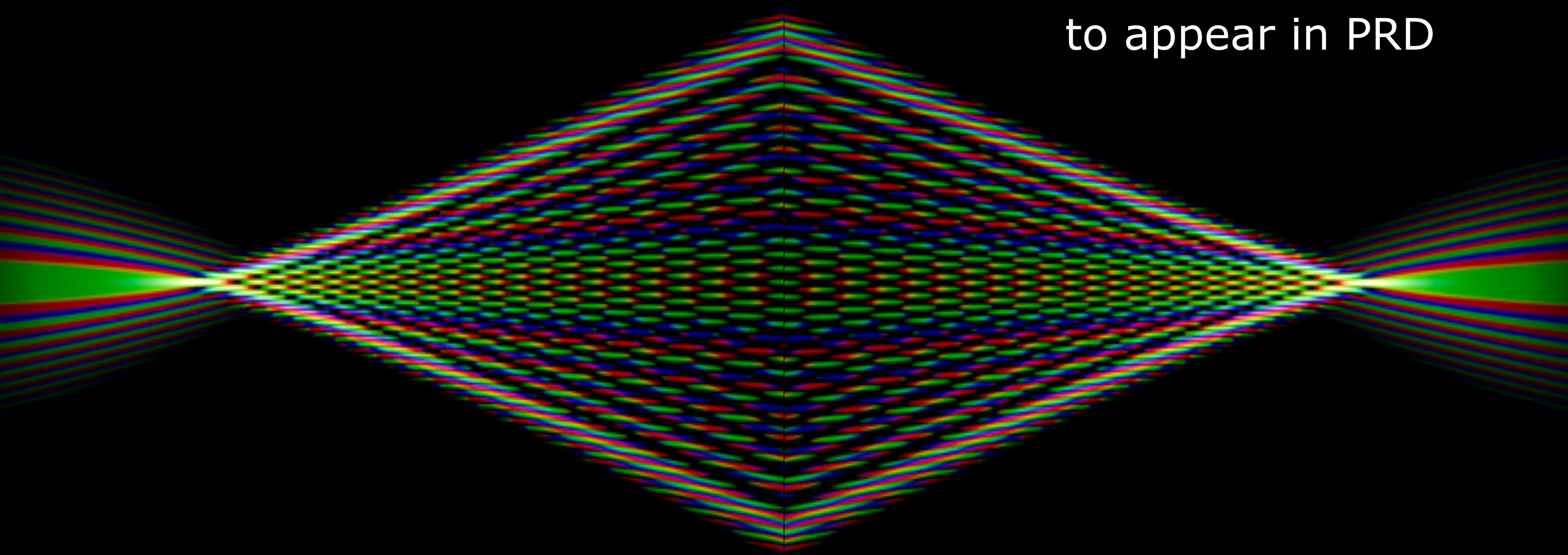
Semiclassical path(s) to large-scale structure

arXiv: 1812.05633

to appear in PRD



Cora Uhlemann

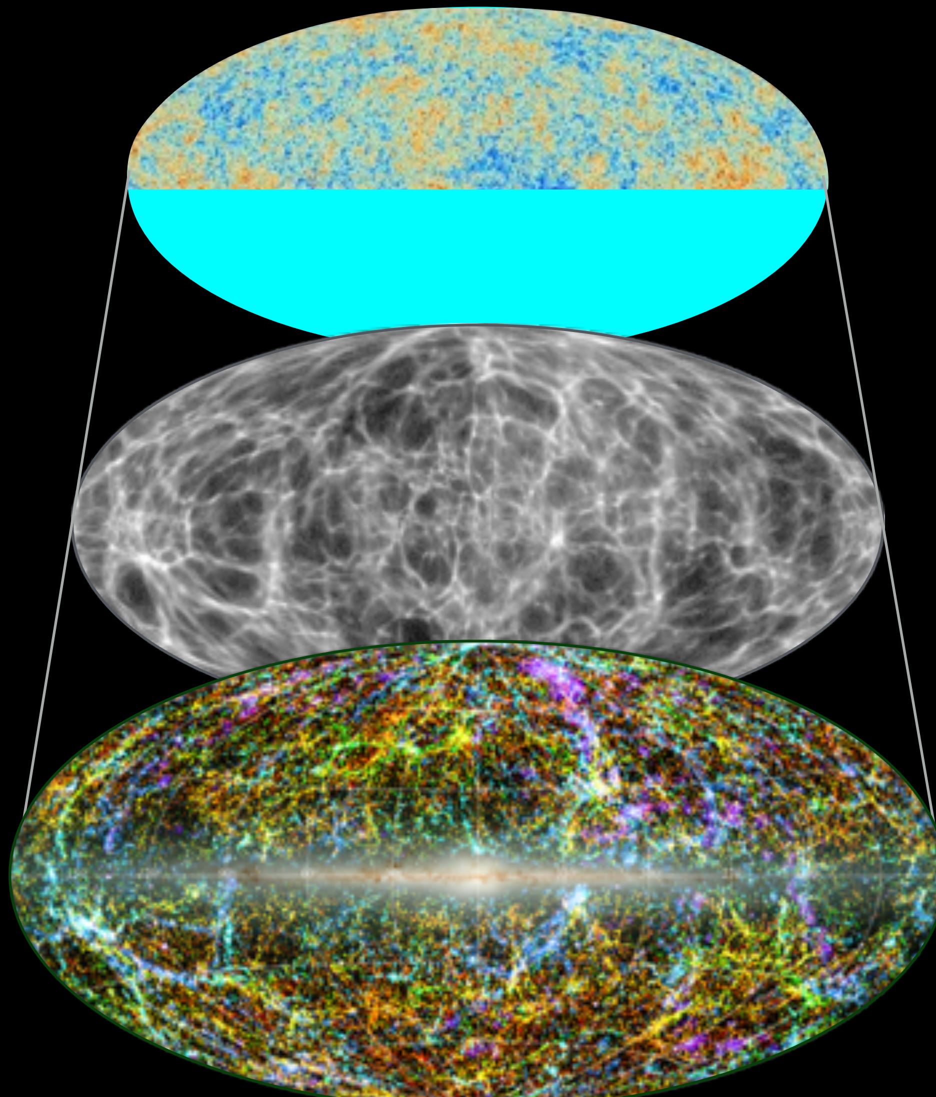
DAMTP & Fitzwilliam College

UNIVERSITY OF CAMBRIDGE

with Oliver Hahn, Cornelius Rampf & Mateja Gosenca

Cosmic Laboratory

beginning
nearly uniform



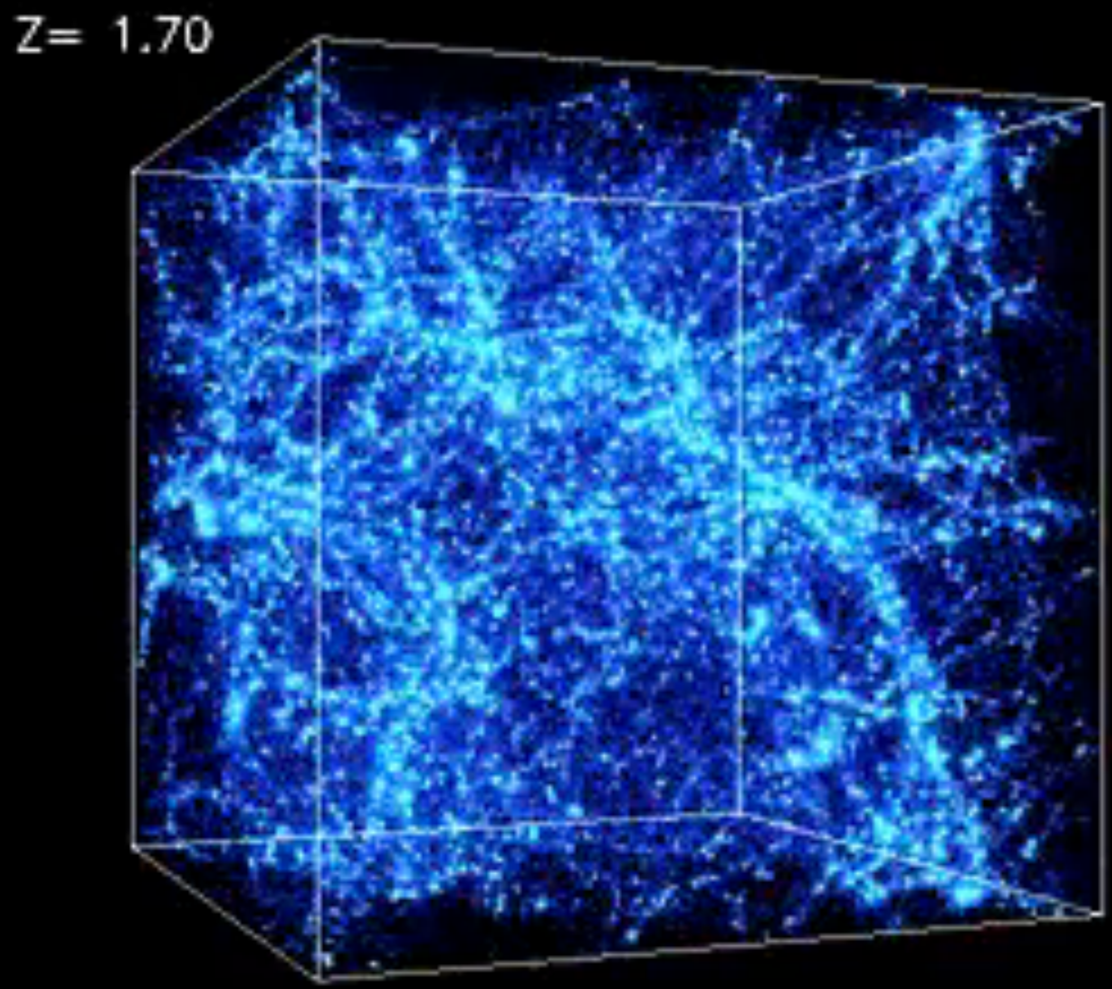
AFTERGLOW
early universe

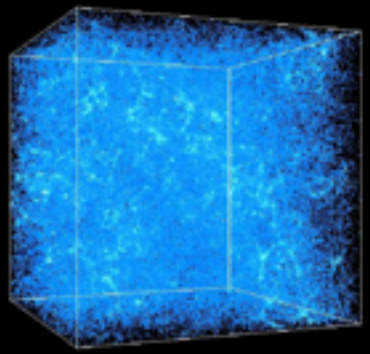
LARGE-SCALE STRUCTURE
Dark Matter

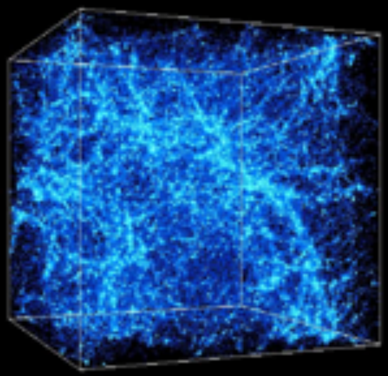
today
rich structure

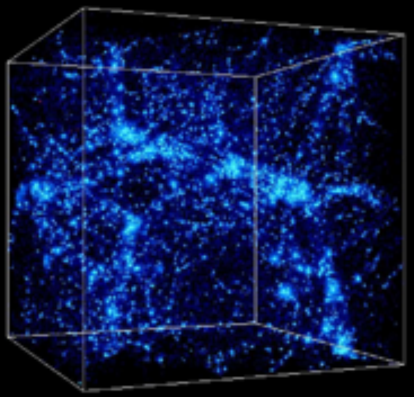
COSMIC WEB
galaxies

Matter Clustering









National Center for Supercomputer Applications (A. Kravtsov & A. Klypin)

Challenges

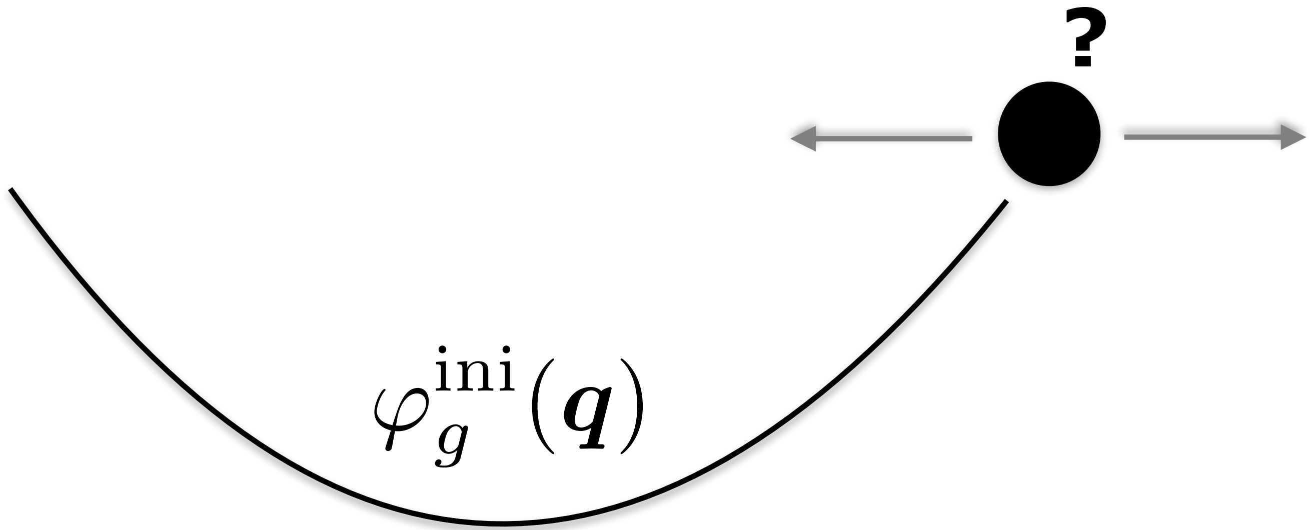
numerical	analytical
N particles	2 fields
large-scales	small-scales
limited power	limited accuracy
limited sampling	limited features

Classical Dynamics

Approximate: Shoot Particles

follow initial gravitational potential

$$\boldsymbol{v}(\boldsymbol{q}, a) = -\boldsymbol{\nabla} \varphi_g^{\text{ini}}(\boldsymbol{q})$$



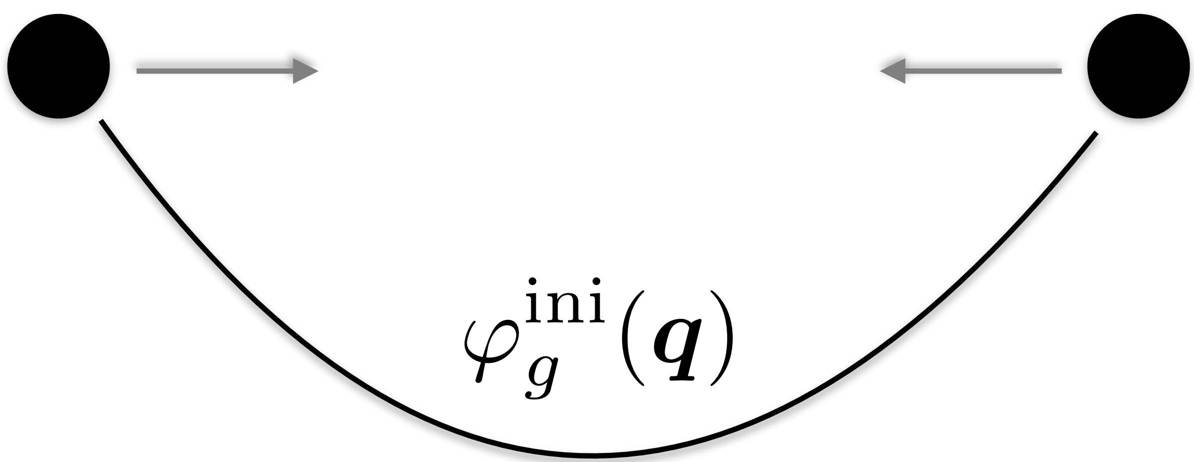
Classical Dynamics

Approximate: Shoot Particles

follow initial gravitational potential

$$\boldsymbol{v}(\boldsymbol{q},a) = -\boldsymbol{\nabla}\varphi_g^{\mathrm{ini}}(\boldsymbol{q})$$

$$\boldsymbol{x}(\boldsymbol{q},a) = \boldsymbol{q} - a\boldsymbol{\nabla}\varphi_g^{\mathrm{ini}}(\boldsymbol{q})$$



Classical Dynamics

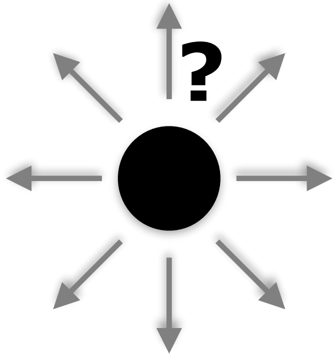
Approximate: Shoot Particles

follow initial gravitational potential

$$\boldsymbol{v}(\boldsymbol{q},a) = -\boldsymbol{\nabla}\varphi_g^{\mathrm{ini}}(\boldsymbol{q}) - a\boldsymbol{\nabla}\varphi_{\mathrm{tidal}}^{\mathrm{ini}}(\boldsymbol{q})$$

$$\boldsymbol{x}(\boldsymbol{q},a) = \boldsymbol{q} - a\boldsymbol{\nabla}\varphi_g^{\mathrm{ini}}(\boldsymbol{q}) - \frac{a^2}{2}\boldsymbol{\nabla}\varphi_{\mathrm{tidal}}^{\mathrm{ini}}(\boldsymbol{q})$$

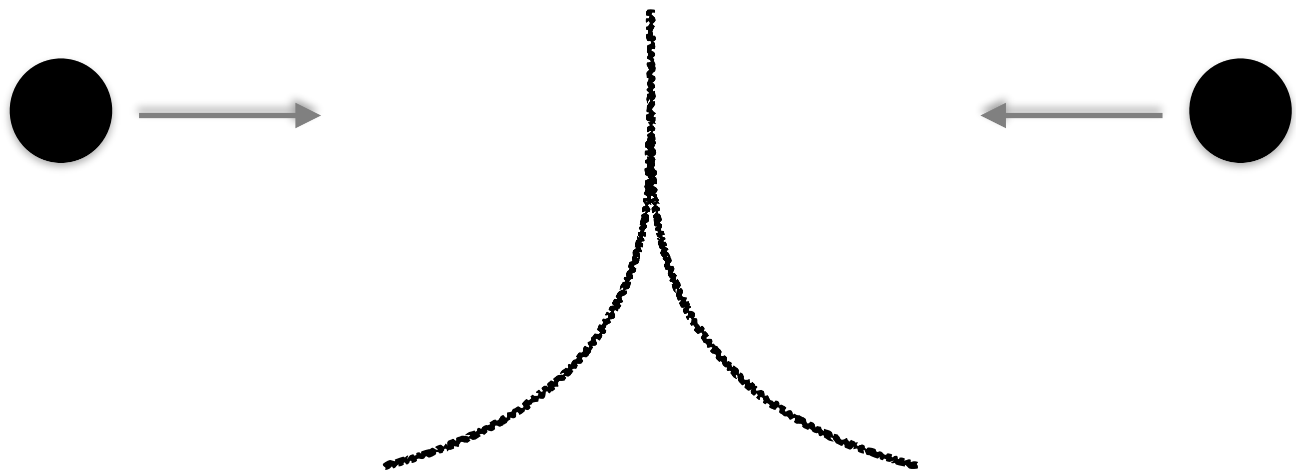
Zel'dovich 2LPT



Classical Dynamics

Problem: Overshooting

shell-crossing: singular Euledensity



no comeback after fly-through

large scale impact: Zvonimir Vlah
LPT @ shell-crossing: Shohei Saga

Classical Dynamics

Free Propagation

classical action: displacement × velocity

$$S_0(\boldsymbol{x}, \boldsymbol{q}, a) = \frac{1}{2}(\boldsymbol{x} - \boldsymbol{q}) \cdot \frac{\boldsymbol{x} - \boldsymbol{q}}{a}$$

background expansion

Semiclassical Dynamics

Translate Free Propagation

transition amplitude

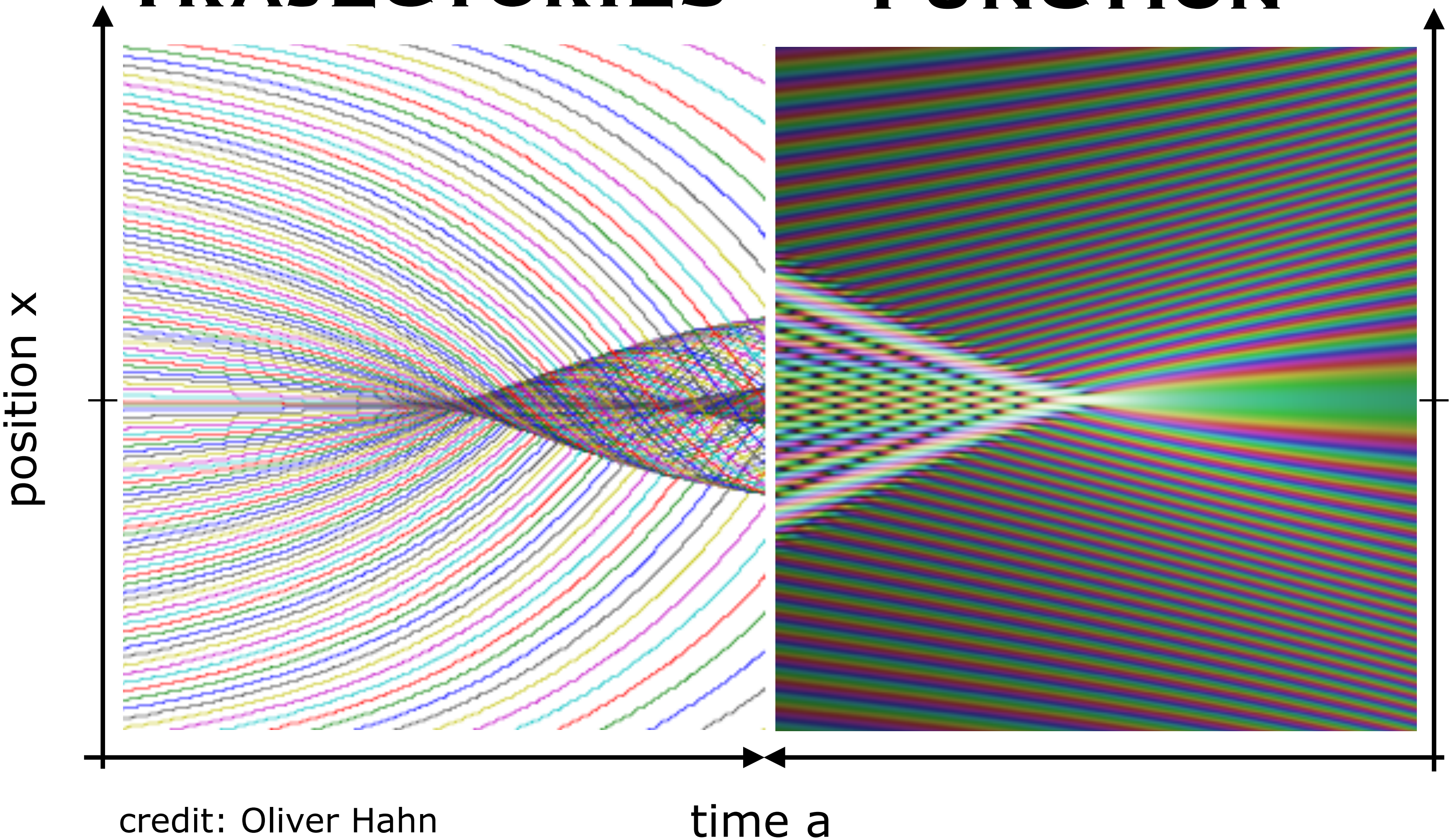
$$\psi_0(\boldsymbol{x}, a) = N \int d^3q \exp\left[\frac{i}{\hbar} S_0(\boldsymbol{x}, \boldsymbol{q}, a)\right] \psi_0^{\mathrm{ini}}(\boldsymbol{q})$$

Schrödinger equation

$$i\hbar\partial_a\psi_0 = -\frac{\hbar^2}{2}\nabla^2\psi_0$$

Semiclassical Dynamics

Particle trajectories | Free Wave function



credit: Oliver Hahn

Semiclassical Dynamics

Propagation with Interaction

$$i\hbar\partial_a\psi = -\frac{\hbar^2}{2}\nabla^2\psi + V_{\rm eff}(\boldsymbol{x}, a)\psi$$

↑

fluid

$$V_{\rm eff} = \frac{3}{2a}\left(\varphi_g - \phi_v\right)$$

$$V_{\rm eff}^{(2)} = \frac{3}{7}\nabla^{-2}\left[\left(\nabla^2\varphi_{\rm g}^{\rm (ini)}\right)^2 - \left(\nabla_i\nabla_j\varphi_{\rm g}^{\rm (ini)}\right)^2\right]$$

2SPT: tidal

Semiclassical Dynamics

Propagator PT: 2PPT

$$i\hbar\partial_a\psi = -\frac{\hbar^2}{2}\nabla^2\psi + V_{\rm eff}^{(2)}\psi$$

solve: free propagator x $\exp\left(\frac{i}{\hbar}S_{\rm tid}\right)$

$$S_{\rm tid} \simeq -\frac{a}{2}\left[V_{\rm eff}^{(2)}(\boldsymbol{q}) + V_{\rm eff}^{(2)}(\boldsymbol{x})\right]$$

Classical Observables

Propagator gone Lagrangian

phase-space $\bar{f}_W[\psi, \hbar \to 0]$

→ displacement: 2LPT

Classical Observables

Propagator gone Lagrangian

phase-space $\bar{f}_W[\psi, \hbar \to 0]$

→ velocity: beyond 2LPT

$$\boldsymbol{v}(\boldsymbol{q}) = -\boldsymbol{\nabla}\varphi_g^{(\mathrm{ini})} - a\boldsymbol{\nabla}V_{\mathrm{eff}}^{(2)}$$

$$+\frac{a^2}{2}\boldsymbol{\nabla}\nabla V_{\mathrm{eff}}^{(2)} \cdot \nabla\varphi_g^{(\mathrm{ini})}$$

vorticity conserver

Classical Observables

Vorticity Conservation

Eulerian $\nabla_x \times \boldsymbol{v} = 0$

pre-shell-crossing

Classical Observables

Vorticity Conservation

Lagrangian: Cauchy invariants

$$\varepsilon_{ijk} x_{l,j} \dot{x}_{l,k} = 0$$

2LPT

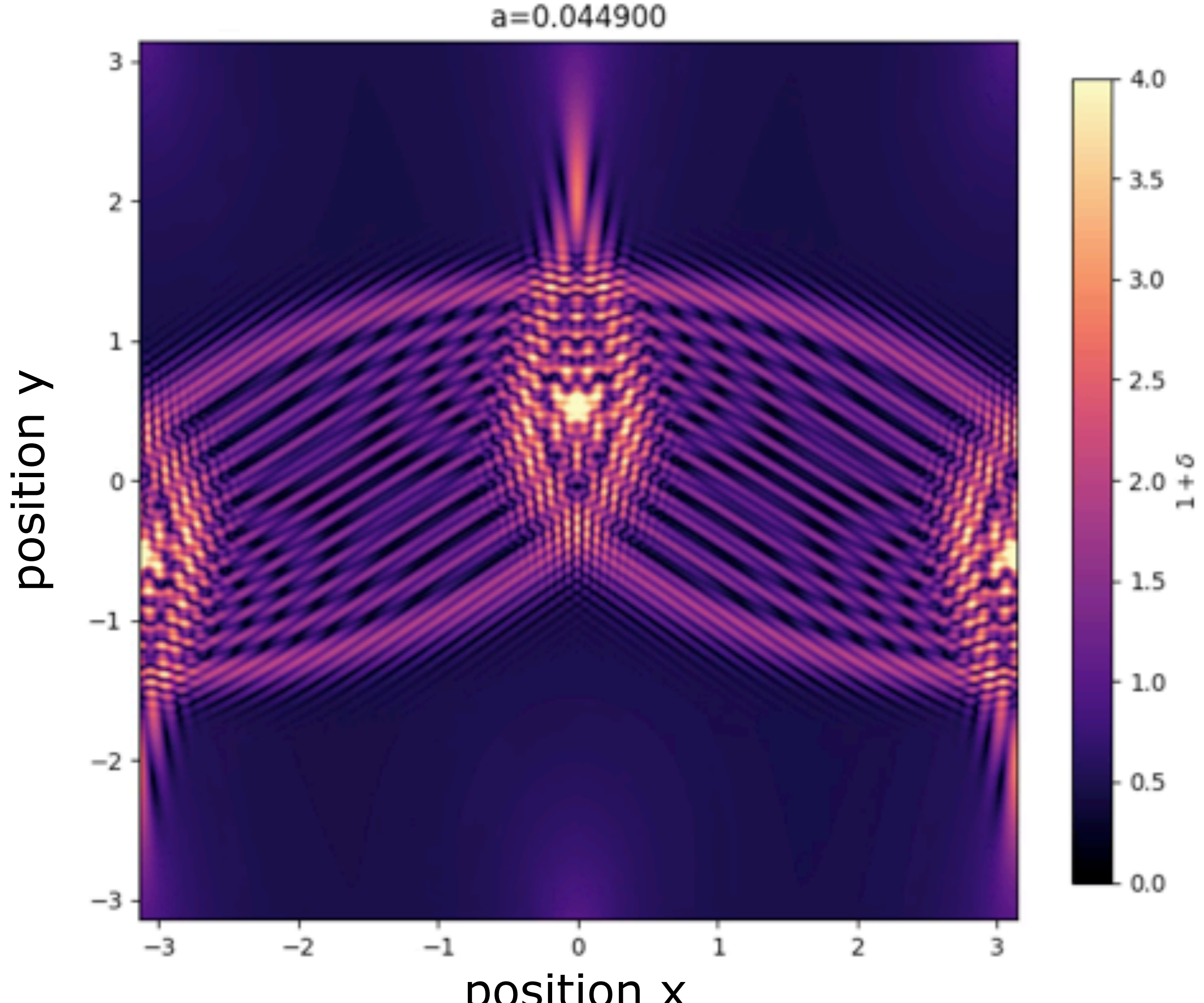
$$= \mathcal{O}(a^2)$$

2PPT

$$= \mathcal{O}(a^3)$$

Phased Wave Example

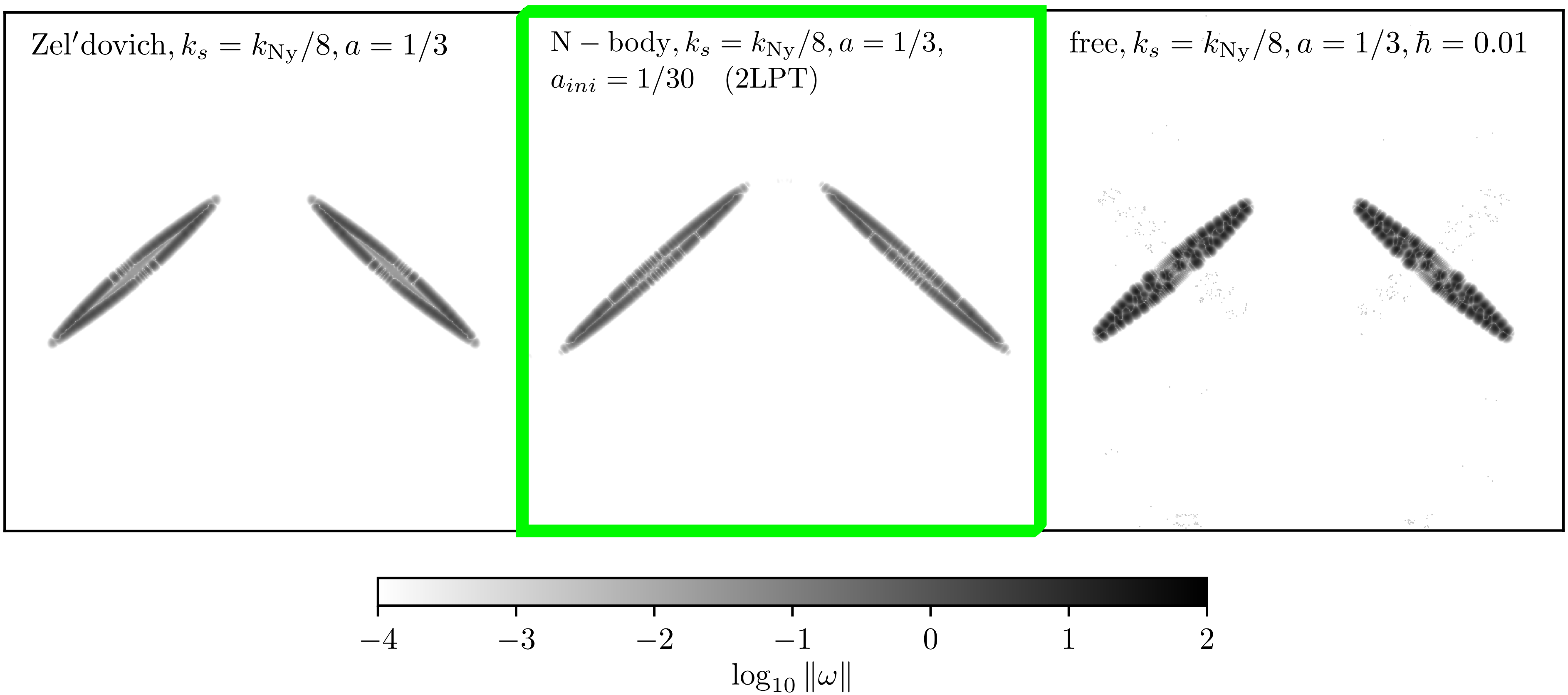
density $1 + \delta(\boldsymbol{x}, a) = |\psi|^2$



Phased Wave Example

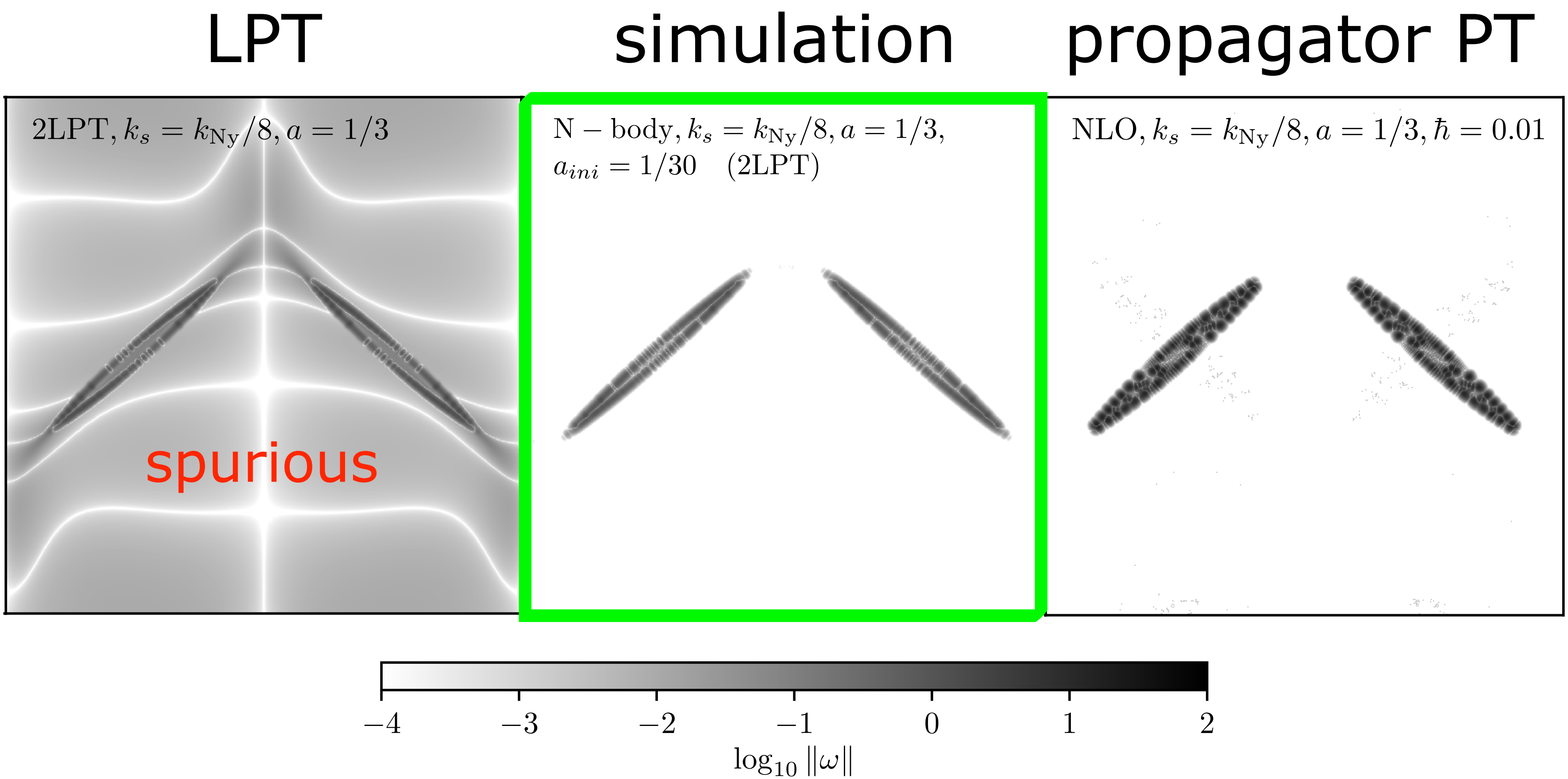
vorticity generation

LPT | simulation | propagator PT



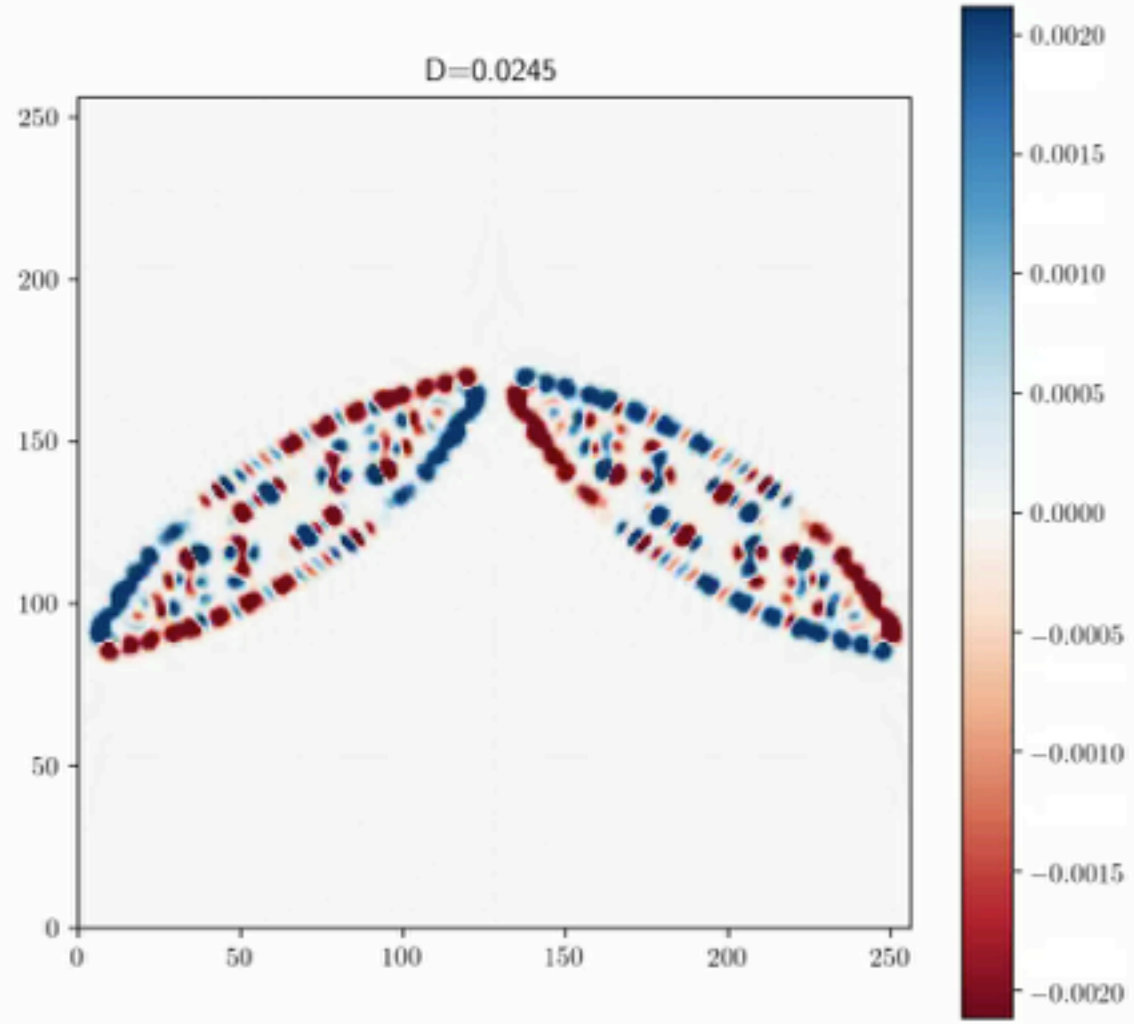
Phased Wave Example

vorticity generation



Phased Wave Example

vorticity from topological defects



Conclusion

Large-scale structure = cosmic laboratory

Challenge: nonlinear dynamics

goal: Zeldovich + tidal + long-term limit

Tool: semiclassical physics

classical action → free propagator

→ free Schrödinger equation

add potential: tidal effects (+long-term?)

new PT avoids spurious vorticity