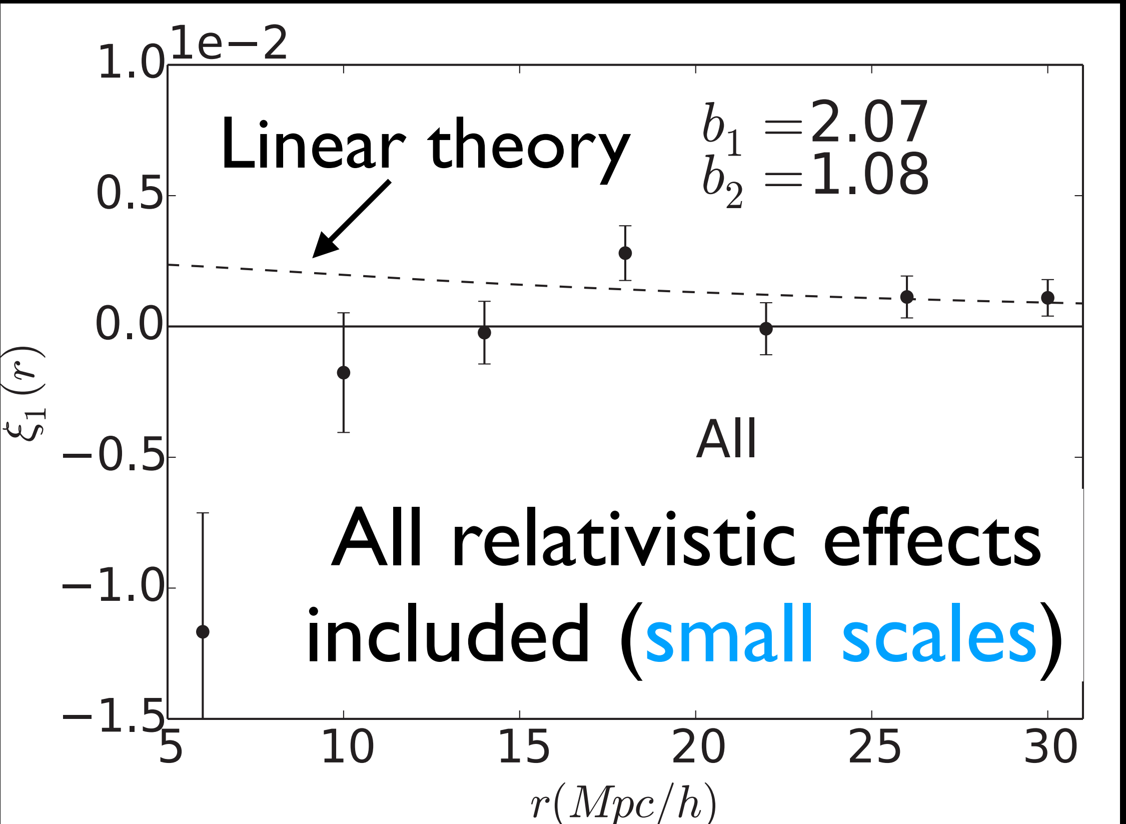


Linear theory

All relativistic effects included (small scales)

Gravitational redshift starts to be dominant, and finally wins (sign flipped)

Linear theory prediction fails

Motivation

Q Can we predict/model these results from analytical calculation ?

Doppler > Potential (large scales)
Doppler < Potential (small scales)

Taking account of

- Wide-angle effects on RSD
- Relativistic effect (gravitational redshift)

Further we need to go beyond linear theory

Related works	Method	Wide-angle	Relativistic
Castorina & White ('18)	Zel'dovich approx. + linear bias	○	N/A
Di Dio & Seljak ('18)	Standard PT 1-loop + nonlinear bias	N/A	○

Motivation

Q Can we predict/model these results from analytical calculation ?

Doppler > Potential (large scales)
Doppler < Potential (small scales)

Taking account of

- Wide-angle effects on RSD
- Relativistic effect (gravitational redshift)

Further we need to go beyond linear theory

In this talk	Method	Wide-angle	Relativistic
Present work	Zel'dovich approx. + linear bias +α	○	○

- consistently reproduce linear theory of wide-angle RSD
- a good agreement with simulation results

Szalay et al. '98,
Papai & Szapudi '08

Modeling dipole cross-correlation

Consider Doppler effect and gravitational redshift: $(c = 1)$

$$\mathbf{s} = \mathbf{x} + \frac{1}{a\,H}\left\{(\mathbf{v}\cdot\hat{x}\,) - \psi\right\}\hat{x}\,; \qquad \hat{x} \equiv \frac{\overrightarrow{x}}{|\,\overrightarrow{x}\,|} \neq \hat{z}$$

Potential

Modeling dipole cross-correlation

Consider Doppler effect and gravitational redshift: $(c = 1)$

$$\mathbf{s} = \mathbf{x} + \frac{1}{a\,H}\left\{(\mathbf{v}\cdot\hat{x}) - \psi\right\}\hat{x}\,; \qquad \hat{x} \equiv \frac{\overrightarrow{x}}{|\,\overrightarrow{x}\,|} \neq \hat{z}$$

Potential

Motion of halos

Zel'dovich approx. (ZA) —1st-oder Lagrangian PT

$$\boldsymbol{x}(\boldsymbol{q},t) = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},t), \qquad \boldsymbol{v}(\boldsymbol{q},t) = a\frac{d\boldsymbol{\Psi}(\boldsymbol{q},t)}{dt}$$

$\boldsymbol{q}$: Lagrangian coordinate

$\boldsymbol{\Psi}$: displacement field $\left(\boldsymbol{\Psi} \xrightarrow{t\to 0} 0\right)$

In ZA,

$$\nabla_q \cdot \boldsymbol{\Psi} = -\,D_+(t)\,\delta_{\rm lin}(q)$$

➡

$$s_i \simeq q_i + \left\{\delta_{ij} + f\,\hat{q}_i\hat{q}_j\right\}\Psi_j(q) - \left(\frac{\psi}{a\,H}\right)\hat{q}_i\,; \qquad f \equiv \frac{d\ln D_+}{d\ln a}$$

Modeling dipole cross-correlation

Consider Doppler effect and gravitational redshift: $(c = 1)$

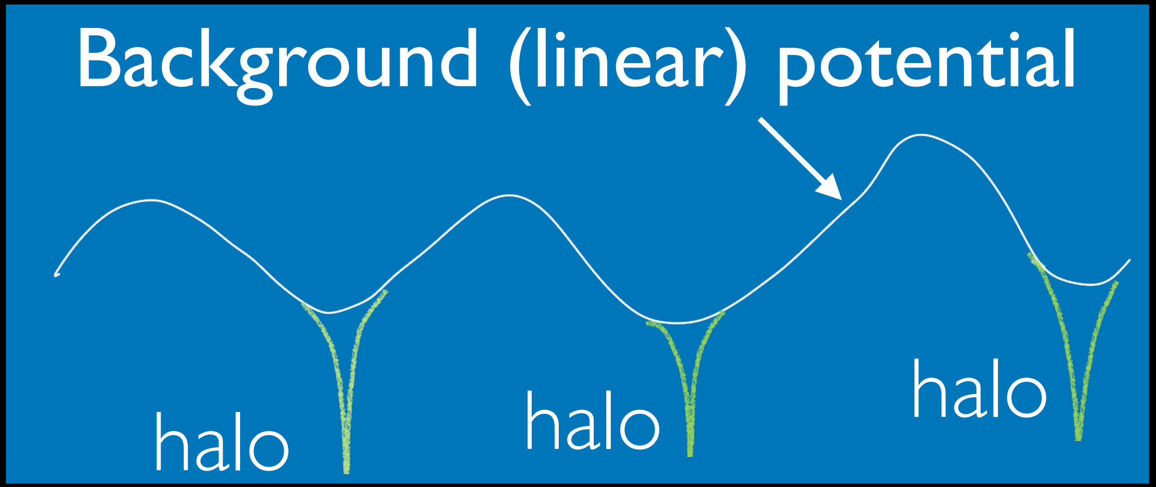
$$\mathbf{s} = \mathbf{x} + \frac{1}{a\,H}\left\{(\mathbf{v}\cdot\hat{x}) - \psi\right\}\hat{x} \qquad \hat{x} \equiv \frac{\overrightarrow{x}}{|\overrightarrow{x}|} \neq \hat{z}$$

Potential

Potential at halos

Perhaps, we need something beyond ZA (linear)

$$\psi \longrightarrow \underline{\psi_{\rm BG}} + \underline{\psi_{\rm halo}}$$



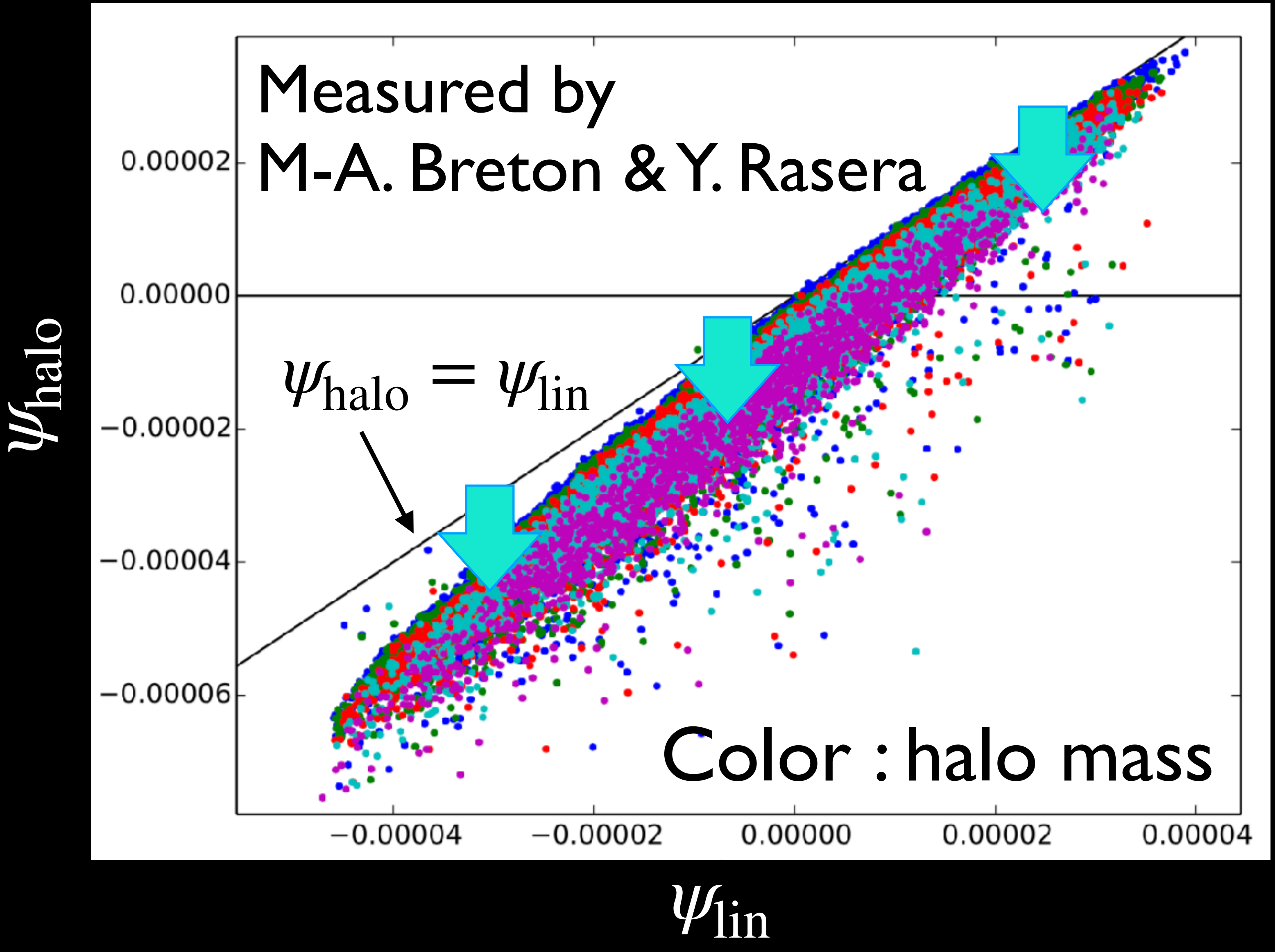
Computed with ZA $\propto (\nabla/\nabla^2)\,\Psi_{\rm ZA}$

Assumed to be constant
(but depend on halo mass)

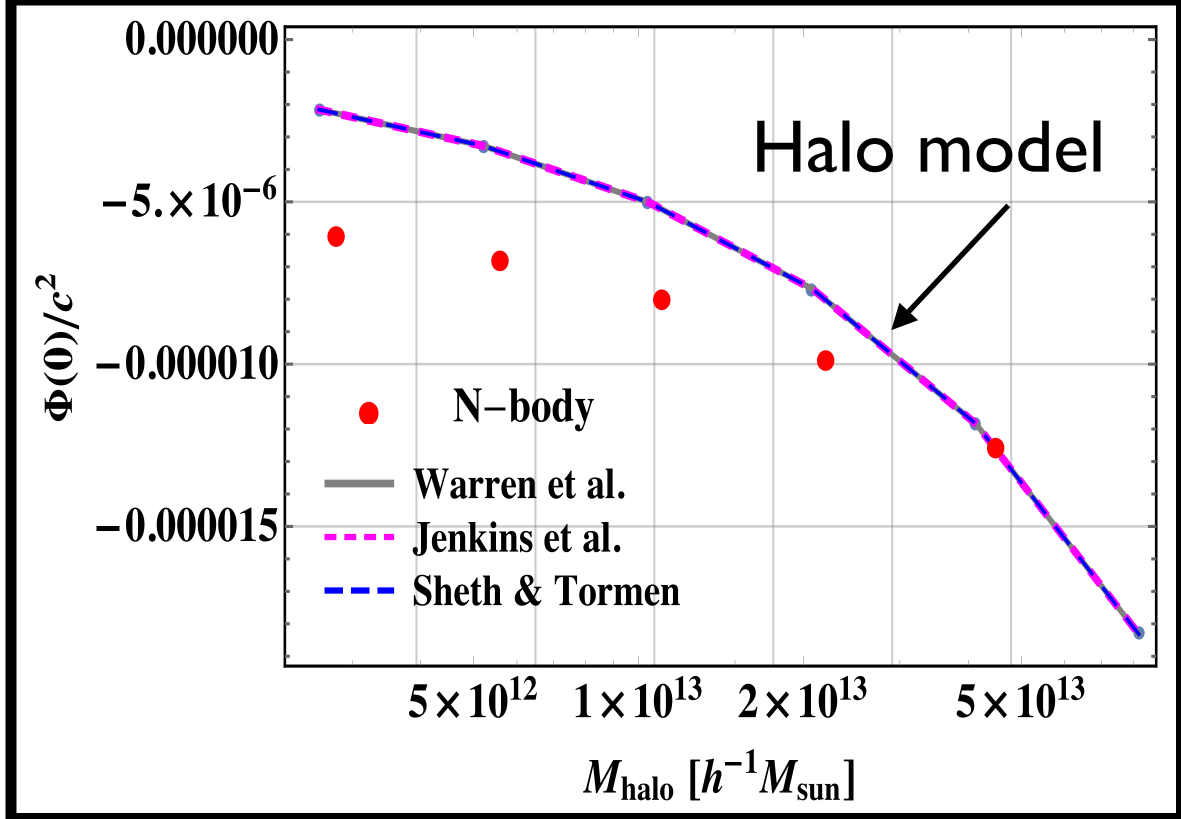
Halo potential

Potential at halo center is systematically deeper than linear potential

(→ Potential offset)


Measured by
M-A. Breton & Y. Rasera

Color : halo mass


Halo model

Measured potential offset shows halo mass dependence, which is roughly consistent with halo model prediction

Modeling dipole cross-correlation

Consider Doppler effect and gravitational redshift: $(c = 1)$

$$\mathbf{s} = \mathbf{x} + \frac{1}{a\,H}\left\{(\mathbf{v}\cdot\hat{x}) - \psi\right\}\hat{x} \qquad \hat{x} \equiv \frac{\vec{x}}{|\,\vec{x}\,|} \neq \hat{z}$$

Potential

Potential at halos

Perhaps, we need something beyond ZA (linear)

$$\boldsymbol{s} = \boldsymbol{q} + \boldsymbol{\Psi}^{(\mathrm{S})}_{\mathrm{ZA}}(\boldsymbol{q}) + \boldsymbol{\Psi}^{(\mathrm{S})}_{\mathrm{halo}}(\boldsymbol{q})$$

$$\Psi^{(\mathrm{S})}_{\mathrm{ZA},i}(\boldsymbol{q}) = (\delta_{ij} + f\,\hat{q}_i\,\hat{q}_j)\,\Psi_{\mathrm{ZA},j}(\boldsymbol{q}) - (\psi_{\mathrm{lin}}/aH)\,\hat{q}_i$$

Perturbative (ZA)
(Doppler + potential)

$$\Psi^{(\mathrm{S})}_{\mathrm{halo}}(\boldsymbol{q}) = -\,(\psi_{\mathrm{halo}}/aH)\,\hat{q}$$

Non-perturbative
(halo potential)

Computing dipole cross-correlation

Provided the relation btw. redshift- & Lagrangian-space positions,

Number density field of object 'X'

$$\boxed{n_{\rm X}^{\rm (S)}(\boldsymbol{s})\,d^3\boldsymbol{s}=n_{\rm X}(\boldsymbol{x})d^3\boldsymbol{x}=\overline{n}_{\rm X}\Bigl\{1+b_{\rm X}^{\rm L}\,\delta_0(\boldsymbol{q})\Bigr\}\,d^3\boldsymbol{q}.}$$

Linear galaxy/halo bias

$$n_{\rm X}^{\rm (S)}(\boldsymbol{s})=\bar{n}_{\rm X}\left|\frac{\partial\boldsymbol{s}}{\partial\boldsymbol{q}}\right|^{-1}\{1+b_{\rm X}^{\rm L}\,\delta_{\rm lin}(\boldsymbol{q})\}$$

$$=\bar{n}_{\rm X}\int d^3\boldsymbol{q}\,\delta_{\rm D}\Bigl[\boldsymbol{s}-\boldsymbol{q}-\Psi_{\rm ZA}^{\rm (S)}(\boldsymbol{q})-\boldsymbol{\Psi}_{\rm X}^{\rm (S)}(\boldsymbol{q})\Bigr]\{1+b_{\rm X}^{\rm L}\,\delta_{\rm lin}(\boldsymbol{q})\}$$

$$=\bar{n}_{\rm X}\int d^3\boldsymbol{q}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\,e^{i\,\boldsymbol{k}\cdot[\boldsymbol{s}-\boldsymbol{q}-\Psi_{\rm ZA}^{\rm (S)}(\boldsymbol{q})-\boldsymbol{\Psi}_{\rm X}^{\rm (S)}(\boldsymbol{q})\,]}\{1+b_{\rm X}^{\rm L}\,\delta_{\rm lin}(\boldsymbol{q})\}$$

➡ Density field of object 'X'

$$1+\delta_{\rm X}^{\rm (S)}(\boldsymbol{s})\equiv\frac{n_{\rm X}^{\rm (S)}(\boldsymbol{s})}{\langle n_{\rm X}^{\rm (S)}(\boldsymbol{s})\rangle}$$

Computing dipole cross-correlation

Correlation between objects ‘X’ and ‘Y’ :

$$\boxed{1+\xi^{(\mathrm{S})}_{\mathrm{XY}}(\boldsymbol{s}_1,\boldsymbol{s}_2)=\Big\langle\{1+\delta^{(\mathrm{S})}_{\mathrm{X}}(\boldsymbol{s}_1)\}\{1+\delta^{(\mathrm{S})}_{\mathrm{Y}}(\boldsymbol{s}_2)\}\Big\rangle=\frac{\langle n^{(\mathrm{S})}_{\mathrm{X}}(\boldsymbol{s}_1)n^{(\mathrm{S})}_{\mathrm{Y}}(\boldsymbol{s}_2)\rangle}{\langle n^{(\mathrm{S})}_{\mathrm{X}}(\boldsymbol{s}_1)\rangle\,\langle n^{(\mathrm{S})}_{\mathrm{Y}}(\boldsymbol{s}_2)\rangle}}$$

$$\langle n^{(\mathrm{S})}_{\mathrm{X}}(\boldsymbol{s}_1)n^{(\mathrm{S})}_{\mathrm{Y}}(\boldsymbol{s}_2)\rangle=\int\frac{d^3\boldsymbol{k}_1d^3\boldsymbol{k}_2}{(2\pi)^6}\int d^3\boldsymbol{q}_1\int d^3\boldsymbol{q}_2$$
$$\times\,e^{i\boldsymbol{k}_1\cdot\{\boldsymbol{s}_1-\boldsymbol{q}_1-\boldsymbol{\Psi}^{(\mathrm{S})}_{\mathrm{X}}(\boldsymbol{q}_1)\}+i\boldsymbol{k}_2\cdot\{\boldsymbol{s}_2-\boldsymbol{q}_2-\boldsymbol{\Psi}^{(\mathrm{S})}_{\mathrm{Y}}(\boldsymbol{q}_2)\}}$$
$$\times\Big\langle e^{-i\boldsymbol{k}_1\cdot\boldsymbol{\Psi}^{(\mathrm{S})}_{\mathrm{ZA}}(\boldsymbol{q}_1)-i\boldsymbol{k}_2\cdot\boldsymbol{\Psi}^{(\mathrm{S})}_{\mathrm{ZA}}(\boldsymbol{q}_2)}\{1+b^{\mathrm{L}}_{\mathrm{X}}\,\delta_{\mathrm{lin}}(\boldsymbol{q}_1)\}\{1+b^{\mathrm{L}}_{\mathrm{X}}\,\delta_{\mathrm{lin}}(\boldsymbol{q}_2)\}\Big\rangle$$

$$\langle n^{(\mathrm{S})}_{\mathrm{X}}(\boldsymbol{s})\rangle=\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\int d^3\boldsymbol{q}\;e^{i\boldsymbol{k}\cdot\{\boldsymbol{s}-\boldsymbol{q}-\boldsymbol{\Psi}^{(\mathrm{S})}_{\mathrm{X}}(\boldsymbol{q})\}}\Big\langle e^{-i\boldsymbol{k}\cdot\boldsymbol{\Psi}^{(\mathrm{S})}_{\mathrm{ZA}}(\boldsymbol{q})}\{1+b^{\mathrm{L}}_{\mathrm{X}}\,\delta_{\mathrm{lin}}(\boldsymbol{q})\}\Big\rangle$$

Distant-observer limit $\langle n^{(\mathrm{S})}_{\mathrm{X}}(\boldsymbol{s}_1)n^{(\mathrm{S})}_{\mathrm{Y}}(\boldsymbol{s}_2)\rangle\rightarrow$ 3D Gaussian integral

(e.g., Carlson et al. ’13, White’14) $\langle n^{(\mathrm{S})}_{\mathrm{X}}(\boldsymbol{s})\rangle\longrightarrow\bar{n}_{\mathrm{X}}$ (mean number density)

Computing dipole cross-correlation

Correlation between objects '*X*' and '*Y*' :

$$1+\xi^{(\mathrm{S})}_{\mathrm{XY}}(\boldsymbol{s}_1,\boldsymbol{s}_2)=\left\langle\{1+\delta^{(\mathrm{S})}_{\mathrm{X}}(\boldsymbol{s}_1)\}\{1+\delta^{(\mathrm{S})}_{\mathrm{Y}}(\boldsymbol{s}_2)\}\right\rangle=\frac{\langle n^{(\mathrm{S})}_{\mathrm{X}}(\boldsymbol{s}_1)n^{(\mathrm{S})}_{\mathrm{Y}}(\boldsymbol{s}_2)\rangle}{\langle n^{(\mathrm{S})}_{\mathrm{X}}(\boldsymbol{s}_1)\rangle\,\langle n^{(\mathrm{S})}_{\mathrm{Y}}(\boldsymbol{s}_2)\rangle}$$

Remarks *In the presence of wide-angle effects,*

$\langle n^{(\mathrm{S})}_{\mathrm{X}}(\boldsymbol{s})\rangle$ cannot be reduced to $\overline{n}_{\mathrm{X}}$ (real-space mean density)

········ Non-trivial scale-dependence from denominator

$\xi^{(\mathrm{S})}_{\mathrm{XY}}$ is function of $s\equiv|\boldsymbol{s}_2-\boldsymbol{s}_1|$ and $|\boldsymbol{s}_1|,|\boldsymbol{s}_2|$

········ One cannot take advantage of symmetry to reduce multi-dim integration → need to evaluate 6D integral

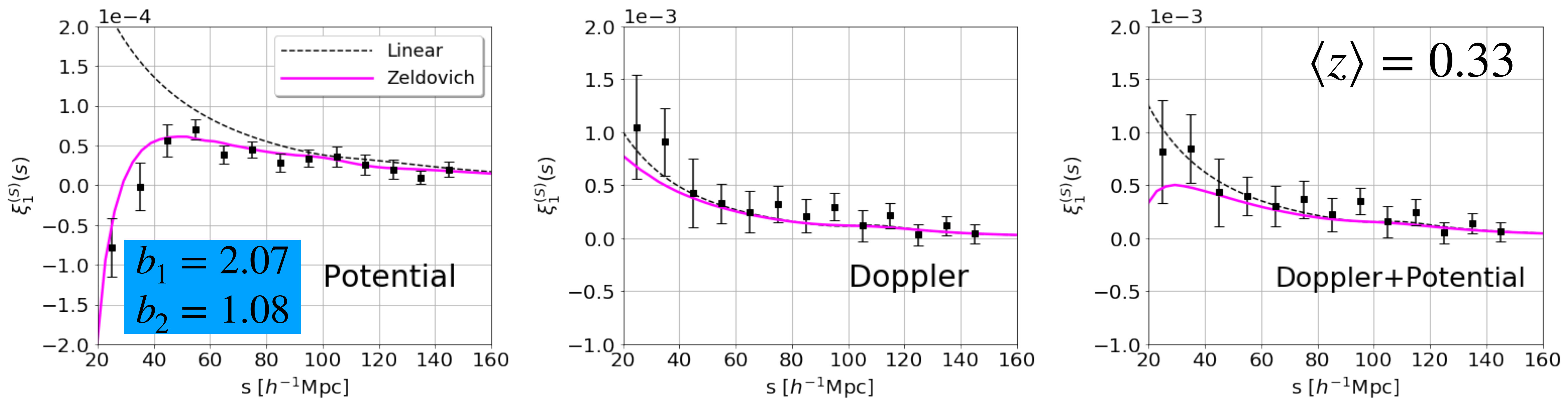
(c.f. Castorina & White '18)

Parameters: b_{X}, b_{Y} (bias) $\psi_{\mathrm{halo,X}}$, $\psi_{\mathrm{halo,Y}}$ (halo potential)

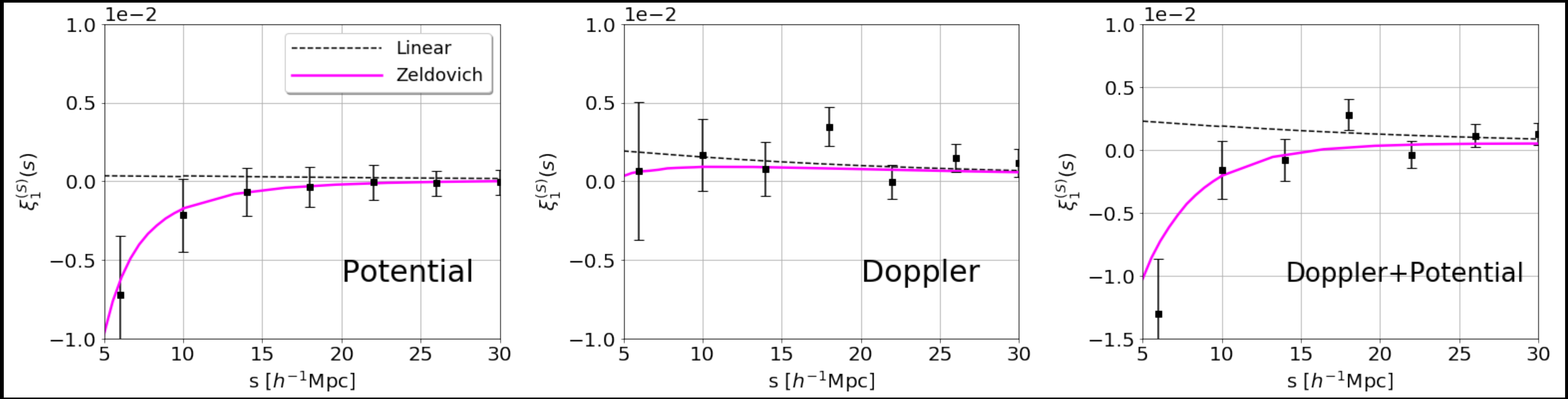
Results: dipole cross correlation

Large scale

Magenta: measured halo potential used



Small scale



Results: dipole cross correlation

Large scale

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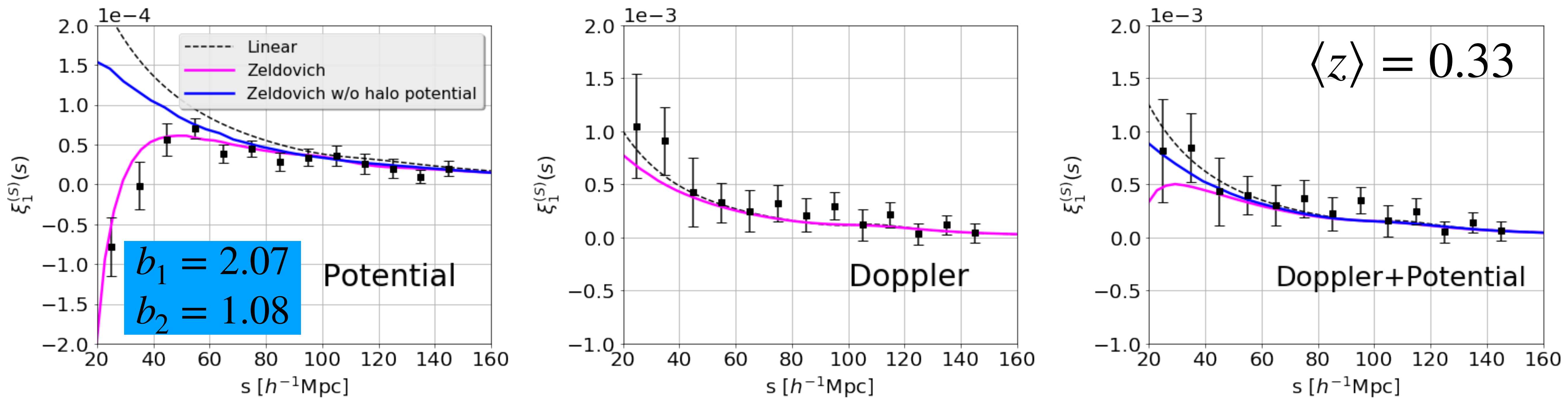
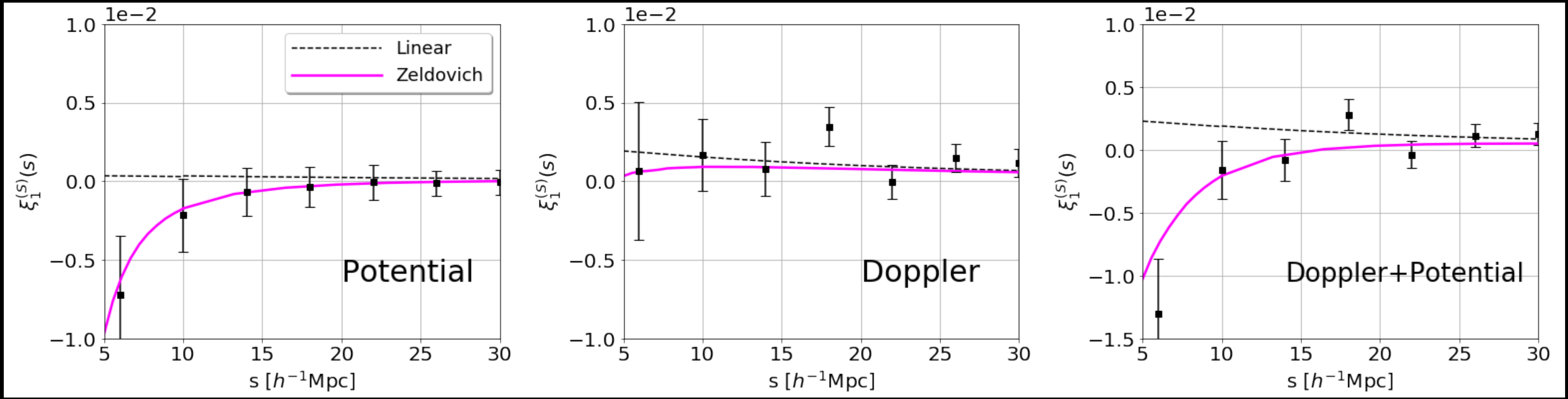


Small scale



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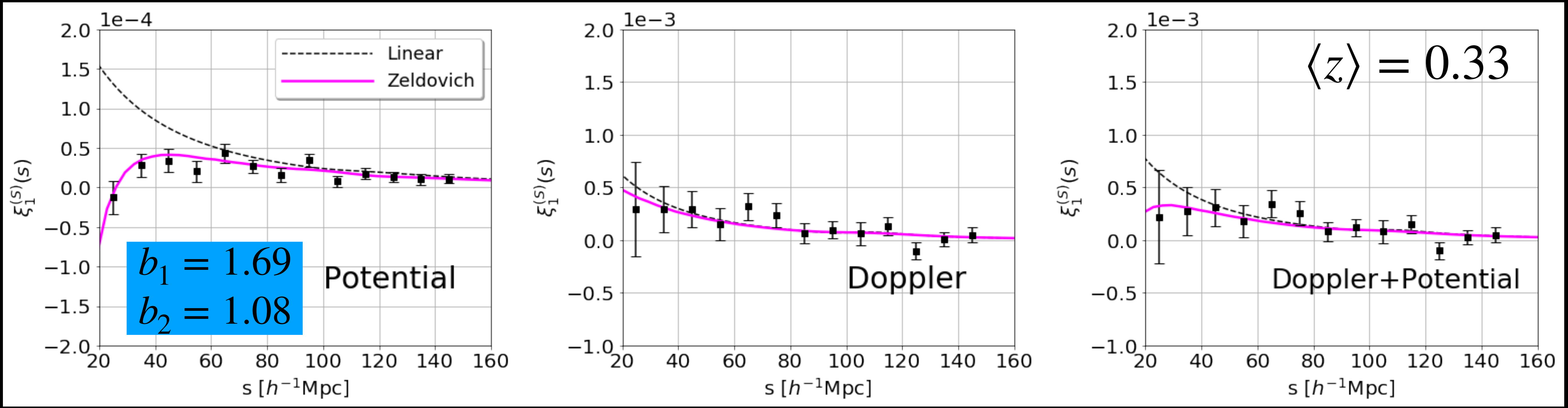
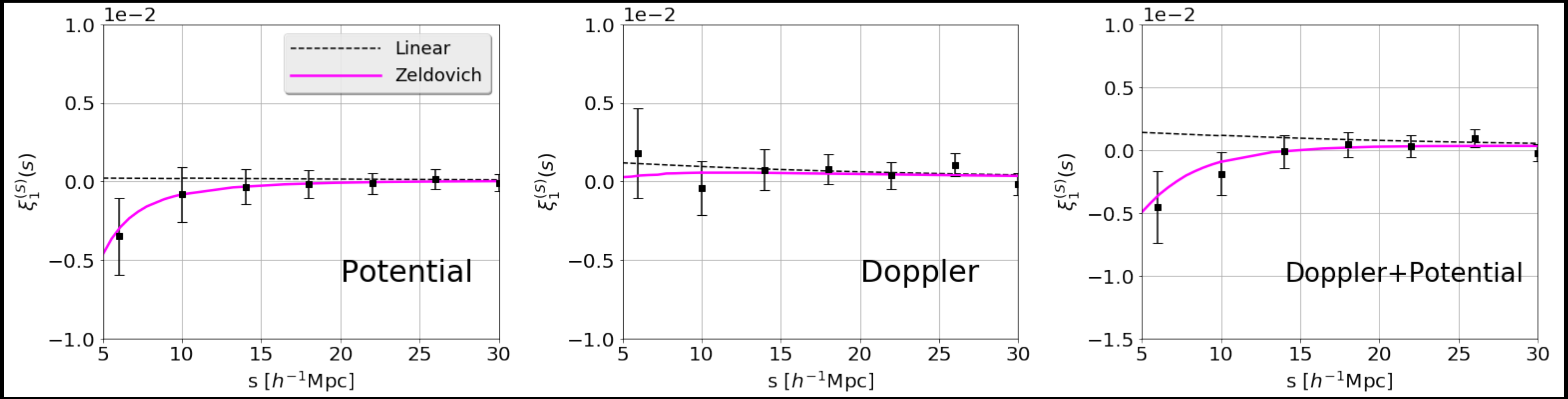


Small scale



Summary

Modeling redshift-space cross-correlation function with wide-angle and relativistic effects at quasi-linear scales

Formulation based on Zel'dovich approximation:

- Doppler + potential (gravitational redshift)
- Linear bias & halo potential (4 parameters)

Consistent with linear theory of wide-angle RSD

(*c.f.* Castorina & White '18)

Good agreement with simulations including relativistic effects

(*c.f.* Di Dio & Seljak '18)

Useful to study impact of wide-angle RSD and feasibility to detect relativistic effects at large scales (e.g., Beutler et al.'18; Alam et al.'17)