Random Galilean Invariance in Renormalised Formalisms

Martin Crocce

Institute for Space Science, Barcelona

Román Scoccimarro, Diego Blas, Ariel Sánchez, Andrea Pezzotta

Nonlinear Gravitational Clustering

scales much smaller than the Horizon (Hubble radius) ⟶ Newtonian gravity

scales larger than strong clustering regime ⟶ *single stream approximation*

no velocity dispersion or pressure
(prior to virialization and shell crossing)

$$\nabla^2\Phi(\mathbf{x},\tau) = \frac{3}{2}\Omega_m(\tau)\,\mathcal{H}^2(\tau)\,\delta(\mathbf{x},\tau)$$

$$\frac{\partial\delta(\mathbf{x},\tau)}{\partial\tau} + \nabla\cdot\{[1+\delta(\mathbf{x},\tau)]\,\mathbf{u}(\mathbf{x},\tau)\} = 0$$

$$\frac{\partial\mathbf{u}(\mathbf{x},\tau)}{\partial\tau} + \mathcal{H}(\tau)\,\mathbf{u}(\mathbf{x},\tau) + \mathbf{u}(\mathbf{x},\tau)\cdot\nabla\mathbf{u}(\mathbf{x},\tau) = -\nabla\Phi(\mathbf{x},\tau) - \frac{1}{\rho}\nabla_j(\rho\,\sigma_{ij})$$

we assume the stress tensor is zero otherwise it generates corrections $\sim k^2 P_{\rm Linear}$

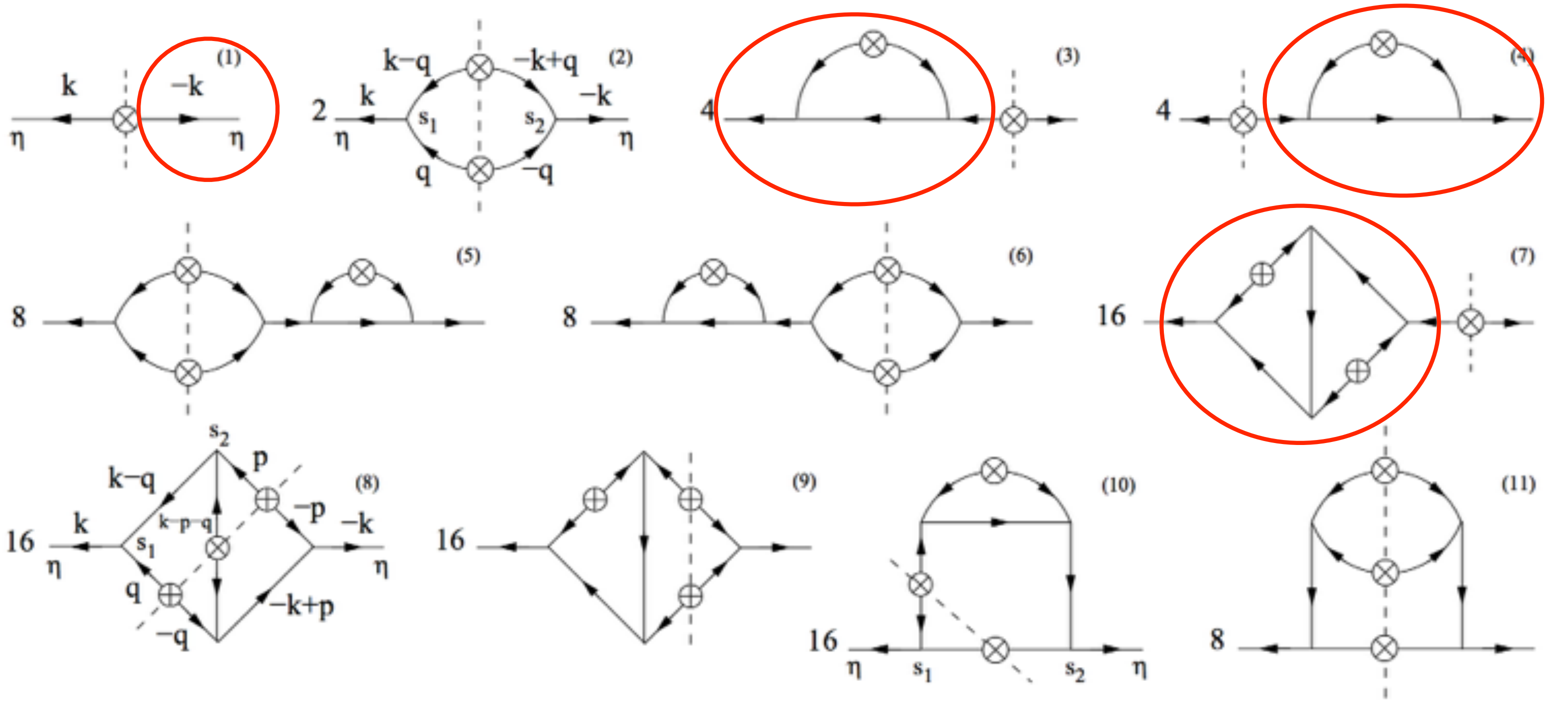
velocity field can be assumed irrotational $\theta(\mathbf{x},\tau) \equiv \nabla\cdot\mathbf{u}(\mathbf{x},\tau)$

$$\frac{\partial\tilde\delta(\mathbf{k},\tau)}{\partial\tau} + \tilde\theta(\mathbf{k},\tau) = -\int d^3k_1 d^3k_2\,\delta_{\rm D}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\,\alpha(\mathbf{k}_1,\mathbf{k}_2)\,\tilde\theta(\mathbf{k}_1,\tau)\,\tilde\delta(\mathbf{k}_2,\tau),$$

$$\frac{\partial\tilde\theta(\mathbf{k},\tau)}{\partial\tau} + \mathcal{H}(\tau)\tilde\theta(\mathbf{k},\tau) + \frac{3}{2}\Omega_m(\tau)\mathcal{H}^2(\tau)\tilde\delta(\mathbf{k},\tau) = -\int d^3k_1 d^3k_2\,\delta_{\rm D}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\beta(\mathbf{k}_1,\mathbf{k}_2)\,\tilde\theta(\mathbf{k}_1,\tau)\,\tilde\theta(\mathbf{k}_2,\tau)$$

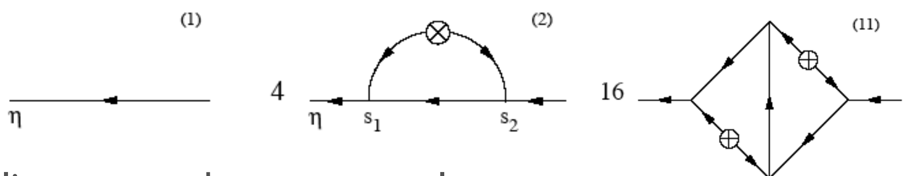
final den or vel field

Power Spectrum : $\langle \Psi(k,a)\Psi(-k,a)\rangle$



all diagrams of this type are systematically put together

$\Gamma^{(1)}(k,z) =$

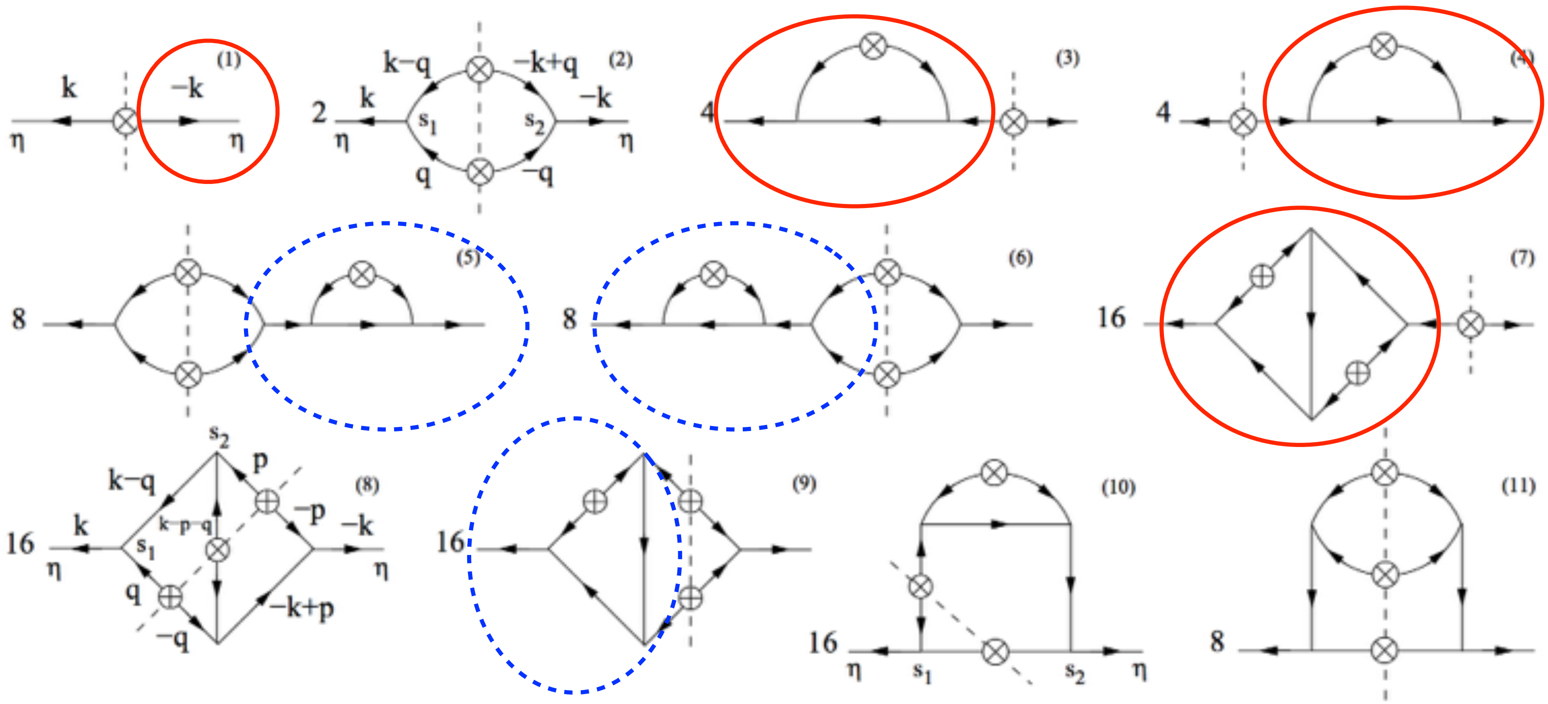


$P_{\delta\delta}(k) = \left[\Gamma^{(1)}_{\delta\delta}(k,z)\right]^2 P_0(k) +$

$$\delta_{\rm D}(k-k_1)\Gamma^{(1)}_a(k,z)P_0(k) = \langle \Psi_a(k,z)\delta_0(-k_1)\rangle$$

- its the cross-correlation with ICs
- it can be measured in n-body

Power Spectrum : $\langle \Psi(k,a)\Psi(-k,a)\rangle$



all diagrams of this type are systematically put together

$$2\,\delta_D(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\;\;\Gamma_a^{(2)}(\mathbf{k}_1,\mathbf{k}_2)P_0(k_1)P_0(k_2),\quad = \langle \Psi_a(\mathbf{k})\delta_0(-\mathbf{k}_1)\delta_0(-\mathbf{k}_2)\rangle$$

$$\rightarrow\quad P_{\delta\delta}(k) = \left[\Gamma^{(1)}_{\delta\delta}(k,z)\right]^2 P_0(k) + 2\int d^3q\left[\Gamma^{(2)}_{\delta\delta}(\mathbf{k}-\mathbf{q},\mathbf{q},z)\right]^2 P_0(|\mathbf{k}-\mathbf{q}|)P_0(q) \quad + ..$$

Multi-point propagator expansions (MPP, .. Bernardeau, Crocce & Sccocimarro 2011)

Resummation of IR-modes

$$\Gamma^{(1)}(k,z) = D(z)\left(1 + \frac{P_{13}}{2P_0} + \frac{P_{15}}{2P_0} + \frac{P_{17}}{2P_0} + \dots\right)$$

It is possible to show that when you consider the high k limit, or in other words the contribution to these integrals of modes q << k (IR-modes) the diagrams simplify to

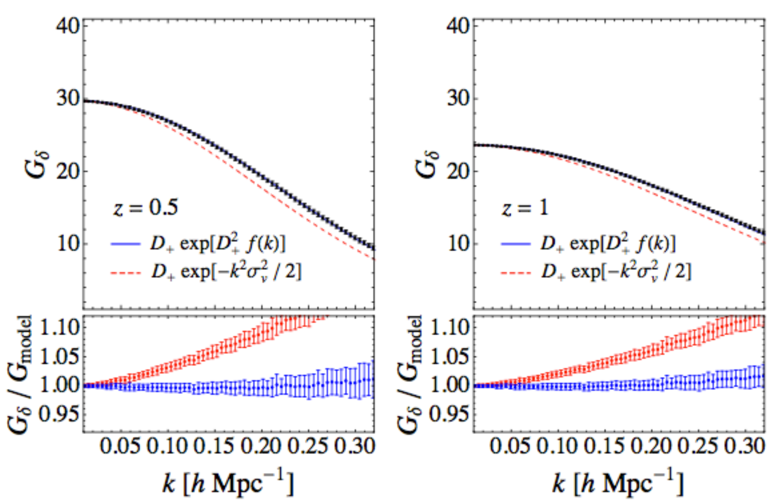
$$n\,\text{loops} \sim \frac{1}{n!}\left(-\frac{k^2\sigma_v^2}{2}\right)^n \xrightarrow{\text{high-k}} \Gamma_\delta^{(1)}(k,z) \approx D(z)\exp(-k^2\sigma_v^2/2)$$

$$\sigma_v^2 = (4\pi/3)\int P(q)/q^2 d^3q$$

This is the variance of the displacement field, its dominated by large scale flow (~ 6Mpc/h)

On very large scales we can use PT to compute corrections (~ P_{13})

$$\xrightarrow{\text{low-k}} \Gamma_\delta^{(1)}(k,z) \approx D(z) - f(k)D^3(z) + \dots$$



Different ansatz for MPP lead to different prescriptions

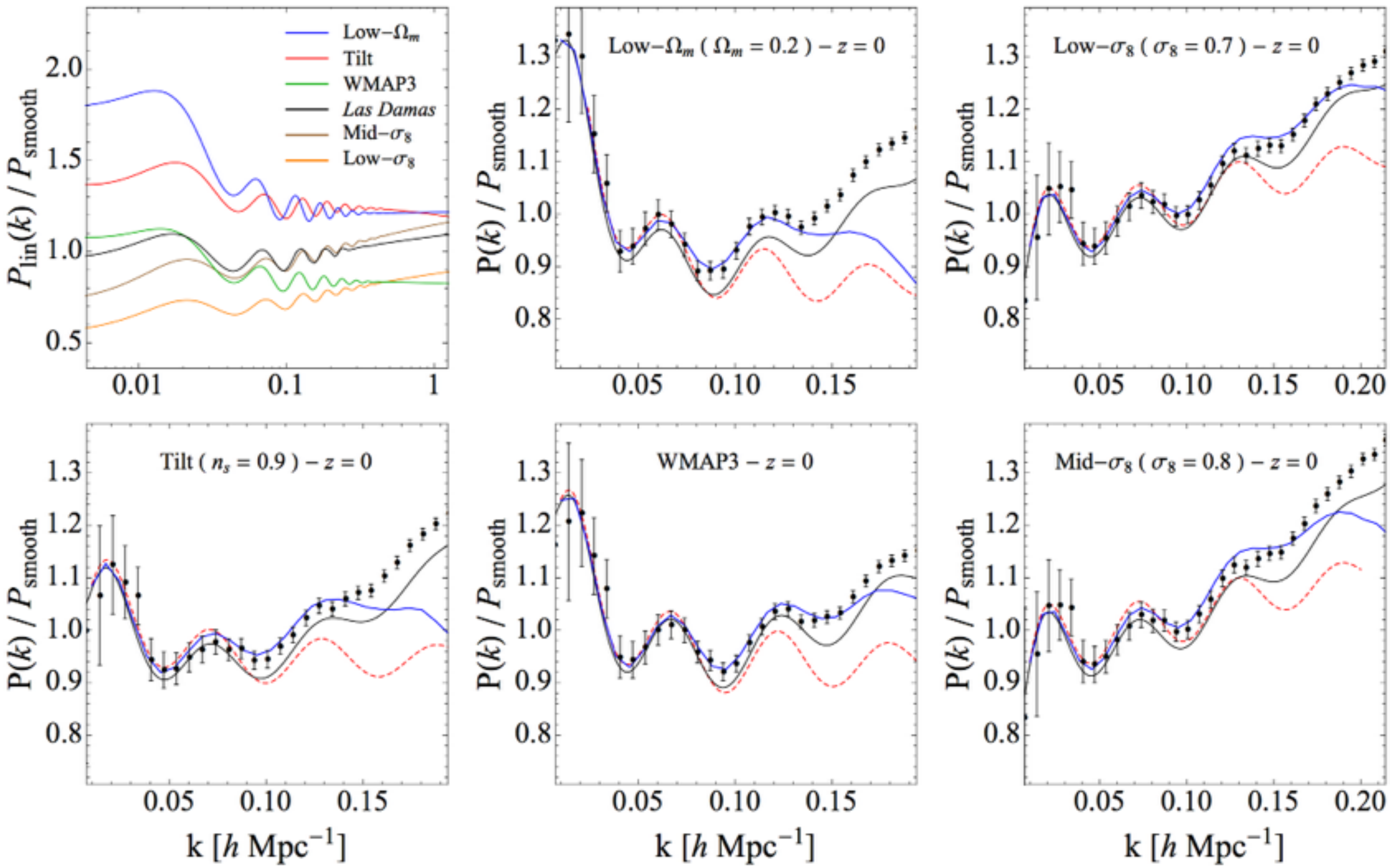
$$\Gamma_\delta^{(1)}(k,z) = D(z)\exp(f(k)D^2(z)) \qquad \textbf{MPT breeze}$$

$$\Gamma_\delta^{(1)} = D(z)\left(1 + \frac{P_{13}}{2P_L} + \frac{k^2\sigma_v^2}{2}\right)\exp(-k^2\sigma_v^2/2) \qquad \textbf{RegPT}$$

.. and similarly for the higher-order propagators.

Power Spectrum (MPTbreeze)

performance for different cosmological models at z = 0 (dedicated sims)

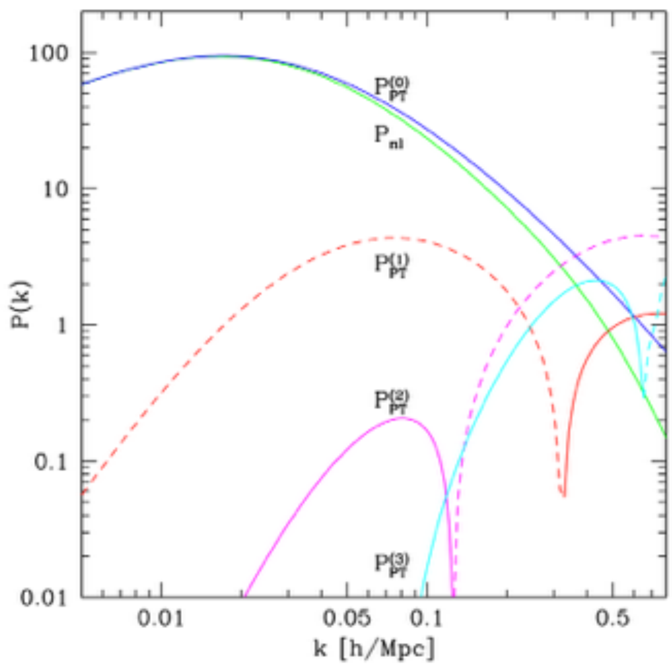


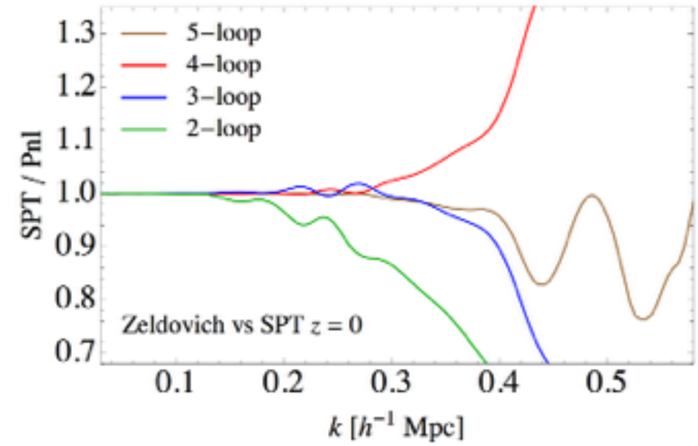
only few seconds of evaluation time

CODES PUBLICLY AVAILABLE

SPT

back to Zeldovich

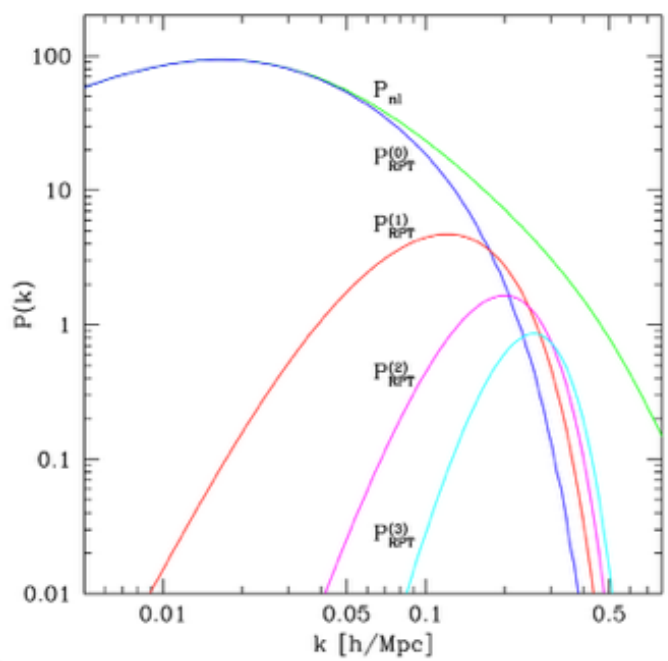


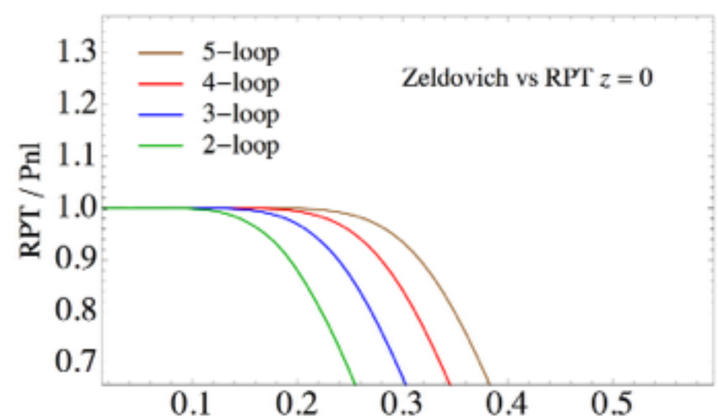


$$P(k) = P_0 + P_{1-loop} + P_{2-loop} + ..$$

- Each term is Galilean Invariant (built of equal time correlators)

RPT



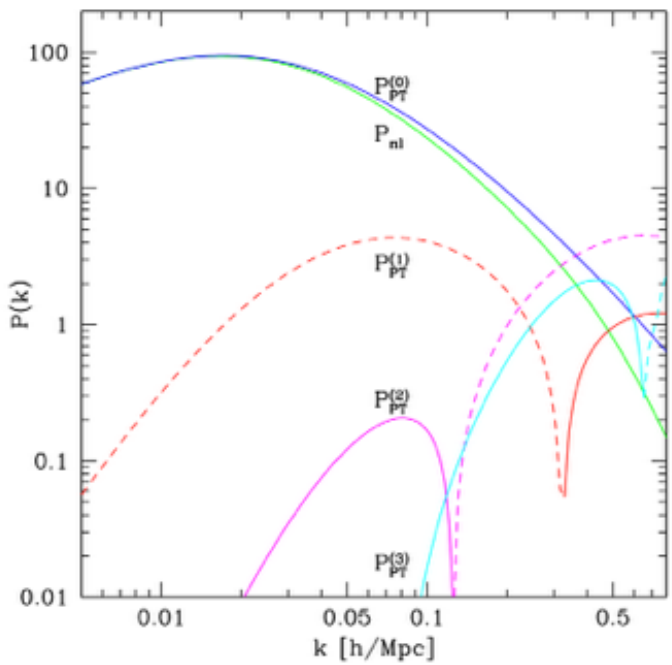


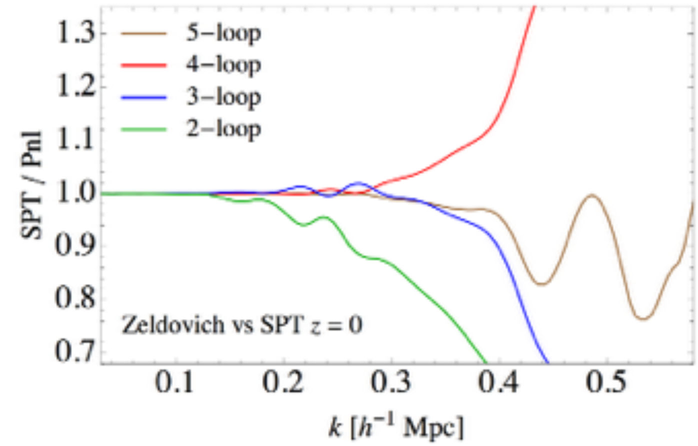
- **Since we only resumed propagator modes, we broke the symmetry**

$$P(k) = \exp(-k^2\sigma_v^2)\left[P_0 + P_{1\ell}^{\rm MC} + P_{2\ell}^{\rm MC}\right]$$

SPT

back to Zeldovich

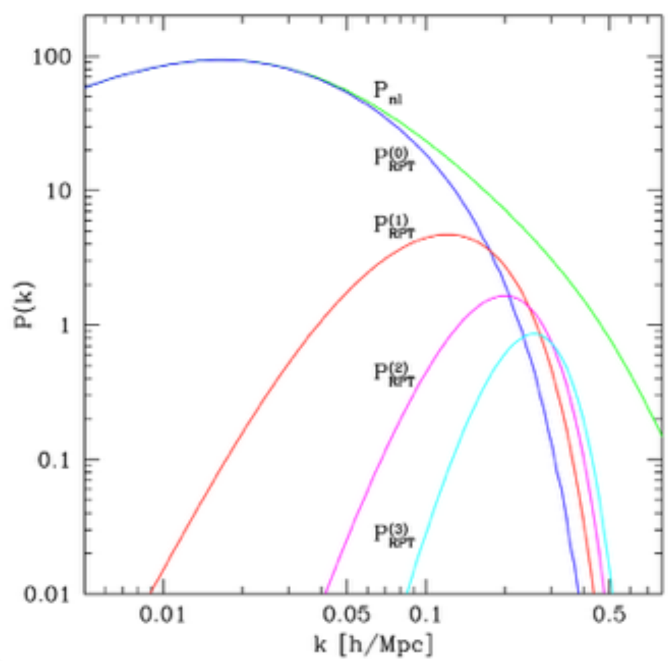


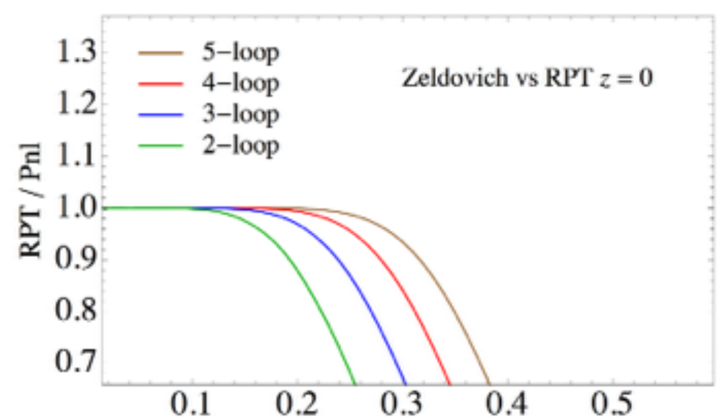


$$P(k) = P_0 + P_{1-loop} + P_{2-loop} + ..$$

- Each term is Galilean Invariant (built of equal time correlators)

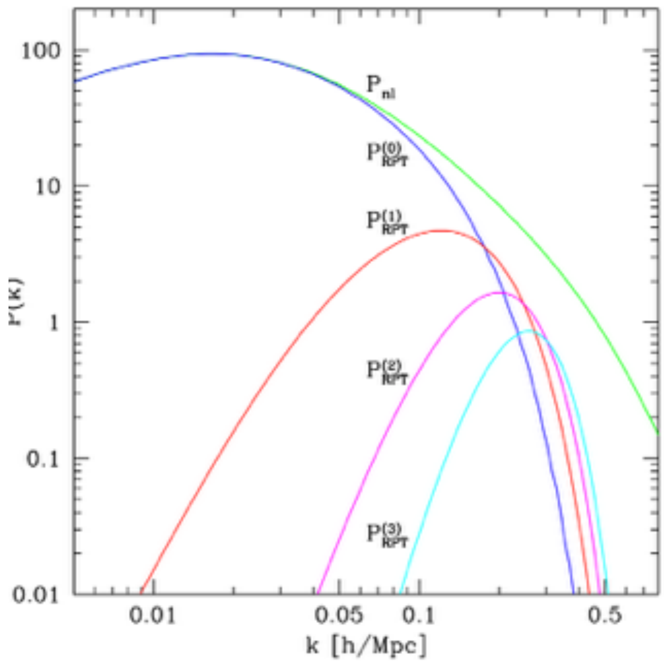
RPT

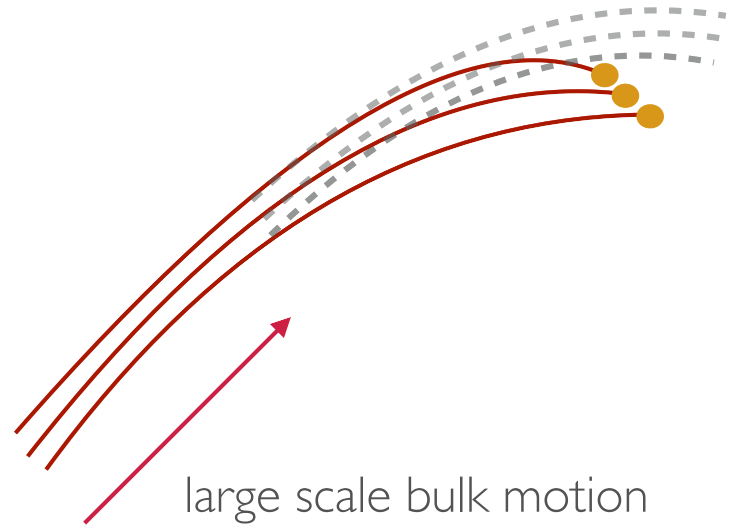




$$P(k) = \exp(-k^2\sigma_v^2)\left[P_0 + P_{1\ell}^{\rm MC} + P_{2\ell}^{\rm MC}\right]$$

- Non-perturbative, non GI
- Perturbative (basically P_{22}, P_{33}) non GI
- Convergence improves (towards large-scales)





- Using the (unequal time) cross-correlations with the initial conditions as basis is not optimal because the bulk flow displacements are large
- These induce strong "gaussian" damping towards high-k, we need many "loops" to restore power

Ideally we want to do RPT in a frame that is moving with the large-scale bulk flow to capture the motion of particles relative to it. Equal time correlations do not depend on uniform displacements (IR safe).

- This problem is related to the breaking of Galilean Invariance in RPT

Galilean Invariance

- Change of coordinates of the form :

$$x^i \mapsto \tilde{x}^i \equiv x^i - u^i T, \quad v^i \mapsto \tilde{v}^i(\tilde{x},\tau) \equiv v^i(x,\tau) - u^i(1 - \mathcal{H}T(\tau)),$$

- The fields change as :

$$T(\tau) \equiv \frac{1}{a}\int^{\tau} a(\tau')\mathrm{d}\tau'.$$

$$\delta(\mathbf{k},\tau) = \tilde{\delta}(\mathbf{k},\tau)e^{i\mathbf{k}\,\mathbf{u}\,T}, \quad v^j(\mathbf{k},\tau) = \tilde{v}^j(\mathbf{k},\tau)e^{i\mathbf{k}\,\mathbf{u}\,T} - u^j(1 - \mathcal{H}T)\delta^{(3)}(\mathbf{k}).$$

- And the velocity variance as : $-k^2\sigma_v^2 \mapsto -k^2\tilde{\sigma}_v^2 - \langle(\mathbf{k}\,\mathbf{u}\,T)^2\rangle$

- The perturbative PT terms as :

$$P_{13} \to P_{13} - \langle(\mathbf{k}\,\mathbf{u}\;T)^2\rangle P_L$$

$$P_{22} \to P_{22} + \langle(\mathbf{k}\,\mathbf{u}\;T)^2\rangle P_L$$

- Equal time correlators are invariant under GT :

$$\langle\delta(\mathbf{k},\tau)\delta(\mathbf{k}',\tau)\rangle = \delta^{(3)}(\mathbf{k}+\mathbf{k}')P_k(\tau).$$

Some qualitative thoughts:

- Our motivation: Find a transformation to field variables that are more efficient for re-summations (or a reference frame).
- We will introduce a family of Random Galilean Transformations (RGT). Since we know how the “resumed propagator” transforms, we look for a Mode-Coupling transformation to counter-act it.
- The Galilean Transformations will be controlled by a stochastic random field, assumed Gaussian. This field will be uniform for each k-value (or in other words we demand that it can only have structure on scales larger than the one we are looking at).
- The Random GT is hence controlled by the variance of this field

The approach in practice:

- We will link this variance to the dynamics of the system in such a way that in practice our final PT expressions will be GI
- Any observable should be independent of this transformation if computed fully non-perturbative. A finite calculation might show some dependence with the GRF.
- Hence we choose the Random Galilean Transformation in a way that, at the perturbative order we are working, the dependence of the observable, e.g. P(k), with the RGT is minimised
- Our intention is that this prescription will bring us closer to the true answer (something that can be show explicitly with ZA).

Let's see how this works in the Zeldovich approximation

$$P(k) = \int \frac{d^3r}{(2\pi)^3}\, e^{i\mathbf{k}\cdot\mathbf{r}} \left[\langle e^{i\mathbf{k}\cdot\Delta\Psi}\rangle - 1\right],$$

$$\mathbf{r} \equiv \mathbf{q} - \mathbf{q}',\ \Delta\Psi \equiv \Psi(\mathbf{q}) - \Psi(\mathbf{q}')$$

displacement field

For ZA : $\Psi = -i(\mathbf{k}/k^2)\delta_L$

$$\langle e^{i\mathbf{k}\cdot\Delta\Psi}\rangle = e^{-\frac{1}{2}\langle(\mathbf{k}\cdot\Delta\Psi)^2\rangle}$$

$$P(k) = \int \frac{d^3r}{(2\pi)^3}\, e^{i\mathbf{k}\cdot\mathbf{r}} \left[e^{-[k^2\sigma_v^2 - I(\mathbf{k},\mathbf{r})]} - 1\right],$$

we pulled out the one-point cumulant (only 2nd order)

vel. field spectrum

$$I(\mathbf{k},\mathbf{r}) \equiv \int d^3q\, \frac{(\mathbf{k}\cdot\mathbf{q})^2}{q^2} \cos(\mathbf{q}\cdot\mathbf{r}) \frac{P(q)}{q^2}$$

$$\sigma_v^2 = I(k,0)/k^2$$

Under a change of the large-scale (non-uniform) velocity field

$$\sigma_v^2 \to \sigma_v^2 + \sigma_u^2$$

$$I \to I + k^2\sigma_u^2$$

If **u** only has structure on large scales (uniform on scales we care) **q << r** then **q r << 1** and σ_u^2 factors out of the I and sig integrals

Zeldovich approximation

The approach we discussed before basically does,

$$P(k) = \int \frac{d^3r}{(2\pi)^3}\, e^{i\mathbf{k}\cdot\mathbf{r}}\, e^{-k^2\sigma_v^2} \sum_{n=1}^{\infty} \frac{[I(\mathbf{k},\mathbf{r})]^n}{n!}$$

And you see how GI is explicitly broken

Let's try to restore GI by imposing a field transformation

$$I \rightarrow I + k^2\sigma_u^2$$

$$\sigma_v^2 \rightarrow \sigma_v^2 + \sigma_u^2$$

$$P(k) = e^{-k^2\sigma_v^2 + I_*} \int \frac{d^3r}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} e^{I(\mathbf{k},\mathbf{r}) - I_*}$$

where $I_* \equiv -k^2\sigma_u^2$

$$P(k) = e^{-k^2\sigma_v^2 + I_*} \int \frac{d^3r}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} \sum_{n=1} \frac{(I(\mathbf{k},\mathbf{r}) - I_*)^n}{n!},$$

We can now pull out the propagator in the new variables, but the above expression will be invariant order by order

Zeldovich approximation

We can now expand this $P(k) = e^{-k^2\sigma_v^2 + I_*} \int \frac{d^3r}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} \sum_{n=1} \frac{(I(\mathbf{k},\mathbf{r}) - I_*)^n}{n!}$, recalling that $FT[I] = P_0$

$$P(k) = G_*^2 \Big[P_0 + (-I_* P_0 + \frac{1}{2} FT[I^2]) + (\frac{I_*^2}{2} P_0 - \frac{I_*}{2} FT[I^2] + \frac{1}{6} FT[I^3])\Big] + \dots$$

$$G_*^2(k) \equiv e^{-k^2\sigma_v^2 + I_*}$$

This is basically the variance of the RGT discussed before

You arrive at :

$$P(k,x) \; = \mathrm{e}^x \left[P_L(1 - x + x^2/2 - \dots) + P_{1L}(1 - x + \dots) + P_{2L} \dots\right]$$

$$x \equiv I_* - k^2\sigma_v^2$$

$$I_* \equiv -k^2\sigma_u^2$$

Since we do not want P(k) to depend on x we require $\partial P(k)/\partial x = 0$

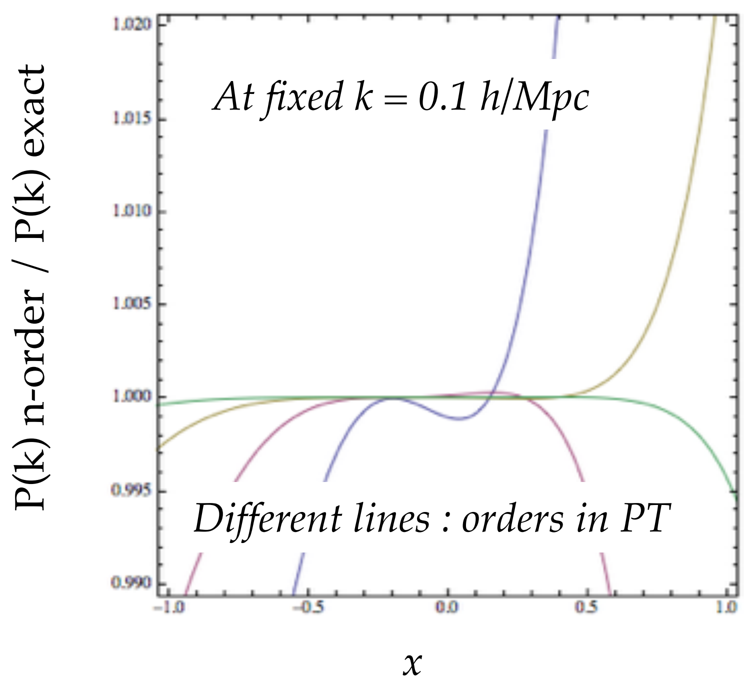
For example at 2nd order : $xP_0 = P_{1L} \pm \sqrt{P_{1L}^2 - 2P_0 P_{2L}}$

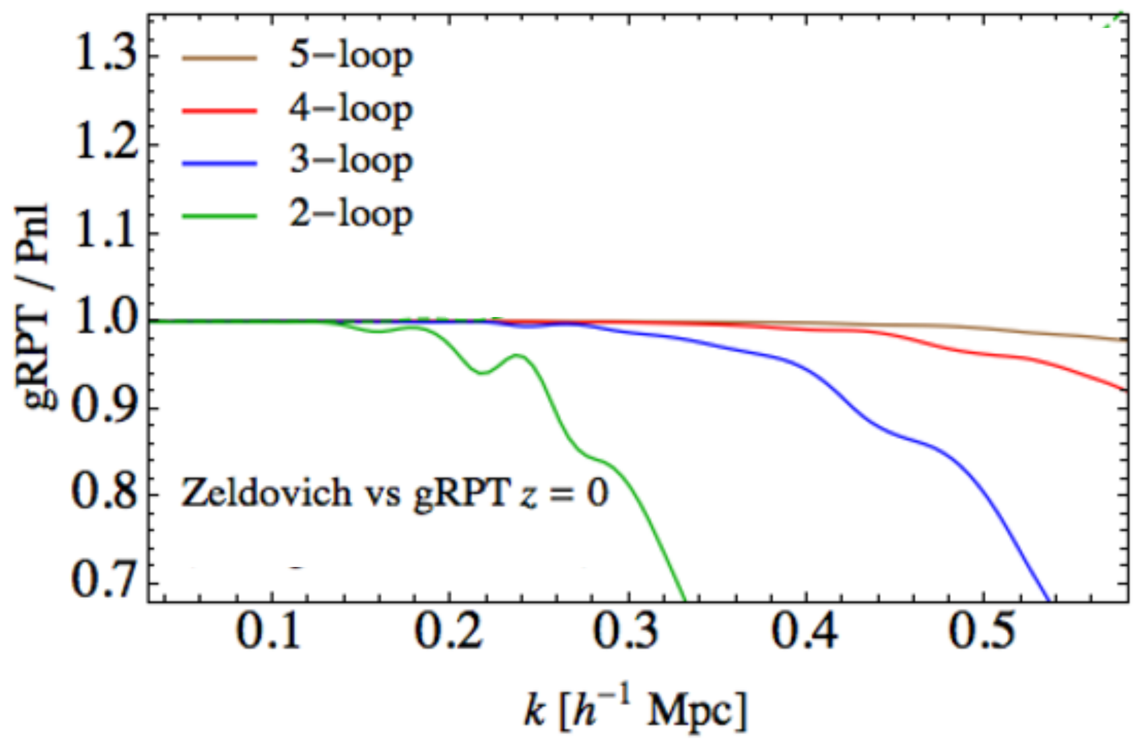
There is some arbitrariness on the root but that's ok

$$P(k) = e^x \left(P_0 + \sqrt{P_{1L}^2 - 2P_0 P_{2L}}\right)$$

At two loops for ZA

Zeldovich approximation





Since we do not want P(k) to depend on x we require

For the exact dynamics is similar: we need to choose an expression for the resumed propagator which we transform

gRPT for the exact dynamics.

For the exact dynamics is similar: we start from a expression for the resumed propagator which we transform. We use the RegPT expression as a starting point.

$$P(k) = e^{-k^2\sigma_v^2}\left(1 + \frac{P_{13}}{P_L} + k^2\sigma_v^2\right)(P_L + P_{22})$$

propagator in reg-PT form

Do the RGT

$$P(k) = e^{x}\left(1 + \frac{P_{13}}{P_L} + k^2\sigma_v^2\right)(P_L(1+x) + P_{22} - k^2\sigma_v^2 P_L)$$

express in terms of invariants

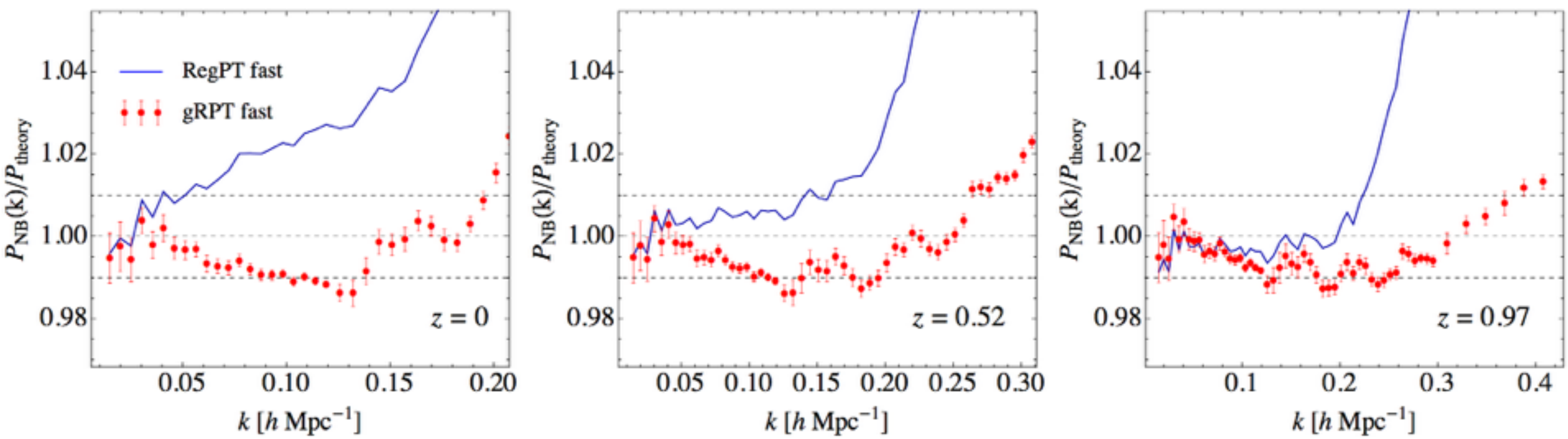
$$P(k,x) = e^{x}\left(1 + \frac{\delta P_{13}^{\rm inv}}{P_L}\right)(P_L(1+x) + P_{22}^{\rm inv})$$

Choose the appropriate value of x (the *boost*) to trace the large-scale flows.

$$\delta P_{13}^{\rm inv} = P_{13} + k^2\sigma_v^2 P_L \qquad P_{22}^{\rm inv} = P_{22} - k^2\sigma_v^2 P_L$$

Comparison to RegPT / MPTbreeze

(the other "state-of-the-art" fast algorithm)



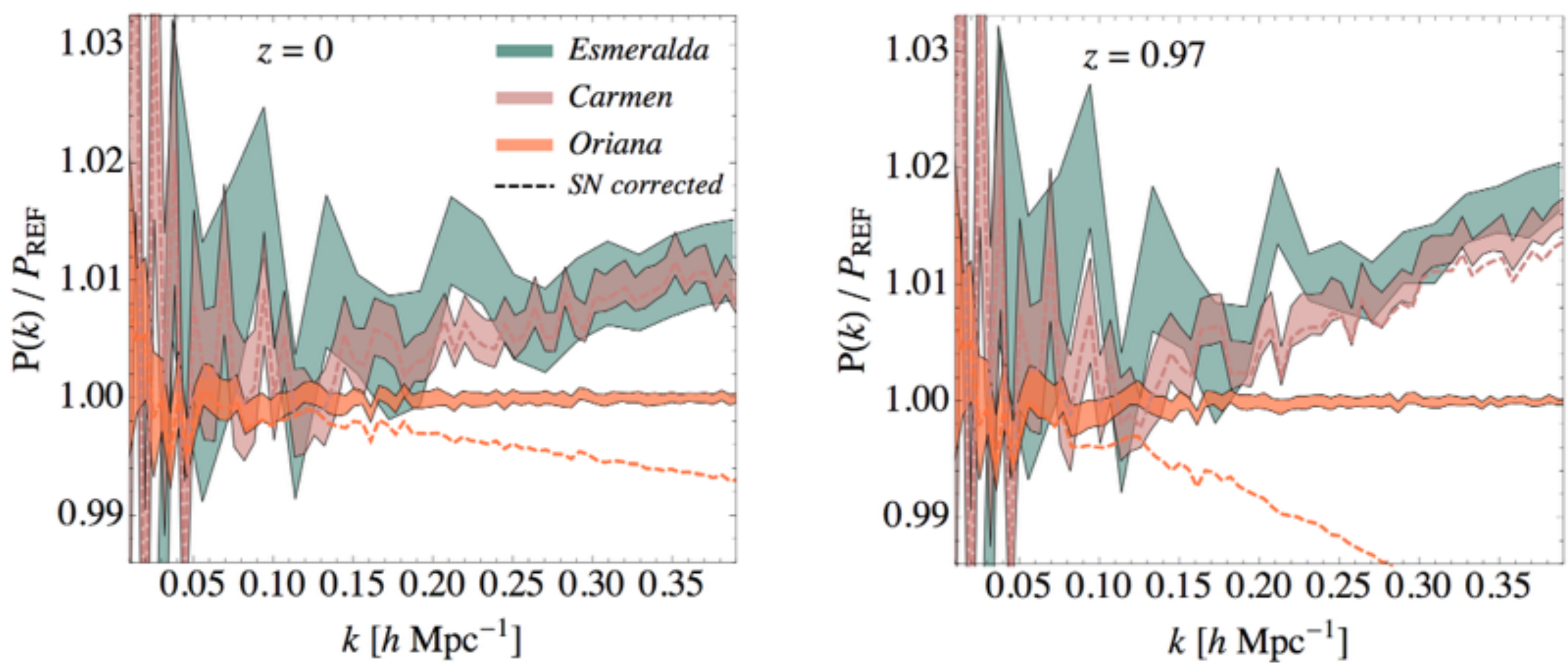
(*) RegPTfast performance ~ MPTbreeze

Improves from 0.2 to 0.4 h^{-1}Mpc in a redshift range useful for Euclid and DESI

Convergence of N-body simulations

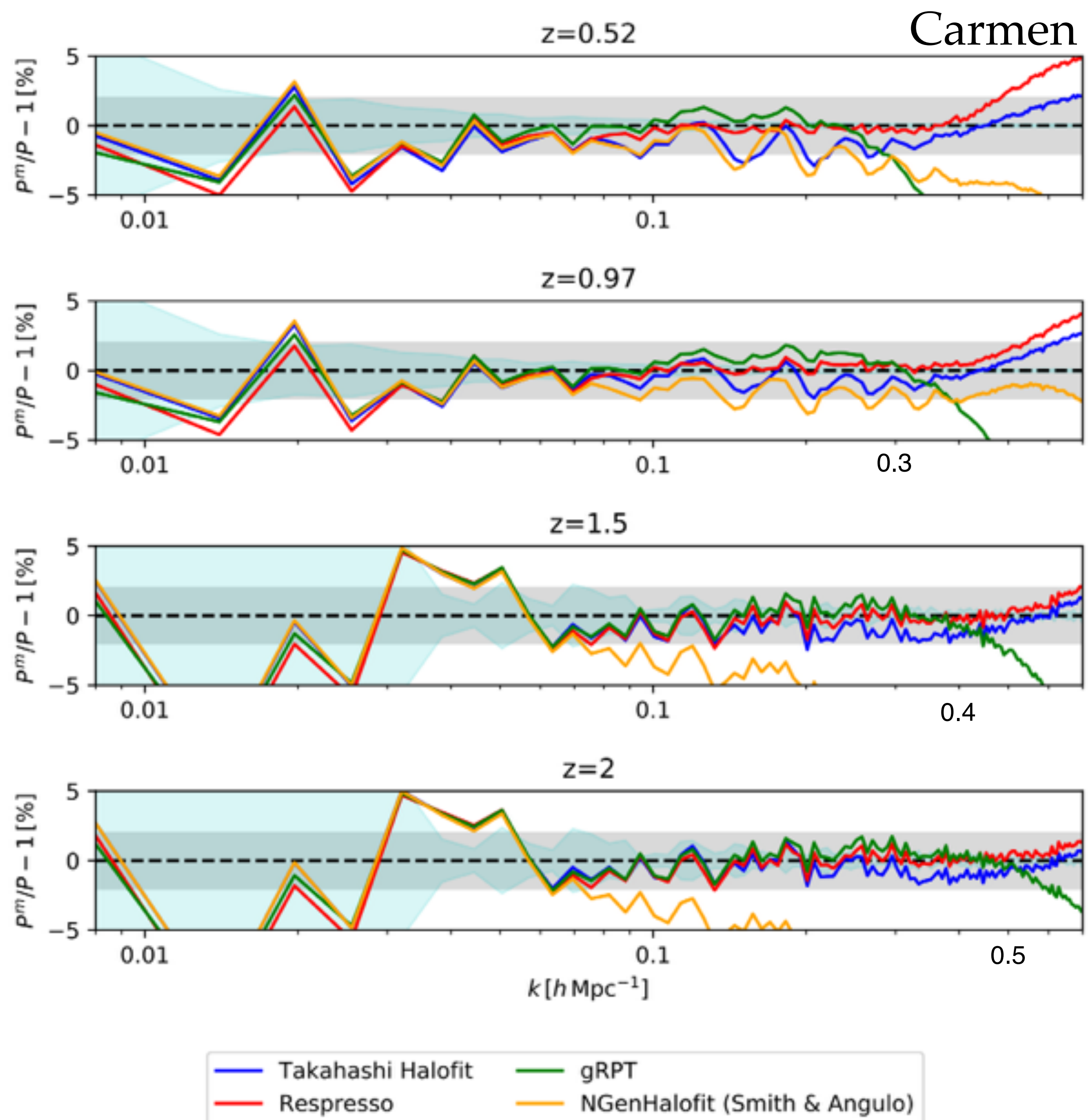
LasDamas suite

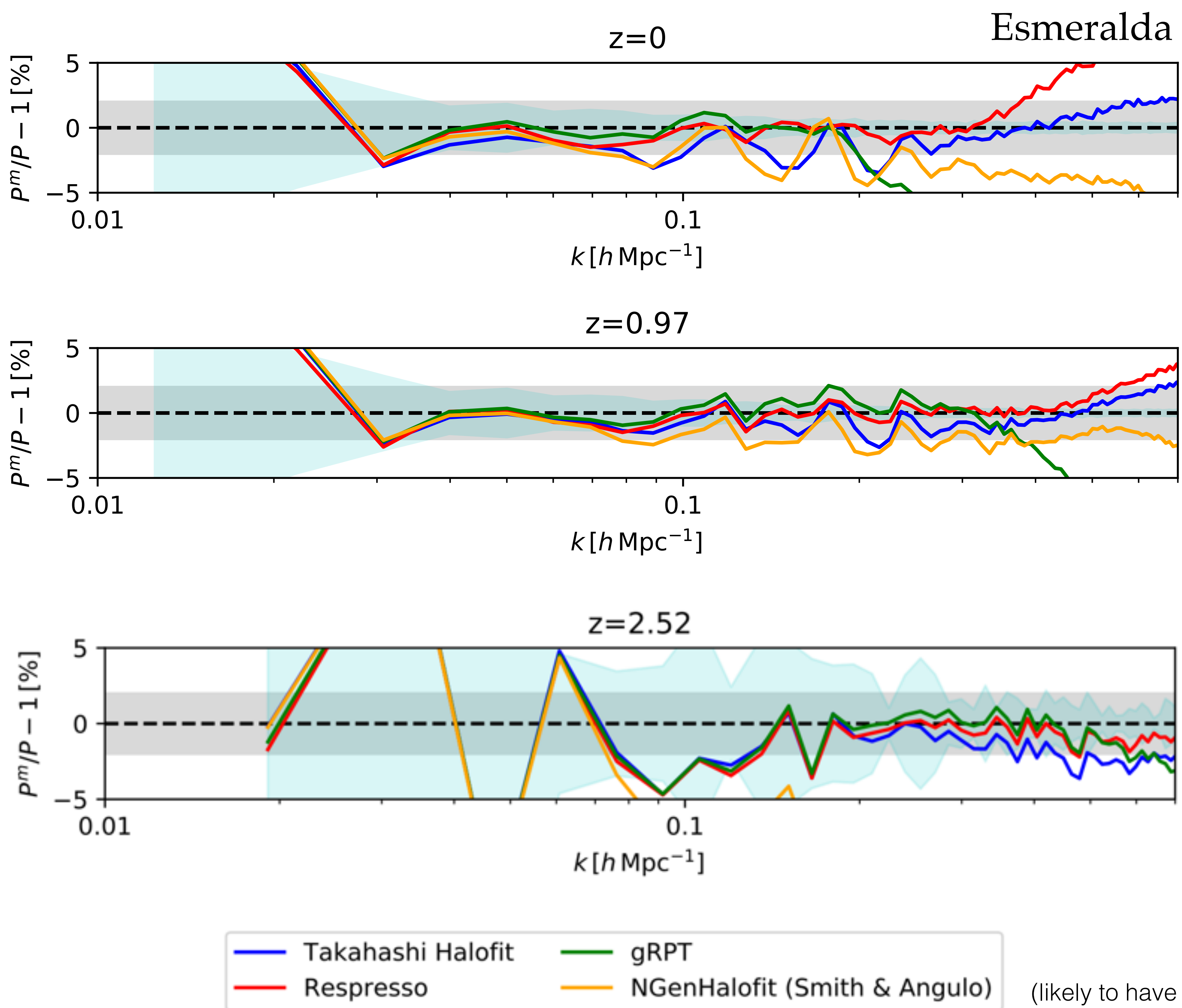
Name	m_p ($h^{-1}\,M_\odot$)	L_b (h^{-1} Mpc)	ϵ (h^{-1} Kpc)	runs
Oriana	45.7×10^{10}	2400	53	40
Carmen	4.94×10^{10}	1000	25	40
Esmeralda	0.93×10^{10}	640	15	30



Overall performance w.r.t. some matter P_k models

(none include free parameters)

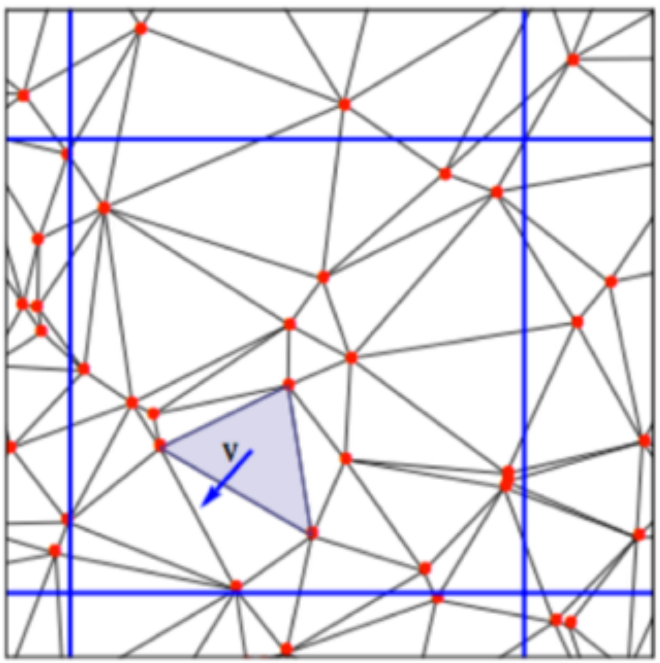




(likely to have some simple bug in the public version)

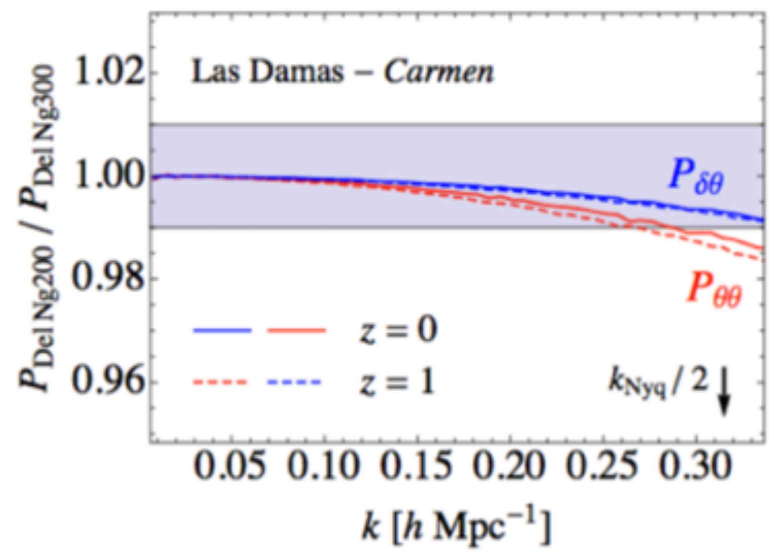
gRPT for Velocity Fields

Measuring velocity fields spectra requires careful estimation of volume weighted quantities (mass resolution important)



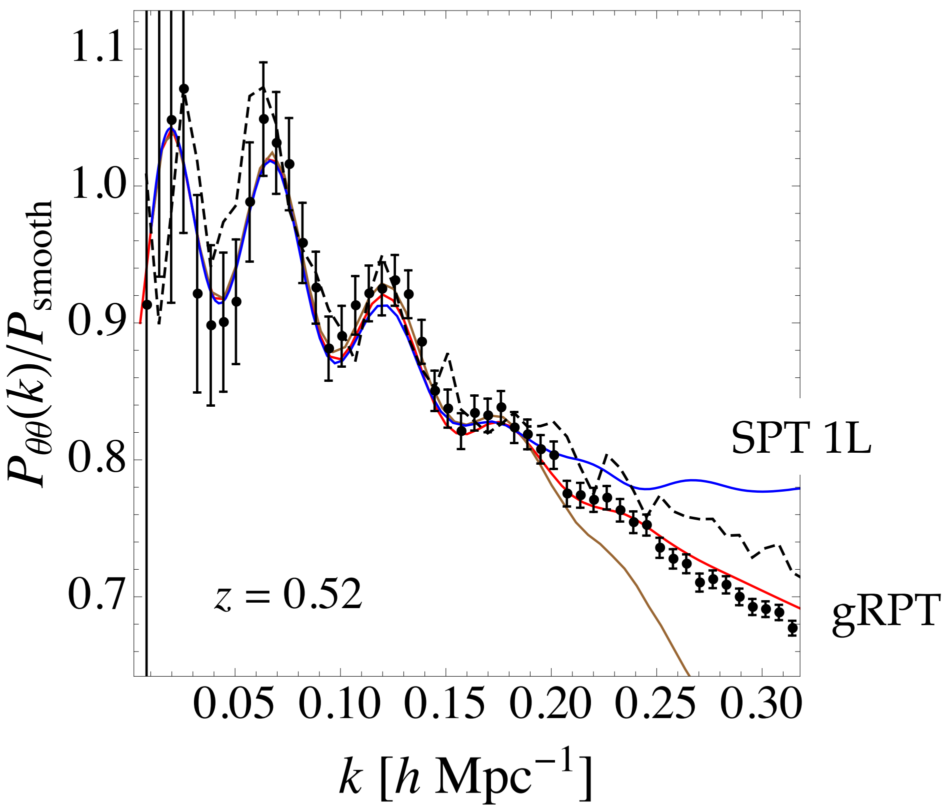
$$\mathbf{v}_i = \frac{\sum_{j=1}^{N_{Del}} \mathbf{v}_j vol_j}{\sum_{j=1}^{N_{Del}} vol_j}$$

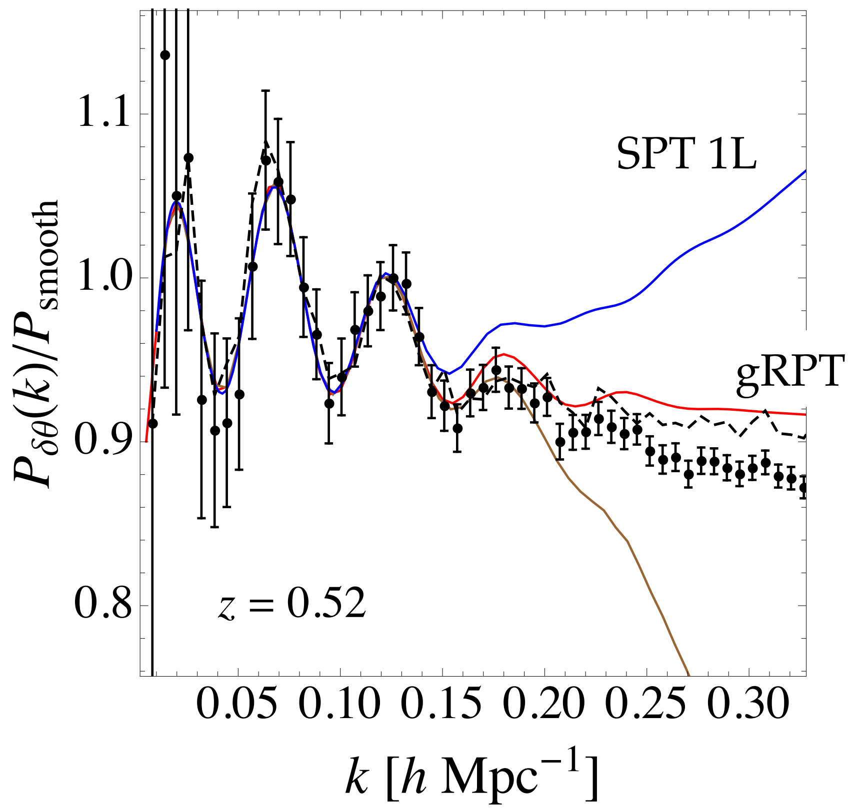
Use of Delaunay Tesselation



gRPT for Velocity Fields

Measuring velocity fields spectra requires careful estimation of volume weighted quantities (mass resolution important)



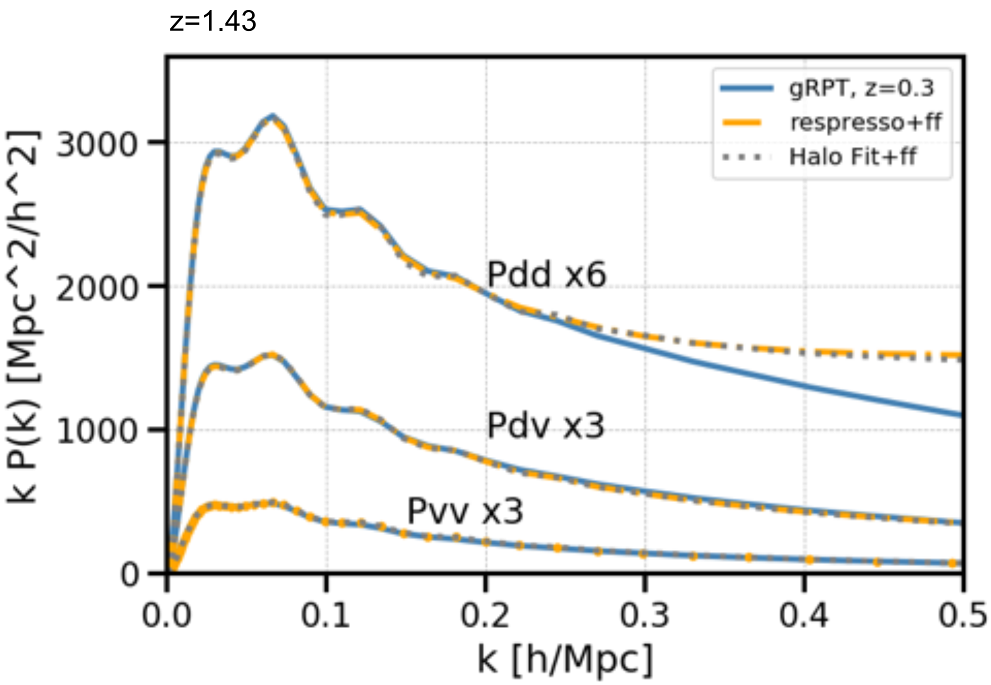


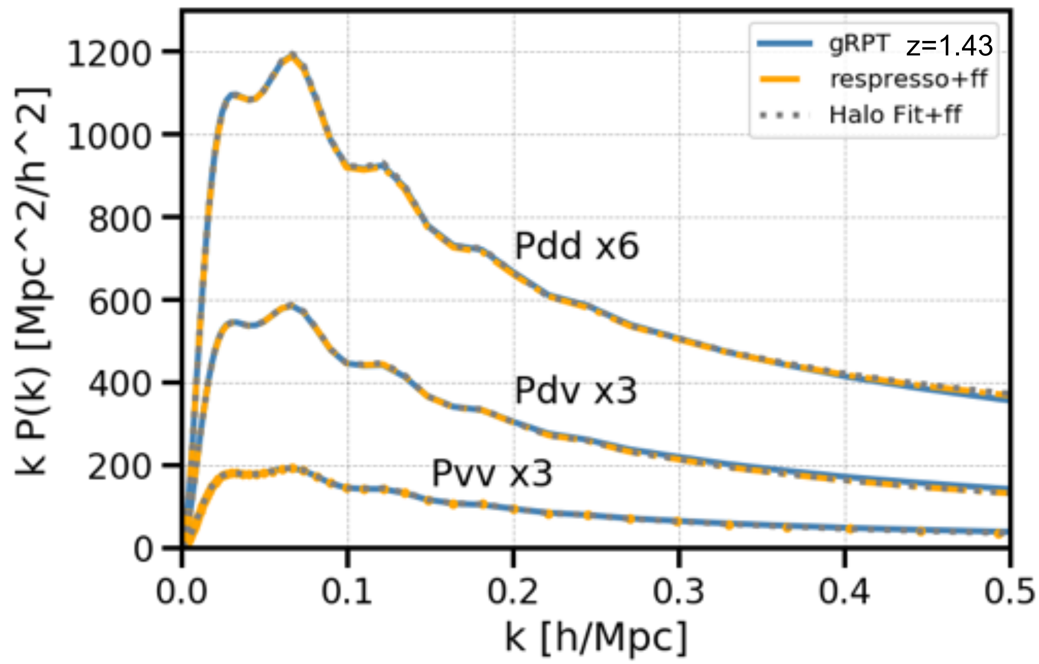
Dashed lines and symbols with error bars are two estimators (dash corrects for mass resolution)

PRELIMINARY
Make sure no issues with the estimator velocity dispersion effects

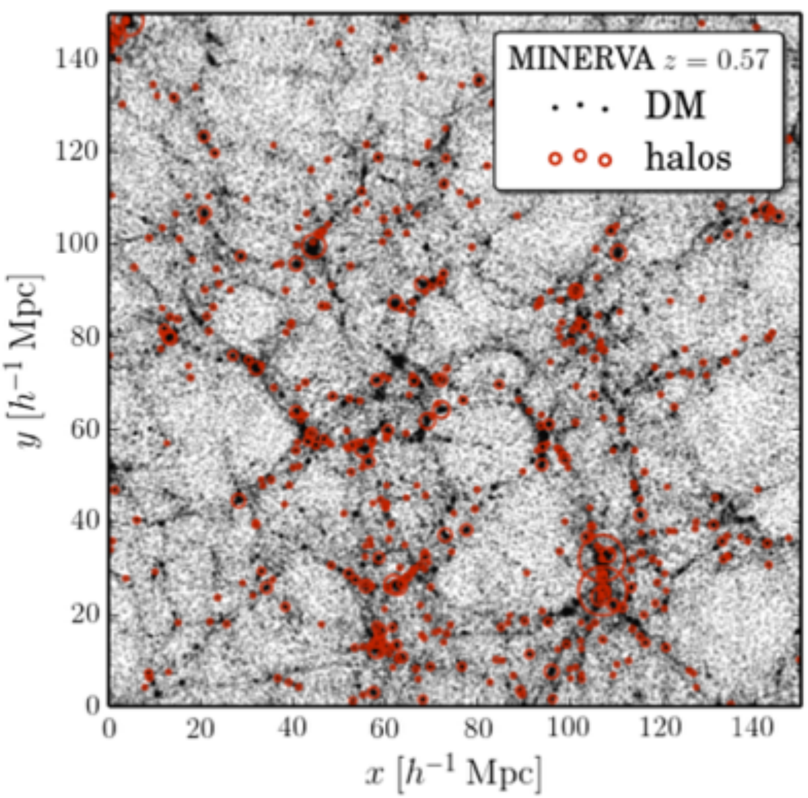
gRPT for Velocity Fields

Alternatively we compare to the fitting functions of Bel et al arxiv/1809.09338 which calibrate the deviation of velocity spectra w.r.t. halo fit matter P(k)



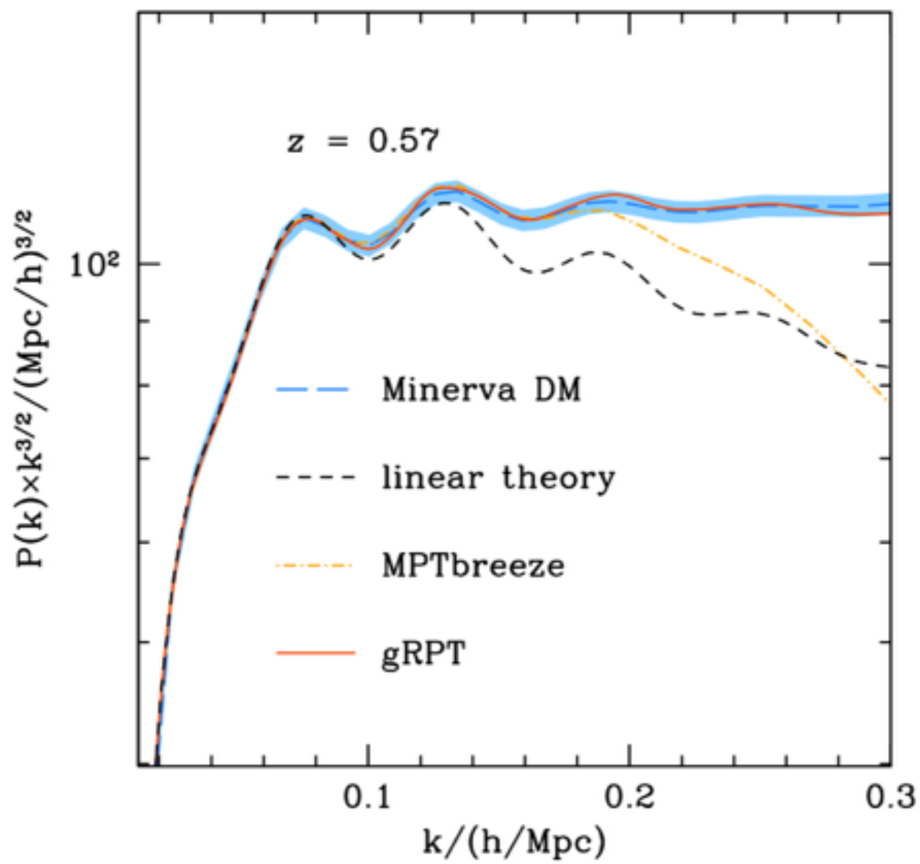


Recovering cosmology with biased tracers - Minerva CMASS mocks



- Minerva: a set of 100 DM N-body simulations.
- Cosmology from WMAP+BOSS DR9
- $L_{\mathrm{BOX}} = 1.5$ Gpc/h, $N = 1000^3$
- Snapshots at $z = 0, 0.3, 0.57, 1$ & 2
- Galaxies with HOD matching CMASS at $z = 0.57$

Recovering cosmology with biased tracers - Minerva matter spectrum



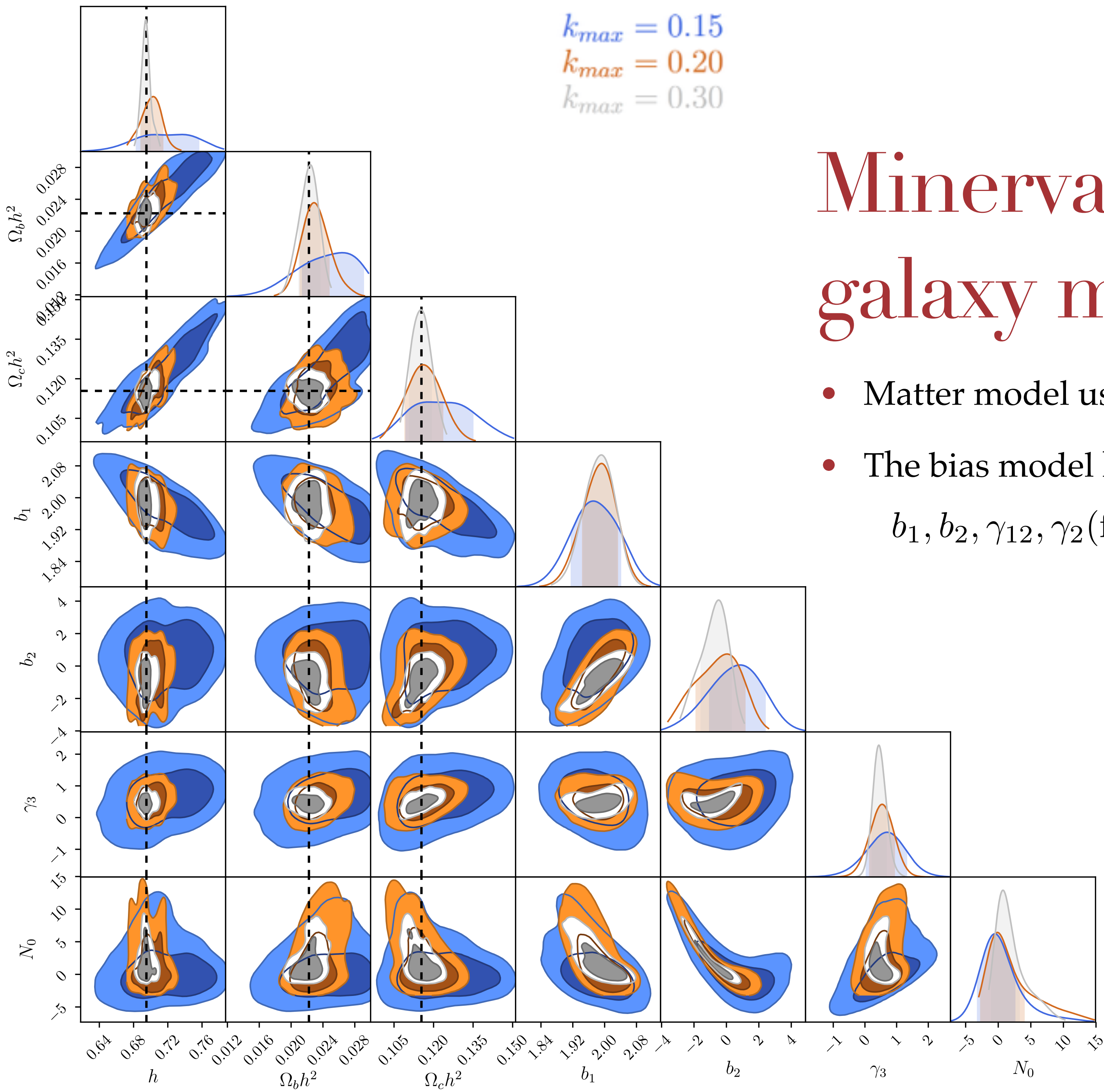
Size of statistical + systematics error bars in BOSS data (~2%)

Minerva CMASS galaxy mocks

- Matter model uses gRPT
- The bias model has 4 nuisance parameters:

  $b_1, b_2, \gamma_{12}, \gamma_2$(fixed to Loc.Lag.)

$$\gamma_{12} \leftrightarrow b_s$$



Conclusions

- We can in principle use GI to find a transformation of the mode-coupling terms in these resumed PT theories which will counter-act the transformation of the propagators
- The new, GI expressions, also improve the k_{max} reach of these theories (maybe to $k_{max} \sim 0.3$ at $z \sim [1\text{-}2]$ at $<\sim 2\%$)
- Using the same transformation seems to work similarly well for velocity fields (which we have used RSD/BOSS)
- We are able to recover underlying cosmology in the mocks for CMASS galaxies (also for other samples).