Random Galilean Invariance in Renormalised Formalisms

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Nonlinear Gravitational Clustering

we assume the stress tensor is zero otherwise it generates corrections.

The velocity field can be assumed irrotational, 

\[ (\perp \cdot \mathbf{x}) \Phi \Delta = (\perp \cdot \mathbf{x}) \Phi \Delta - (\perp \cdot \mathbf{x}) \Phi \Delta = (\perp \cdot \mathbf{x}) \Delta \cdot (\perp \cdot \mathbf{x}) \mathbf{n} + (\perp \cdot \mathbf{x}) \mathbf{n} (\perp \cdot \mathbf{n}) \mathbf{H} + \frac{\perp \cdot \mathbf{p}}{(\perp \cdot \mathbf{x}) \Phi \Delta} \]

\[ 0 = \{ (\perp \cdot \mathbf{x}) \mathbf{H} [ (\perp \cdot \mathbf{x}) \Phi + 1] \} \cdot \Delta + \frac{\perp \cdot \mathbf{p}}{(\perp \cdot \mathbf{x}) \Phi \Delta} \]

\[ (\perp \cdot \mathbf{x}) \mathbf{H} (\perp \cdot \mathbf{u}) \Phi \Delta = (\perp \cdot \mathbf{x}) \Phi \Delta \]

prior to virialization and shell crossing

(\perp \cdot \mathbf{H}) \Phi \Delta

no velocity dispersion or pressure

\[ \text{single stream approximation} \quad \text{single stream approximation} \quad \text{single stream approximation} \]

scales larger than strong clustering regime

Newtonian gravity

 scales much smaller than the horizon (Hubble Radius)
\[ \langle \Phi(z) \rangle \big| \big( \Phi(z) \big) \big| \big( 1 \big) \rangle = \langle \Phi(z) \big| \big( 1 \big) \big| \big( \Phi(z) \big) \rangle \]
Power Spectrum:

\[
\langle (\alpha \cdot \gamma \cdot \Psi \cdot \Phi \cdot \Psi \cdot \Phi) \rangle = (\gamma)^{2\delta} (k^2 - k^2 - 1)^0 \langle 3 \rangle \langle 2 \rangle (\Psi \cdot \Phi \cdot \Phi \cdot \Phi) \langle 2 \rangle (k^2 - k^2 - 1)^0 \langle 3 \rangle
\]
Resummation of IR-modes

It is possible to show that when you consider the high-\(k\) limit, or in other words the contribution to these integrals of modes \(q \ll k\) (IR-modes) the diagrams simplify to

\[
\left( \frac{\partial}{\partial z} \right) (z) D = \left( \frac{\partial}{\partial z} \right) (z) D = (z', y)_{(1)}
\]

\[
D(z) = D(z) \exp\left( \frac{\partial}{\partial z} y \right) (z) D = (z', y)_{(1)}
\]

Different ansätze for MFP lead to different prescriptions

\[
\cdots + (z) D(y) f - (z) D \approx (z', y)_{(1)}
\]

On very large scales we can use PT to compute corrections (\(P^{13}\))

The variance of the displacement field is dominated by large scale flow (~6Mpc/h)

This is the variance of the displacement field.

\[
\left( \frac{\partial}{\partial z} \right) (z) D \approx (z', y)_{(1)}
\]

It is possible to show that when you consider the high-\(k\) limit, or in other words the contribution to these integrals of modes \(k \gg q\) (IR-modes) the diagrams simplify to

\[
\left( \cdots + \frac{0 \partial z}{\partial t} + \frac{0 \partial z}{\partial t} + \frac{0 \partial z}{\partial t} + 1 \right) (z) D = (z', y)_{(1)}
\]

Resummation of IR-modes
Power Spectrum (MPTbreeze)

only few seconds of evaluation time

Performance for different cosmological models at $z = 0$ (dedicated sims)

CODES PUBLICLY AVAILABLE
Each term is Galilean Invariant (built of equal time correlators)

$$P(k) = P_0 + P_1 + P_2 + \ldots$$

Since we only resumed propagator modes, we broke the symmetry.

---

back to Zeldovich

$$\left[ \frac{\tau}{N_{MC}} \frac{d}{dt} + \frac{1}{N_{MC}} + 0 \right] \exp(-\frac{\alpha}{2} - \frac{\gamma}{2} - \lambda) = (\frac{\gamma}{2})$$

---

Each term is Galilean Invariant.

$$\ldots + \text{loop} - \text{loop} - \ldots$$

---

back to Zeldovich

$$\left[ \frac{\tau}{N_{MC}} \frac{d}{dt} + \frac{1}{N_{MC}} + 0 \right] \exp(-\frac{\alpha}{2} - \frac{\gamma}{2} - \lambda) = (\frac{\gamma}{2})$$
\[ \frac{\pi^2}{12} - \frac{\pi^2}{2} + 0 = \left( \frac{\pi^2}{12} - \frac{\pi^2}{2} + 0 \right) \exp(-\frac{\pi^2}{12} - \frac{\pi^2}{2} + 0) = (\gamma) d \]
Using the (unequal time) cross-correlations with the initial conditions as basis is not optimal because the bulk flow displacements are large.

Ideally we want to do RPT in a frame that is moving with the large-scale bulk flow to restore power.

These induce strong "gaussian" damping towards high-\(k\), we need many "loops" to optimize because the bulk flow displacements are large.

This problem is related to the breaking of Galilean Invariance in RPT.
Galilean Invariance

Equal time correlators are invariant under GT:

\[ \langle \tau \psi \rho (L \mathbf{u} + \mathbf{k}) \psi \rangle = \langle \psi (L \mathbf{u} + \mathbf{k}) \rho \psi \rangle \]

The perturbative PT terms as:

\[ \tau \rho \langle \tau \frac{\mathbf{n}}{\mathbf{k}} \rangle + \frac{\mathbf{n}}{\mathbf{k}} \rho \left\langle \tau \frac{\mathbf{n}}{\mathbf{k}} \right\rangle - \frac{\mathbf{n}}{\mathbf{k}} \rho \left\langle \tau \frac{\mathbf{n}}{\mathbf{k}} \right\rangle - \frac{\mathbf{n}}{\mathbf{k}} \rho \]

The velocity variance as:

And the velocity change as:

\[ (L \mathbf{u} - \mathbf{H}) \mathbf{n} = (L \mathbf{u} - \mathbf{H}) \mathbf{n} \]

The fields change as:

\[ \tau \rho (L \mathbf{u} - \mathbf{H}) \mathbf{n} = (L \mathbf{u} - \mathbf{H}) \mathbf{n} \]

Change of coordinates of the form:

Galilean Invariance
Our motivation: Find a transformation to field variables that are more efficient for re-summations (or a reference frame).

We will introduce a family of Random Galilean Transformations (RGT). Since we know how the “resumed propagator” transforms, we look for a Mode-Coupling transformation to counter-act it.

The Galilean Transformations will be controlled by a stochastic random field, assumed Gaussian. This field will be uniform for each $k$-value (or in other words we demand that it can only have structure on scales larger than the one we are looking at).

The Random Galilean Transformations will be controlled by the variance of this field.

Some qualitative thoughts:
We will link this variance to the dynamics of the system in such a way that, in practice, our final PT expressions will be GI.

Our intention is that this prescription will bring us closer to the true answer (something that can be shown explicitly with ZA).

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Hence we choose the Random Galilean Transformation in a way that, at the perturbative order we are working, the dependence of the observable, e.g. \( P(k) \), with the RGT is minimized.

Any observable that is dependent with the CRF, computed fully non-perturbative, a finite calculation might show some dependence with the CRF.

We will link this variance to the dynamics of the system in such a way that in practice our final PT expressions will be GI.

The approach in practice:
For $\mathbf{Z}_A$:

- We pulled out the one-point cumulant (only 2nd order).

Under a change of the large-scale (non-uniform) velocity field, if $\mathbf{u}$ only has structure on large scales (uniform on scales we care) $q \ll r$ then $q/r \ll 1$ and we pull out of the $I$ and $\sigma$ integrals.

Let's see how this works in the Zeldovich approximation.
Invariant order by order

We can now pull out the propagator in the new variables, but the above expression will be

\[ \sum_{I=0}^{\infty} e^{i\frac{\nu z}{\lambda}} \int *_{I+\frac{\nu z}{\lambda}}^{\infty} = (\gamma)d \]

\[ n \phi + \frac{\alpha c}{\phi} \leftarrow \frac{\alpha c}{\phi} \]

\[ n \phi \gamma + I \leftarrow I \]

\[ n \phi \gamma - \equiv \leftarrow I \]

Let's try to restore GI by imposing a field transformation

\[ I - \frac{\nu z}{\lambda} \int *_{I+\frac{\nu z}{\lambda}}^{\infty} = (\gamma)d \]

And you see how GI is explicitly broken

\[ \sum_{I=0}^{\infty} e^{i\frac{\nu z}{\lambda}} \int *_{I+\frac{\nu z}{\lambda}}^{\infty} = (\gamma)d \]

The approach we discussed before basically does

\[ \text{Zeldovich approximation} \]
We can now expand this recalling that $P(k) = e^{\frac{7}{2}d(1 - x + x^2/2 + \ldots)} + P_1L(1 - x + \ldots) + P_2L\ldots$

You arrive at:

\[ 0 = x^2/(\langle y \rangle d^2) \]

Since we do not want $P(k)$ to depend on $x$ we require

\[ \frac{n_{\varphi} \gamma}{\gamma} - \equiv *I \]
\[ \frac{\alpha_{\varphi} \gamma}{\gamma} - *I \equiv x \]

This is basically the variance of the RGT discussed before.

\[ \square \]

Zeldovich approximation
For the exact dynamics is similar: we need to choose an expression for the resumed propagator which we transform

$$P(k) \text{ n-order } / P(k) \text{ exact}$$

At fixed $k = 0.1 \, h/\text{Mpc}$

Different lines: orders in PT

Since we do not want $P(k)$ to depend on $x$ we require
GRT for the exact dynamics.

For the exact dynamics is similar: we start from a expression for the resumed propagator in reg-PT form

\( (\frac{\tau}{\nu}d + xI) (\frac{a_{\nu} c}{\nu} \gamma + \frac{\tau d}{\nu + d} + I) \gamma \varphi \gamma = (\gamma, \gamma) d \)

The boost (to trace the large-scale flows) chooses the appropriate value of \( x \).

Choose the appropriate value of \( x \) (the boost) to trace the large-scale flows.
Comparison to RegPT / MPTbreeze (the other "state-of-the-art" fast algorithm)

Euclid and DESI in a redshift range useful for Imroves from 0.2 to 0.4 $h^{-1} \text{Mpc}$

$\kappa \left| y_{\text{Mpc}^{-1}} \right|$

$P_{\text{NL}}(k)/P_{\text{theory}}$

$0.05$ $0.02$

$0.1$ $0.05$

$0.2$ $0.12$

$0.3$ $0.08$

$0.4$ $0.04$

$0.5$ $0.02$

$0.6$ $0.01$

$0.7$ $0.00$

$0.8$ $0.00$

$0.9$ $0.00$

$1.0$ $0.00$

$1.1$ $0.00$

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$11.0$ $0.00$
Convergence of N-body simulations
LasDamas suite

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<th>Runs</th>
<th>SN connected</th>
<th>Carmen</th>
<th>Oriona</th>
<th>Esmeralda</th>
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<td>45.7 x 10^10</td>
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<td>15</td>
<td>25</td>
<td>40</td>
<td>38</td>
<td>15 Kpc</td>
</tr>
</tbody>
</table>

$P(k) / P_{REF}$
Overall performance w.r.t. some matter models (none include free parameters; P/H models Carmen)
Use of Delaunay Tessellation

Measuring velocity fields spectra requires careful estimation of volume weighted quantities (mass resolution important).
Measuring velocity fields spectra requires careful estimation of volume-weighted quantities (mass resolution important). Dashed lines and symbols with error bars are two estimators (dash corrects for mass resolution). Make sure no issues with the estimator.

Preliminary

\[ \gamma \]

\[ \mathcal{M}_c \]

0.05 0.10 0.15 0.20 0.25 0.30

0.05 0.10 0.15 0.20 0.25 0.30

\[ P(k) \]

\[ P_{\text{smooth}}(k) \]

\[ z = 0.52 \]

\[ z = 0.52 \]

\[ P(k) \]

\[ P_{\text{smooth}}(k) \]

\[ \mathcal{R}^{\text{gRPT}} \]

\[ \mathcal{R}^{\text{gRPT}} \]

\[ \mathcal{S}^{\text{SPT}} \]

\[ \mathcal{S}^{\text{SPT}} \]

\[ \mathcal{S}^{\text{gRPT}} \]

\[ \mathcal{S}^{\text{gRPT}} \]
Alternatively we compare to the fitting functions of Bel et al. [arXiv:1809.09338] which calibrate the deviation of velocity spectra w.r.t. halo fit matter $P(k)$. 

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Recovering cosmology with biased tracers - Minerva CMASS mocks

- Minerva: a set of 100 DM N-body simulations.
- Cosmology from WMAP+BOSS DR9 simulations.
- Snapshots at $z = 0, 0.3, 0.57, 1 \& 2$
- $L_{box} = 1.5 \text{ Gpc}/h, N = 1000^3$
- Galaxies with HOD matching CMASS at $z = 0.57$

at $z = 0.57$
Recovering cosmology with biased tracers - Minceria matter spectrum

Size of statistical + systematic error bars in BOSS data ($\sim 2\%$)
\[ k_{\text{max}} = 0 \quad \Delta 15 \quad k_{\text{max}} = 0 \quad \Delta 20 \quad k_{\text{max}} = 0 \quad \Delta 30 \]

- The bias model has 4 nuisance parameters: \( b_1, b_2, \Delta_{12}, \Delta_{23} \) (fixed to Loc. Lag.)
- Matter model uses GRPT
- \( \psi_{12} \leftrightarrow y_{12} \)
- Minerva CMASS galaxy mocks
Conclusions

• We can in principle use GI to find a transformation of the mode-coupling terms in these resumed PT theories which will counter-act the transformation of the propagators.

• The new, GI expressions also improve the $k_{\text{max}}$ reach of these theories (maybe to $k_{\text{max}} \sim 0.3$ at $z \approx 1-2$, at $>2\%$)

• Using the same transformation seems to work similarly well for velocity fields (which we have used RSD/Boss mocks for CMASS galaxies (also for other samples)).

• We are able to recover underlying cosmology in the mocks.

• We can in principle use GI to find a transformation of the propagators which will counter-act the transformation of the mode-coupling terms in these resumed PT theories.