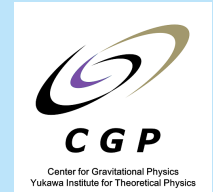
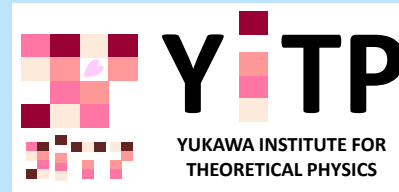


Lagrangian perturbation theory at **shell-crossing**

Shohei Saga



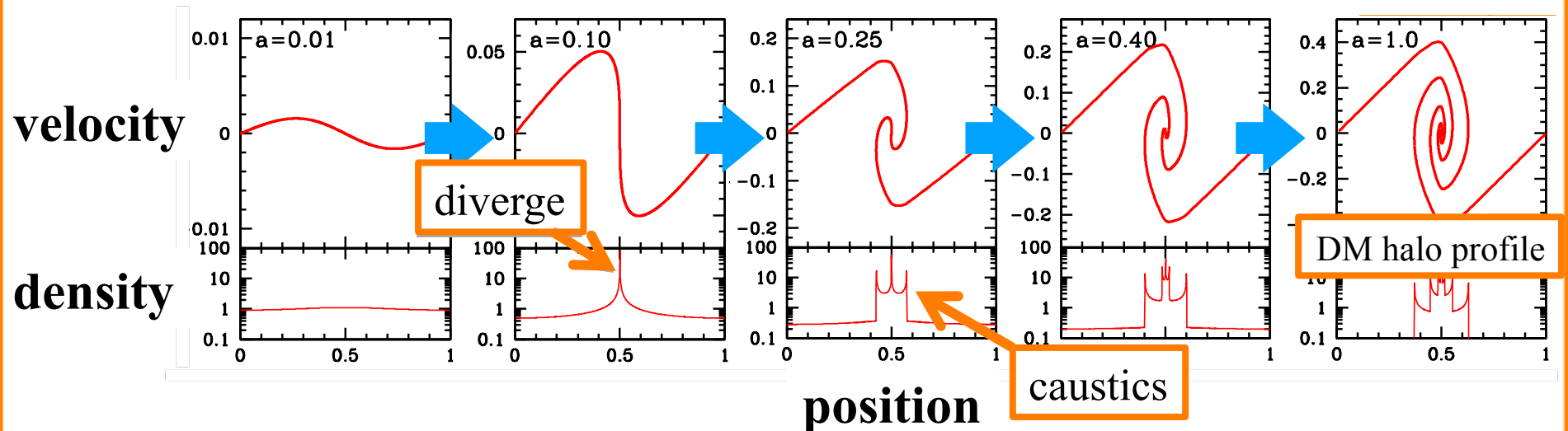
Based on Phys.Rev.Lett. 121, 241302 (2018)

Atsushi Taruya (YITP) and Stéphane Colombi (IAP)

1 Cold Dark Matter

- initially **negligible velocity dispersion**
- single-stream flow \sim fluid approximation

Phase space sheet (line) in 1D cosmology



A.Taruya and S.Colombi [MNRAS 470, 4 (2017)]

- ① Initial time: small density peak at the origin
- ② Shell crossing
- ③ Multi-stream flow (multi-valued function)

1.1 Motivation and goal

Background

- Shell crossing is inevitable if dark matter is cold
- Little knowledge of 3D shell crossing
- Recently, high-resolution Vlasov-Poisson has been developing

Goal

- On the basis of **perturbation theory**, we obtain the information about shell crossing in 3D cosmology
- Convergence study of perturbation theory at shell crossing

Method

Lagrangian perturbation theory

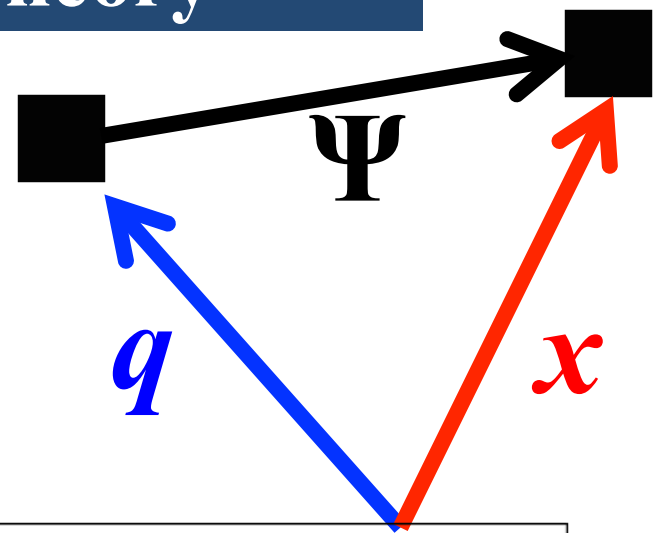
2. Lagrangian Perturbation Theory

Eulerian position

Displacement field

$$\mathbf{x}(\mathbf{q}, t) = \mathbf{q} + \Psi(\mathbf{q}, t)$$

Lagrangian (initial) position



Nonlinear equation for the displacement field Ψ

Longitudinal part: $(\hat{\mathcal{T}} - 4\pi G \bar{\rho}_m) \Psi_{i,i} = -\epsilon_{ijk} \epsilon_{ipq} \Psi_{j,p} (\hat{\mathcal{T}} - 2\pi G \bar{\rho}) \Psi_{k,q}$
 $\hat{\mathcal{T}} \equiv \frac{\partial^2}{\partial t^2} + 2H \frac{\partial}{\partial t}$
 $-\frac{1}{2} \epsilon_{ijk} \epsilon_{pqr} \Psi_{i,p} \Psi_{j,q} \left(\hat{\mathcal{T}} - \frac{4\pi G}{3} \bar{\rho} \right) \Psi_{k,r}$

Transverse part: $\hat{\mathcal{T}} \epsilon_{ijk} \Psi_{j,k} = -\epsilon_{ijk} \Psi_{a,j} \hat{\mathcal{T}} \Psi_{a,k}$

$$\Psi_i = \nabla^{-2} [\nabla_i \Psi_{a,a} + \epsilon_{iab} \nabla_a (\epsilon_{bcd} \Psi_{c,d})]$$

T.Matsubara [PRD92 023534 (2015)]

$$\Psi(\mathbf{q}, t) = \sum_{n=1}^{\infty} (D_+(t))^n \Psi^{(n)}(\mathbf{q}) \quad \longrightarrow \quad \Psi^{(10)}(\mathbf{q})$$

2.1 Initial condition and Simulation

Initial condition:

Three sine waves

$$\Psi^{(1)}(\mathbf{q}, t) = D_+(t) \begin{pmatrix} \epsilon_x \sin q_x \\ \epsilon_y \sin q_y \\ \epsilon_z \sin q_z \end{pmatrix}$$

→ Small density peak at the origin

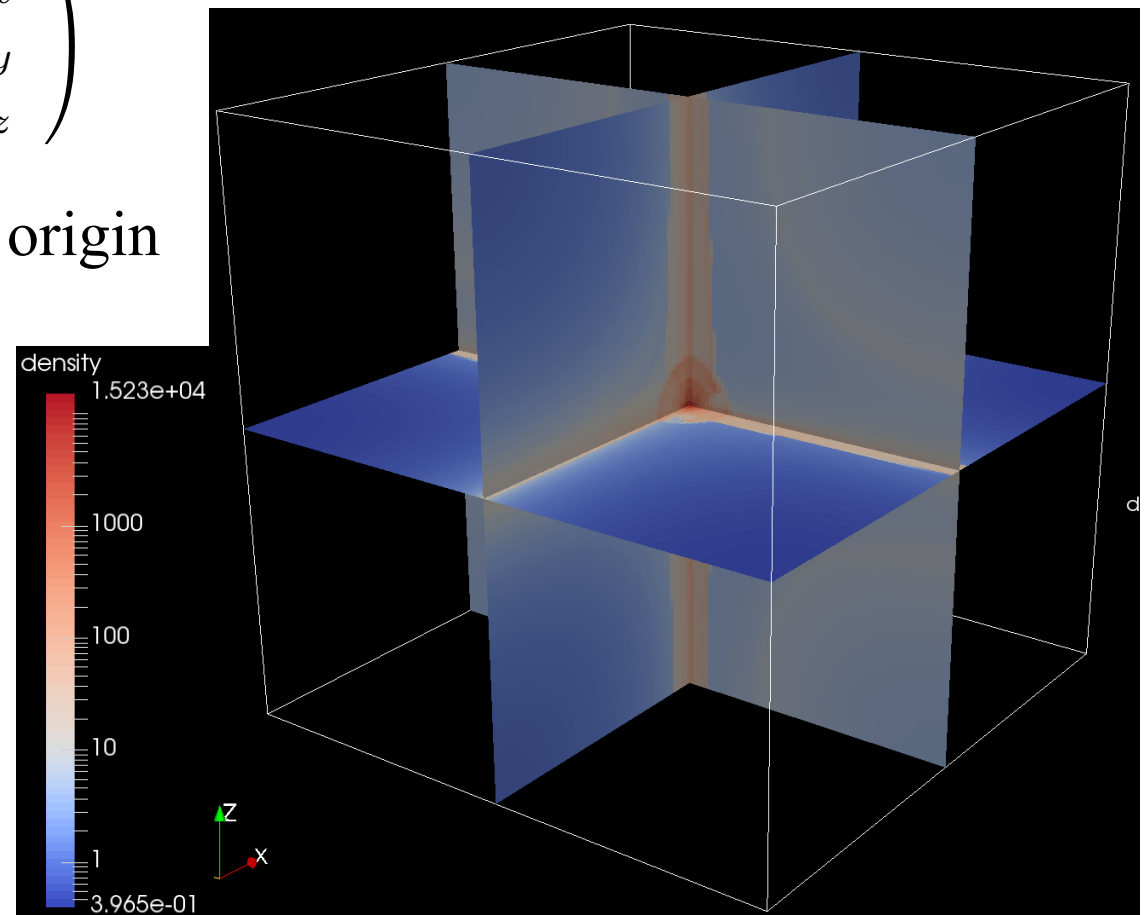
6D Vlasov-Poisson solver:
CoIDICE

T.Sousbie and S.Colombi
[J.Comput.Phys321, 644(2016)]

<https://vlasix.org/>

F.Moutarde et al. [ApJ, 382, 377-381 (1991)]

T.Buchert et al. [A&A, 318, 1-10 (1997)]



2.2 Expressions

x-component of the displacement field

1st order (Zel'dovich approximation)

$$\epsilon_x \sin(q_x)$$

2nd order

$$-\frac{3}{14} \epsilon_x \sin(q_x) (\epsilon_y \cos(q_y) + \epsilon_z \cos(q_z))$$

3rd order

$$\frac{\epsilon_x \sin(q_x) (78 \epsilon_x \cos(q_x) (\epsilon_y \cos(q_y) + \epsilon_z \cos(q_z)) + 160 \epsilon_y \epsilon_z \cos(q_y) \cos(q_z) - 3 \epsilon_y^2 (\cos(2 q_y) - 25) - 3 \epsilon_z^2 (\cos(2 q_z) - 25))}{1260}$$

1260

4th order

$$-\frac{1}{3880800} \epsilon_x \sin(q_x) (42 \epsilon_x^2 (101 \cos(2 q_x) + 2123) (\epsilon_y \cos(q_y) + \epsilon_z \cos(q_z)) + 60 \epsilon_x \cos(q_x) (6010 \epsilon_y \epsilon_z \cos(q_y) \cos(q_z) + \epsilon_y^2 (1274 \cos(2 q_y) + 2039) + \epsilon_z^2 (1274 \cos(2 q_z) + 2039)) + 50 \epsilon_y^2 \epsilon_z (571 \cos(2 q_y) + 4177) \cos(q_z) + 50 \epsilon_y \epsilon_z^2 \cos(q_y) (571 \cos(2 q_z) + 4177) + 21 \epsilon_y^3 (2715 \cos(q_y) - 443 \cos(3 q_y)) + 21 \epsilon_z^3 (2715 \cos(q_z) - 443 \cos(3 q_z)))$$

2.3 Expressions

5th order

$$\frac{1}{18\,396\,738\,360\,000} \epsilon_x \sin(q_x) (54\,054 \epsilon_x^3 (1\,970\,751 \cos(q_x) - 126\,335 \cos(3q_x)) (\epsilon_y \cos(q_y) + \epsilon_z \cos(q_z)) +$$

$$765 \epsilon_x^2 (1040 \epsilon_y \epsilon_z (203\,958 \cos(2q_x) + 1\,102\,715) \cos(q_y) \cos(q_z) + 33 \epsilon_y^2 (2\,273\,783 \cos(2(q_x - q_y)) + 2\,273\,783 \cos(2(q_x + q_y)) + 1\,476\,930 \cos(2q_x) + 9\,562\,610 \cos(2q_y) + 9\,599\,590) +$$

$$33 \epsilon_z^2 (2\,273\,783 \cos(2(q_x - q_z)) + 2\,273\,783 \cos(2(q_x + q_z)) + 1\,476\,930 \cos(2q_x) + 9\,562\,610 \cos(2q_z) + 9\,599\,590)) + 1870 \epsilon_x \cos(q_x) (26 \epsilon_y^2 \epsilon_z (11\,522\,545 \cos(2q_y) + 20\,694\,771) \cos(q_z) +$$

$$26 \epsilon_y \epsilon_z^2 \cos(q_y) (11\,522\,545 \cos(2q_z) + 20\,694\,771) + 27 \epsilon_y^3 (11\,773\,099 \cos(q_y) - 369\,227 \cos(3q_y)) + 27 \epsilon_z^3 (11\,773\,099 \cos(q_z) - 369\,227 \cos(3q_z))) -$$

$$13 (214\,200 \epsilon_y^3 \epsilon_z (19\,521 \cos(3q_y) - 180\,697 \cos(q_y)) \cos(q_z) - 1870 \epsilon_y^2 \epsilon_z^2 (1\,919\,335 \cos(2(q_y - q_z)) + 1\,919\,335 \cos(2(q_y + q_z)) + 4\,759\,758 \cos(2q_y) + 4\,759\,758 \cos(2q_z) + 21\,165\,750) +$$

$$214\,200 \epsilon_y \epsilon_z^3 \cos(q_y) (19\,521 \cos(3q_z) - 180\,697 \cos(q_z)) + 2079 \epsilon_y^4 (1\,297\,372 \cos(2q_y) + 59\,485 \cos(4q_y) - 3\,461\,625) + 2079 \epsilon_z^4 (1\,297\,372 \cos(2q_z) + 59\,485 \cos(4q_z) - 3\,461\,625))$$

6th order

$$\frac{1}{811\,296\,161\,676\,000\,000} \sin(q_x) \epsilon_x (-305\,613 (2\,659\,396\,870 \cos(q_y) - 962\,623\,441 \cos(3q_y) + 28\,312\,075 \cos(5q_y)) \epsilon_y^5 + 4\,873\,050 (266\,598\,492 \cos(2q_y) + 76\,018\,045 \cos(4q_y) - 1\,901\,498\,841) \cos(q_z) \epsilon_z \epsilon_y^4 -$$

$$4420 \cos(q_y) (-516\,334\,675\,626 \cos(2q_y) + 52\,110\,906\,915 \cos(2(q_y - q_z)) + 1\,838\,044\,474\,430 \cos(2q_z) + 52\,110\,906\,915 \cos(2(q_y + q_z)) + 3\,604\,788\,111\,078) \epsilon_z^2 \epsilon_y^3 -$$

$$4420 \cos(q_z) (1\,838\,044\,474\,430 \cos(2q_y) + 52\,110\,906\,915 \cos(2(q_y - q_z)) - 516\,334\,675\,626 \cos(2q_z) + 52\,110\,906\,915 \cos(2(q_y + q_z)) + 3\,604\,788\,111\,078) \epsilon_z^3 \epsilon_y^2 +$$

$$4\,873\,050 \cos(q_y) (266\,598\,492 \cos(2q_z) + 76\,018\,045 \cos(4q_z) - 1\,901\,498\,841) \epsilon_z^4 \epsilon_y - 305\,613 (2\,659\,396\,870 \cos(q_z) - 962\,623\,441 \cos(3q_z) + 28\,312\,075 \cos(5q_z)) \epsilon_z^5 +$$

$$1\,833\,678 (116\,189\,716 \cos(2q_x) + 7\,558\,375 \cos(4q_x) - 839\,506\,587) \epsilon_x^4 (\cos(q_y) \epsilon_y + \cos(q_z) \epsilon_z) + 8820 \cos(q_x) \epsilon_x^3$$

$$(-33 (-1\,499\,341\,220 \cos(2q_x) + 745\,268\,925 \cos(2(q_x - q_y)) + 34\,417\,340\,854 \cos(2q_y) + 745\,268\,925 \cos(2(q_x + q_y)) + 15\,828\,123\,428) \epsilon_y^2 + 4420 (32\,910\,965 \cos(2q_x) - 531\,579\,429)$$

$$\cos(q_y) \cos(q_z) \epsilon_z \epsilon_y - 33 (-1\,499\,341\,220 \cos(2q_x) + 745\,268\,925 \cos(2(q_x - q_z)) + 34\,417\,340\,854 \cos(2q_z) + 745\,268\,925 \cos(2(q_x + q_z)) + 15\,828\,123\,428) \epsilon_z^2) -$$

$$340 \epsilon_x^2 (2079 \cos(q_y) (8\,156\,203\,254 \cos(2q_x) + 2\,768\,321\,325 \cos(2(q_x - q_y)) + 2\,763\,034\,018 \cos(2q_y) + 2\,768\,321\,325 \cos(2(q_x + q_y)) + 21\,975\,421\,138) \epsilon_y^3 +$$

$$52 (386\,519\,295\,393 \cos(2q_x) + 295\,586\,482\,530 \cos(2(q_x - q_y)) + 1\,129\,466\,363\,210 \cos(2q_y) + 295\,586\,482\,530 \cos(2(q_x + q_y)) + 1\,479\,850\,604\,694) \cos(q_z) \epsilon_z \epsilon_y^2 +$$

$$52 \cos(q_y) (386\,519\,295\,393 \cos(2q_x) + 295\,586\,482\,530 \cos(2(q_x - q_z)) + 1\,129\,466\,363\,210 \cos(2q_z) + 295\,586\,482\,530 \cos(2(q_x + q_z)) + 1\,479\,850\,604\,694) \epsilon_z^2 \epsilon_y +$$

$$2079 \cos(q_z) (8\,156\,203\,254 \cos(2q_x) + 2\,768\,321\,325 \cos(2(q_x - q_z)) + 2\,763\,034\,018 \cos(2q_z) + 2\,768\,321\,325 \cos(2(q_x + q_z)) + 21\,975\,421\,138) \epsilon_z^3) -$$

$$20 \cos(q_x) \epsilon_x (14\,553 (5\,904\,767\,794 \cos(2q_y) - 2\,780\,948\,475 \cos(4q_y) + 39\,272\,806\,105) \epsilon_y^4 + 4420 (571\,117\,785\,469 \cos(q_y) - 1\,357\,574\,367 \cos(3q_y)) \cos(q_z) \epsilon_z \epsilon_y^3 +$$

$$510\,510 (1\,660\,817\,391 \cos(2q_y) + 538\,260\,884 \cos(2(q_y - q_z)) + 1\,660\,817\,391 \cos(2q_z) + 538\,260\,884 \cos(2(q_y + q_z)) + 2\,964\,557\,714) \epsilon_z^2 \epsilon_y^2 +$$

$$4420 \cos(q_y) (571\,117\,785\,469 \cos(q_z) - 1\,357\,574\,367 \cos(3q_z)) \epsilon_z^3 \epsilon_y + 14\,553 (5\,904\,767\,794 \cos(2q_z) - 2\,780\,948\,475 \cos(4q_z) + 39\,272\,806\,105) \epsilon_z^4))$$

2.4 Expressions

7th order

$$\begin{aligned}
& \frac{1}{1\ 004\ 810\ 024\ 548\ 828\ 360\ 142\ 280\ 000\ 000} \sin(q_x) \epsilon_x (3\ 034\ 100\ 803\ 734 (1\ 32\ 504\ 410\ 196\ 934 \cos(q_x) - 22\ 415\ 880\ 538\ 705 \cos(3 q_x) + 383\ 378\ 540\ 875 \cos(5 q_x)) (\cos(q_y) \epsilon_y + \cos(q_z) \epsilon_z) \epsilon_x^5 + \\
& 553\ 391\ 055 (363 (277\ 129\ 358\ 703\ 288 \cos(2 q_x) - 221\ 569\ 332\ 279\ 750 \cos(4 q_x) - 310\ 443\ 780\ 648\ 625 \cos(4 q_x - 2 q_y) + 4\ 043\ 457\ 776\ 606\ 492 \cos(2 (q_x - q_y)) + 26\ 302\ 111\ 171\ 118\ 442 \cos(2 q_y) + \\
& 4\ 043\ 457\ 776\ 606\ 492 \cos(2 (q_x + q_y)) - 310\ 443\ 780\ 648\ 625 \cos(2 (2 q_x + q_y)) + 8\ 639\ 970\ 224\ 710\ 734) \epsilon_y^2 + 392\ 080 (2\ 849\ 830\ 451\ 562 \cos(2 q_x) - 818\ 223\ 510\ 775 \cos(4 q_x) + 44\ 804\ 309\ 653\ 645) \\
& \cos(q_y) \cos(q_z) \epsilon_x \epsilon_y + 363 (277\ 129\ 358\ 703\ 288 \cos(2 q_x) - 221\ 569\ 332\ 279\ 750 \cos(4 q_x) - 310\ 443\ 780\ 648\ 625 \cos(4 q_x - 2 q_z) + 4\ 043\ 457\ 776\ 606\ 492 \cos(2 (q_x - q_z)) + \\
& 26\ 302\ 111\ 171\ 118\ 442 \cos(2 q_z) + 4\ 043\ 457\ 776\ 606\ 492 \cos(2 (q_x + q_z)) - 310\ 443\ 780\ 648\ 625 \cos(2 (2 q_x + q_z)) + 8\ 639\ 970\ 224\ 710\ 734) \epsilon_z^2 \epsilon_x^4 + \\
& 4\ 485\ 140 \cos(q_x) (87\ 318 \cos(q_y) (17 (317\ 200\ 143\ 950\ 625 \cos(2 (q_x - q_y)) + 1\ 423\ 122\ 370\ 938\ 650 \cos(2 q_y) + 317\ 200\ 143\ 950\ 625 \cos(2 (q_x + q_y)) + 2\ 162\ 942\ 532\ 366\ 438) - 2\ 323\ 267\ 677\ 375\ 190 \cos(2 q_x) \epsilon_x^3 + \\
& 130 (17 (137\ 983\ 685\ 717\ 505\ 525 \cos(2 (q_x - q_y)) + 2\ 969\ 669\ 587\ 394\ 985\ 734 \cos(2 q_y) + 137\ 983\ 685\ 717\ 505\ 525 \cos(2 (q_x + q_y)) + 2\ 215\ 505\ 888\ 579\ 262\ 006) - 1\ 048\ 826\ 181\ 631\ 251\ 114 \cos(2 q_x) \\
& \cos(q_z) \epsilon_x \epsilon_y^2 + 130 \cos(q_y) \\
& (17 (137\ 983\ 685\ 717\ 505\ 525 \cos(2 (q_x - q_z)) + 2\ 969\ 669\ 587\ 394\ 985\ 734 \cos(2 q_z) + 137\ 983\ 685\ 717\ 505\ 525 \cos(2 (q_x + q_z)) + 2\ 215\ 505\ 888\ 579\ 262\ 006) - 1\ 048\ 826\ 181\ 631\ 251\ 114 \cos(2 q_x) \epsilon_z^2 \epsilon_y + \\
& 87\ 318 \cos(q_z) (17 (317\ 200\ 143\ 950\ 625 \cos(2 (q_x - q_z)) + 1\ 423\ 122\ 370\ 938\ 650 \cos(2 q_z) + 317\ 200\ 143\ 950\ 625 \cos(2 (q_x + q_z)) + 2\ 162\ 942\ 532\ 366\ 438) - 2\ 323\ 267\ 677\ 375\ 190 \cos(2 q_x) \epsilon_z^3 \epsilon_x^3 + \\
& 75\ 110 (1\ 303\ 533 (58\ 152\ 516\ 016\ 142\ 730 \cos(2 q_x) - 1\ 031\ 493\ 519\ 759\ 625 \cos(2 (q_x - 2 q_y)) + 28\ 800\ 768\ 510\ 344\ 900 \cos(2 (q_x - q_y)) + 48\ 549\ 817\ 895\ 634\ 696 \cos(2 q_y) - \\
& 7\ 124\ 418\ 190\ 183\ 710 \cos(4 q_y) + 28\ 800\ 768\ 510\ 344\ 900 \cos(2 (q_x + q_y)) - 1\ 031\ 493\ 519\ 759\ 625 \cos(2 (q_x + 2 q_y)) + 104\ 493\ 594\ 962\ 874\ 390) \epsilon_y^4 + 1\ 127\ 100 \cos(q_y) \\
& (226\ 760\ 047\ 458\ 931\ 834 \cos(2 q_x) + 63\ 226\ 570\ 697\ 650\ 635 \cos(2 (q_x - q_y)) + 85\ 203\ 185\ 368\ 922\ 438 \cos(2 q_y) + 63\ 226\ 570\ 697\ 650\ 635 \cos(2 (q_x + q_y)) + 536\ 755\ 303\ 105\ 443\ 074) \cos(q_z) \epsilon_x \epsilon_y^3 + \\
& 461\ 890 (241\ 590\ 221\ 276\ 456\ 052 \cos(2 q_x) + 145\ 469\ 210\ 965\ 983\ 954 \cos(2 (q_x - q_y)) + 547\ 592\ 158\ 923\ 252\ 492 \cos(2 q_y) + 145\ 469\ 210\ 965\ 983\ 954 \cos(2 (q_x + q_y)) + \\
& 145\ 469\ 210\ 965\ 983\ 954 \cos(2 (q_x - q_z)) + 61\ 752\ 265\ 120\ 742\ 625 \cos(2 (q_x - q_y - q_z)) + 190\ 476\ 735\ 598\ 070\ 350 \cos(2 (q_y - q_z)) + \\
& 61\ 752\ 265\ 120\ 742\ 625 \cos(2 (q_x + q_y - q_z)) + 547\ 592\ 158\ 923\ 252\ 492 \cos(2 q_z) + 145\ 469\ 210\ 965\ 983\ 954 \cos(2 (q_x + q_z)) + 61\ 752\ 265\ 120\ 742\ 625 \cos(2 (q_x - q_y + q_z)) + \\
& 190\ 476\ 735\ 598\ 070\ 350 \cos(2 (q_y + q_z)) + 61\ 752\ 265\ 120\ 742\ 625 \cos(2 (q_x + q_y + q_z)) + 722\ 801\ 721\ 130\ 204\ 716) \epsilon_z^2 \epsilon_y^2 + 1\ 127\ 100 \cos(q_y) \cos(q_z) \\
& (226\ 760\ 047\ 458\ 931\ 834 \cos(2 q_x) + 63\ 226\ 570\ 697\ 650\ 635 \cos(2 (q_x - q_z)) + 85\ 203\ 185\ 368\ 922\ 438 \cos(2 q_z) + 63\ 226\ 570\ 697\ 650\ 635 \cos(2 (q_x + q_z)) + 536\ 755\ 303\ 105\ 443\ 074) \epsilon_z^3 \epsilon_y + \\
& 1\ 303\ 533 (58\ 152\ 516\ 016\ 142\ 730 \cos(2 q_x) - 1\ 031\ 493\ 519\ 759\ 625 \cos(2 (q_x - 2 q_z)) + 28\ 800\ 768\ 510\ 344\ 900 \cos(2 (q_x - q_z)) + 48\ 549\ 817\ 895\ 634\ 696 \cos(2 q_z) - \\
& 7\ 124\ 418\ 190\ 183\ 710 \cos(4 q_z) + 28\ 800\ 768\ 510\ 344\ 900 \cos(2 (q_x + q_z)) - 1\ 031\ 493\ 519\ 759\ 625 \cos(2 (q_x + 2 q_z)) + 104\ 493\ 594\ 962\ 874\ 390) \epsilon_z^4 \epsilon_x^4 + \\
& 1\ 314\ 610 \cos(q_x) (-305\ 613 (-22\ 055\ 935\ 460\ 016\ 250 \cos(q_y) + 5\ 283\ 563\ 479\ 587\ 967 \cos(3 q_y) + 218\ 655\ 867\ 954\ 875 \cos(5 q_y)) \epsilon_y^5 + \\
& 3770 (1\ 730\ 167\ 322\ 158\ 143\ 988 \cos(2 q_y) - 429\ 044\ 746\ 416\ 822\ 975 \cos(4 q_y) + 6\ 421\ 215\ 185\ 231\ 777\ 931) \cos(q_z) \epsilon_x \epsilon_y^4 + \\
& 52\ 780 \cos(q_y) (24\ 468\ 715\ 615\ 587\ 642 \cos(2 q_y) + 5 (6\ 222\ 005\ 410\ 021\ 245 \cos(2 (q_y - q_z)) + 82\ 009\ 905\ 587\ 163\ 710 \cos(2 q_z) + 6\ 222\ 005\ 410\ 021\ 245 \cos(2 (q_y + q_z)) + 135\ 736\ 584\ 119\ 121\ 846) \epsilon_z^2 \epsilon_y^3 + \\
& 52\ 780 \cos(q_z) (410\ 049\ 527\ 935\ 818\ 550 \cos(2 q_y) + 31\ 110\ 027\ 050\ 106\ 225 \cos(2 (q_y - q_z)) + 24\ 468\ 715\ 615\ 587\ 642 \cos(2 q_z) + 31\ 110\ 027\ 050\ 106\ 225 \cos(2 (q_y + q_z)) + 678\ 682\ 920\ 595\ 609\ 230) \epsilon_z^3 \epsilon_y^2 + \\
& 3770 \cos(q_y) (1\ 730\ 167\ 322\ 158\ 143\ 988 \cos(2 q_z) - 429\ 044\ 746\ 416\ 822\ 975 \cos(4 q_z) + 6\ 421\ 215\ 185\ 231\ 777\ 931) \epsilon_z^4 \epsilon_y - \\
& 305\ 613 (-22\ 055\ 935\ 460\ 016\ 250 \cos(q_z) + 5\ 283\ 563\ 479\ 587\ 967 \cos(3 q_z) + 218\ 655\ 867\ 954\ 875 \cos(5 q_z)) \epsilon_z^5 \epsilon_x + \\
& 377 (1\ 341\ 335\ 457 (-443\ 913\ 672\ 901\ 179 \cos(2 q_y) + 6\ 195\ 867\ 562\ 590 \cos(4 q_y) + 425 (2\ 828\ 142\ 515 \cos(6 q_y) + 1\ 790\ 837\ 852\ 258)) \epsilon_y^6 + \\
& 143\ 881\ 674\ 300 (83\ 805\ 177\ 792\ 886 \cos(q_y) - 24\ 017\ 169\ 020\ 137 \cos(3 q_y) + 105\ 584\ 014\ 195 \cos(5 q_y)) \cos(q_z) \epsilon_x \epsilon_y^5 - \\
& 193\ 325 (3\ 108\ 729\ 593\ 158\ 423\ 416 \cos(2 q_y) + 5\ 743\ 038\ 318\ 082\ 194\ 402 \cos(4 q_y) + 17 (134\ 073\ 949\ 961\ 514\ 975 \cos(4 q_y - 2 q_z) - 825\ 388\ 193\ 136\ 040\ 604 \cos(2 (q_y - q_z)) - \\
& 4\ 531\ 039\ 763\ 474\ 630\ 406 \cos(2 q_z) - 825\ 388\ 193\ 136\ 040\ 604 \cos(2 (q_y + q_z)) + 134\ 073\ 949\ 961\ 514\ 975 \cos(2 (2 q_y + q_z)) - 6\ 805\ 749\ 817\ 327\ 430\ 970) \epsilon_z^4 \epsilon_y^4 + 449\ 106\ 000 \cos(q_y) \cos(q_z) \\
& (-2\ 893\ 688\ 202\ 539\ 726 \cos(2 q_y) + 7\ 989\ 174\ 900\ 132\ 525 \cos(2 (q_y - q_z)) - 2\ 893\ 688\ 202\ 539\ 726 \cos(2 q_z) + 7\ 989\ 174\ 900\ 132\ 525 \cos(2 (q_y + q_z)) + 130\ 989\ 210\ 802\ 201\ 794) \epsilon_z^3 \epsilon_y^3 + \\
& 193\ 325 (77\ 027\ 675\ 979\ 068\ 716\ 902 \cos(2 q_y) - 2\ 279\ 257\ 149\ 345\ 754\ 575 \cos(2 (q_y - 2 q_z)) + 14\ 031\ 599\ 283\ 312\ 690\ 268 \cos(2 (q_y - q_z)) - 3\ 108\ 729\ 593\ 158\ 423\ 416 \cos(2 q_z) - \\
& 5\ 743\ 038\ 318\ 082\ 194\ 402 \cos(4 q_z) + 14\ 031\ 599\ 283\ 312\ 690\ 268 \cos(2 (q_y + q_z)) - 2\ 279\ 257\ 149\ 345\ 754\ 575 \cos(2 (q_y + 2 q_z)) + 115\ 697\ 746\ 894\ 566\ 326\ 490) \epsilon_z^4 \epsilon_y^2 + \\
& 143\ 881\ 674\ 300 \cos(q_x) (83\ 805\ 177\ 792\ 886 \cos(q_z) - 24\ 017\ 169\ 020\ 137 \cos(3 q_z) + 105\ 584\ 014\ 195 \cos(5 q_z)) \epsilon_z^5 \epsilon_y + \\
& 1\ 341\ 335\ 457 (-443\ 913\ 672\ 901\ 179 \cos(2 q_z) + 6\ 195\ 867\ 562\ 590 \cos(4 q_z) + 425 (2\ 828\ 142\ 515 \cos(6 q_z) + 1\ 790\ 837\ 852\ 258)) \epsilon_z^6)
\end{aligned}$$

2.5 Expressions

8th order

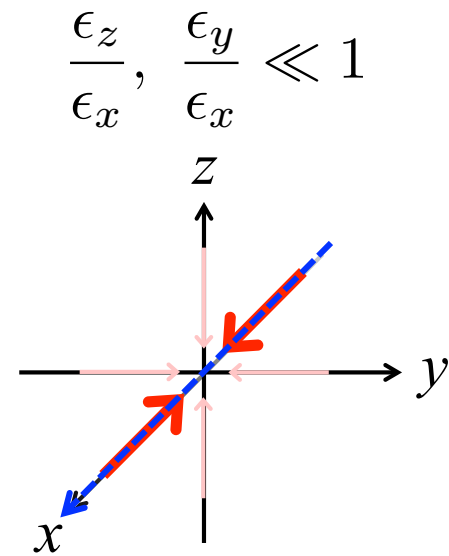
$$\begin{aligned}
& 1069117866119953375191385920000000 \\
& \sin(q_x) \epsilon_x (1.633746586626 - 245713485247929951 \cos(2q_x) - 2195281540970250 \cos(4q_x) + 494330346828175 \cos(6q_x) + 865027121950584922) (\cos(q_x) \epsilon_y + \cos(q_x) \epsilon_z) \epsilon_x^2 - 7779240 \cos(q_x) (627(428992591700823991100 \cos(2q_x) + 1571794373829512625 \cos(4q_x) + \\
& \quad 33352805228223745550 \cos(6q_x - 2q_y) + 515931466598138198040 \cos(2(q_x - q_y)) - 7784580247782371085868 \cos(2q_x) + 515931466598138198040 \cos(2(q_x + q_y)) + 33352805228223745550 \cos(2(2q_x + q_y)) - 1529203372519416608365) \epsilon_x^2 + \\
& \quad 2371330(560926390417289924 \cos(2q_x) + 2752127183932775 \cos(4q_x) - 2607108359711238507) \cos(q_x) \cos(q_x) \epsilon_x \epsilon_y + 627(428992591700823991100 \cos(2q_x) + 1571794373829512625 \cos(4q_x) + 33352805228223745550 \cos(6q_x - 2q_y) + \\
& \quad 515931466598138198040 \cos(2(q_x - q_y)) - 7784580247782371085868 \cos(2q_x) + 515931466598138198040 \cos(2(q_x + q_y)) + 33352805228223745550 \cos(2(2q_x + q_y)) - 1529203372519416608365) \epsilon_x^2 \epsilon_x^2 + \\
& \quad 3515(4584195(1037796098184111135612 \cos(2q_x - 3q_y) + 22425047626060301625 \cos(4q_x - 3q_y) + 1065982814286662929188 \cos(2q_x - q_y) - 71045518667401407385 \cos(4q_x - q_y) + 5558973502654370204522 \cos(q_y) + \\
& \quad 2436591489369587854102 \cos(3q_y) + 1065982814286662929188 \cos(2q_x + q_y) - 71045518667401407385 \cos(4q_x + q_y) + 1037796098184111135612 \cos(2q_x + 3q_y) + 22425047626060301625 \cos(4q_x + 3q_y) \epsilon_x^2 \epsilon_x^2 + \\
& \quad 58(8572509073745045865079448 \cos(2q_x) - 11788106125102093582384050 \cos(4q_x) - 74093859475845652593375 \cos(4q_x - 2q_y) + 169968401151646360846947780 \cos(2(q_x - q_y)) + 805731257316020840390800630 \cos(2q_y) + 169968401151646360846947780 \cos(2(q_x + q_y)) - \\
& \quad 74093859475845652593375 \cos(2(2q_x + q_y)) + 471288301537253467508400474) \cos(q_x) \epsilon_x^2 \epsilon_x^2 + 58 \cos(q_x) (8572509073745045865079448 \cos(2q_x) - 11788106125102093582384050 \cos(4q_x) - 74093859475845652593375 \cos(4q_x - 2q_y) + \\
& \quad 169968401151646360846947780 \cos(2(q_x - q_y)) + 805731257316020840390800630 \cos(2q_y) + 1065982814286662929188 \cos(2q_x - q_y) - 71045518667401407385 \cos(4q_x - q_y) + 5558973502654370204522 \cos(q_y) + \\
& \quad 4584195(1037796098184111135612 \cos(2q_x - 3q_y) + 22425047626060301625 \cos(4q_x - 3q_y) + 1065982814286662929188 \cos(2q_x - q_y) - 71045518667401407385 \cos(4q_x - q_y) + 5558973502654370204522 \cos(q_y) + \\
& \quad 2436591489369587854102 \cos(3q_y) + 1065982814286662929188 \cos(2q_x + q_y) - 71045518667401407385 \cos(4q_x + q_y) + 1037796098184111135612 \cos(2q_x + 3q_y) + 22425047626060301625 \cos(4q_x + 3q_y) \epsilon_x^2 \epsilon_x^2 + \\
& \quad 214600(2488563(2404131967594317394930 \cos(q_x) + 14699297937925103750 \cos(3q_x) + 1271785013595974822 \cos(4q_x - 4q_y) + 4399990193891960850 \cos(3q_x - 4q_y) + 133664268006839401079 \cos(q_x - 2q_y) + 23125200373177343889 \cos(3q_x - 2q_y) + \\
& \quad 133664268006839401079 \cos(q_x + 2q_y) + 23125200373177343889 \cos(3q_x + 2q_y) + 1271785013595974822 \cos(q_x + 4q_y) + 4399990193891960850 \cos(3q_x + 4q_y) \epsilon_x^2 \epsilon_x^2 + \\
& \quad 680 \cos(q_x) \cos(q_x) (26377362711437143406448 \cos(2q_x) + 1724350177131876798541955 \cos(2(q_x - q_y)) + 103537114811431170053964 \cos(2q_y) + 24350177131876798541955 \cos(2(q_x + q_y)) + 189521250296157892651058) \cos(q_x) \epsilon_x \epsilon_x^2 + \\
& \quad 190190 \cos(q_x) (73150733551296492000 \cos(2q_x) + 394609048551392352993 \cos(2(q_x - q_y)) + 6492905696156135980254 \cos(2q_y) + 394609048551392352993 \cos(2(q_x + q_y)) + \\
& \quad 394609048551392352993 \cos(2(q_x - q_y)) + 290989365058303439205 \cos(2(q_x - q_y) + 290989365058303439205 \cos(2(q_x + q_y) - q_z)) + 6492905696156135980254 \cos(2q_x) + \\
& \quad 394609048551392352993 \cos(2(q_x + q_z)) + 290989365058303439205 \cos(2(q_x - q_y + q_z)) + 2718224708508735619054 \cos(2(q_x + q_z)) + 290989365058303439205 \cos(2(q_x + q_y + q_z)) + 5481903536908993663560) \epsilon_x^2 \epsilon_x^2 + \\
& \quad 680 \cos(q_x) \cos(q_x) \cos(q_x) (26377362711437143406448 \cos(2q_x) + 1724350177131876798541955 \cos(2(q_x - q_y)) + 103537114811431170053964 \cos(2q_x) + 24350177131876798541955 \cos(2(q_x + q_z)) + 189521250296157892651058) \epsilon_x^2 \epsilon_x^2 + \\
& \quad 2488563(2404131967594317394930 \cos(q_x) + 14699297937925103750 \cos(3q_x) + 1271785013595974822 \cos(q_x - 4q_y) + 4399990193891960850 \cos(3q_x - 4q_y) + 133664268006839401079 \cos(q_x - 2q_y) + \\
& \quad 23125200373177343889 \cos(3q_x - 2q_y) + 133664268006839401079 \cos(q_x + 2q_y) + 23125200373177343889 \cos(3q_x + 2q_y) + 1271785013595974822 \cos(q_x + 4q_y) + 4399990193891960850 \cos(3q_x + 4q_y) \epsilon_x^2 \epsilon_x^2 + \\
& \quad 1850(174191941 \cos(q_x) (2694647836666346056502 \cos(2q_x) - 79399637804625869575 \cos(2(q_x - 2q_y)) + 75315853526608759468 \cos(2(q_x - q_y)) - 6312646270370160227256 \cos(2q_y) - 1402382546049784852450 \cos(4q_x) + 75315853526608759468 \cos(2(q_x + q_y)) - \\
& \quad 79399637804625869575 \cos(2(q_x + 2q_y)) + 3516837643125107421146) \epsilon_x^2 \epsilon_x^2 + 2465(37237442991970252628982294 \cos(2q_x) - 312093211837576432167135 \cos(2(q_x - 2q_y)) + 17345506396856886439397076 \cos(2(q_x - q_y)) + \\
& \quad 3373524641470653978366808 \cos(2q_x) - 3487168087521395361696570 \cos(4q_x) + 17345506396856886439397076 \cos(2(q_x + q_y)) - 312093211837576432167135 \cos(2(q_x + 2q_y)) + 6489411721032327720719330) \cos(q_x) \epsilon_x \epsilon_x^2 + \\
& \quad 121818 \cos(q_x) (8011371695323945952255196 \cos(2q_x) + 2017084716808033597921350 \cos(2(q_x - q_y)) + 3303096819059364071793132 \cos(2q_y) + 2017084716808033597921350 \cos(2(q_x + q_y)) + 3564496447098991842491490 \cos(2(q_x - q_z)) + \\
& \quad 769814407616991363782925 \cos(2(q_x - q_y) - q_z)) + 152933797910081005590610 \cos(2(q_x - q_y) + q_z) + 769814407616991363782925 \cos(2(q_x + q_y - q_z)) + 12481850044517386286724980 \cos(2q_x) + \\
& \quad 3564496447098991842491490 \cos(2(q_x + q_z)) + 769814407616991363782925 \cos(2(q_x - q_y + q_z)) + 152933797910081005590610 \cos(2(q_x + q_z)) + 769814407616991363782925 \cos(2(q_x + q_y + q_z)) + 17289778089122435681309532) \epsilon_x^2 \epsilon_x^2 + \\
& \quad 121818 \cos(q_x) (8011371695323945952255196 \cos(2q_x) + 3564496447098991842491490 \cos(2(q_x - q_y)) + 12481850044517386286724980 \cos(2q_y) + 3564496447098991842491490 \cos(2(q_x + q_z)) + 2017084716808033597921350 \cos(2(q_x - q_z)) + \\
& \quad 769814407616991363782925 \cos(2(q_x - q_y) - q_z)) + 152933797910081005590610 \cos(2(q_x - q_z)) + 769814407616991363782925 \cos(2(q_x + q_y - q_z)) + 3303096819059364071793132 \cos(2q_x) + \\
& \quad 2017084716808033597921350 \cos(2(q_x + q_z)) + 769814407616991363782925 \cos(2(q_x - q_y + q_z)) + 152933797910081005590610 \cos(2(q_x + q_z)) + 769814407616991363782925 \cos(2(q_x + q_y + q_z)) + 17289778089122435681309532) \epsilon_x^2 \epsilon_x^2 + \\
& \quad 2465 \cos(q_x) (37237442991970252628982294 \cos(2q_x) - 312093211837576432167135 \cos(2(q_x - 2q_y)) + 17345506396856886439397076 \cos(2(q_x - q_z)) + 33737524641470653978366808 \cos(2q_x) - 3487168087521395361696570 \cos(4q_x) + \\
& \quad 17345506396856886439397076 \cos(2(q_x + q_z)) - 312093211837576432167135 \cos(2(q_x + 2q_y)) + 6489411721032327720719330) \epsilon_x^2 \epsilon_x^2 + 174191941 \cos(q_x) (2694647836666346056502 \cos(2q_x) - 79399637804625869575 \cos(2(q_x - 2q_y)) + \\
& \quad 75315853526608759468 \cos(2(q_x - q_z)) - 6312646270370160227256 \cos(2q_x) - 1402382546049784852450 \cos(4q_x) + 75315853526608759468 \cos(2(q_x + q_z)) - 79399637804625869575 \cos(2(q_x + 2q_y)) + 3516837643125107421146) \epsilon_x^2 \epsilon_x^2 - \\
& \quad 40 \cos(q_x) (121939587(1402580864994528556478 \cos(2q_x) + 945250976433680996345 \cos(4q_x) - 5525(4261854929259466 \cos(6q_x) + 1482430507417107313) \epsilon_x^2 \epsilon_x^2 + \\
& \quad 22527635(-306582770556862288653010 \cos(q_x) + 54767957989838564683227 \cos(3q_x) + 4307453213356403211255 \cos(5q_x)) \cos(q_x) \epsilon_x \epsilon_x^2 - 510184675(4639444862944510447176 \cos(2q_x) - 838687869143105762277 \cos(4q_x) - \\
& \quad 17(13099400197237968510 \cos(4q_x - 2q_z) - 148900726126399338934 \cos(2(q_y - q_z)) - 556208371442325201600 \cos(2q_z) - 148900726126399338934 \cos(2(q_y + q_z)) + 13099400197237968510 \cos(2(2q_x + q_z)) - 817160160332733313005) \epsilon_x^2 \epsilon_x^2 - \\
& \quad 2371330 \cos(q_x) \cos(q_x) (77415446480467820945874 \cos(2q_x) + 413269909554286670675025 \cos(2(q_x - q_z)) + 77415446480467820945874 \cos(2q_x) + 413269909554286670675025 \cos(2(q_x + q_z)) + 720165127414798218506546) \epsilon_x^2 \epsilon_x^2 - \\
& \quad 510184675(9455542314519528427200 \cos(2q_x) - 222689803353045464670 \cos(2(q_x - 2q_z)) + 2531312344148788761878 \cos(2(q_x - q_z)) + 4639444862944510447176 \cos(2q_x) - 838687869143105762277 \cos(4q_x) + 2531312344148788761878 \cos(2(q_x + q_z)) - \\
& \quad 222689803353045464670 \cos(2(q_x + 2q_z)) + 13891722725656466321085) \epsilon_x^2 \epsilon_x^2 + 22527635 \cos(q_x) (-306582770556862288653010 \cos(q_x) + 54767957989838564683227 \cos(3q_x) + 4307453213356403211255 \cos(5q_x)) \epsilon_x^2 \epsilon_x^2 + \\
& \quad 121939587(1402580864994528556478 \cos(2q_x) + 945250976433680996345 \cos(4q_x) - 5525(4261854929259466 \cos(6q_x) + 1482430507417107313) \epsilon_x^2 \epsilon_x^2 + \\
& \quad 29(28168044597(827808374798014805 \cos(q_x) - 407511682759452297 \cos(3q_x) + 32682412580801125 \cos(5q_x) - 335214948722225 \cos(7q_x)) \epsilon_x^2 \epsilon_x^2 + 79513556850(-3199521946356060169 \cos(2q_x) - 426269439903081070 \cos(4q_x) + 22157903009071225 \cos(6q_x) + 8222380144549761790) \\
& \quad \cos(q_x) \epsilon_x \epsilon_x^2 - 7030 \cos(q_x) (10793525853032550940917704 \cos(2q_x) + 1988748601768714806578250 \cos(4q_x) + 3488726031704474627341275 \cos(4q_x - 2q_z) + 3479121831999070815324380 \cos(2(q_x - q_z)) - \\
& \quad 302741637586991729892000430 \cos(2q_x) + 3479121831999070815324380 \cos(2(q_x + q_z)) + 3488726031704474627341275 \cos(2(2q_x + q_z)) - 272208306739742605716514002) \epsilon_x^2 \epsilon_x^2 + \\
& \quad 78625(3699848237896519803002532 \cos(2q_x - 3q_z) + 198728133191062035450685 \cos(4q_x - 3q_z) + 9188393949480738286320572 \cos(2q_x - q_z) - 1558299818480711534986485 \cos(4q_x - q_z) + 55806125520765647027674674 \cos(q_x) + \\
& \quad 3508054663308705034350462 \cos(3q_x) + 9188393949480738286320572 \cos(2q_x + q_z) - 1558299818480711534986485 \cos(4q_x + q_z) + 3699848237896519803002532 \cos(2q_x + 3q_z) + 198728133191062035450685 \cos(4q_x + 3q_z) \epsilon_x^2 \epsilon_x^2 + \\
& \quad 78625(55806125520765647027674674 \cos(q_x) + 3508054663308705034350462 \cos(3q_x) - 1558299818480711534986485 \cos(q_x - 4q_z) + 198728133191062035450685 \cos(3q_x - 4q_z) + 9188393949480738286320572 \cos(q_x - 2q_z) + \\
& \quad 3699848237896519803002532 \cos(3q_x - 2q_z) + 9188393949480738286320572 \cos(q_x + q_z) + 3699848237896519803002532 \cos(3q_x + 2q_z) - 1558299818480711534986485 \cos(q_x + 4q_z) + 198728133191062035450685 \cos(3q_x + 4q_z) \epsilon_x^2 \epsilon_x^2 - \\
& \quad 7030 \cos(q_x) (3(1162908677234824875780425 \cos(2(q_x - 2q_z)) + 11597072773330236238441460 \cos(2(q_x - 2q_z)) + 359784195108503146927568 \cos(2q_x) + 662916200589571602192750 \cos(4q_x) + 11597072773330236238441460 \cos(2(q_x + q_z)) + \\
& \quad 1162908677234824875780425 \cos(2(q_x + 2q_z)) - 90736102246580868572171334) - 302741637586991729892000430 \cos(2q_x)) \epsilon_x^2 \epsilon_x^2 + 79513556850 \cos(q_x) \\
& \quad (-3199521946356060169 \cos(2q_x) - 426269439903081070 \cos(4q_x) + 22157903009071225 \cos(6q_x) + 8222380144549761790) \epsilon_x^2 \epsilon_x^2 + 22157903009071225 \cos(6q_x) + 8222380144549761790) \epsilon_x^2 \epsilon_x^2 +
\end{aligned}$$

2.7 Expressions

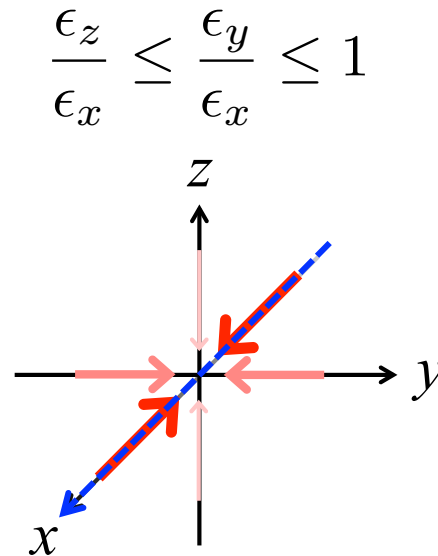
10th order

$$\sin(q_x) \epsilon_x \left(91\,305\,563\,116\,476\,345\,919\,879\,482\,171\,966\,519\,111\,359\,819\,301\,570 \cos(q_y) \epsilon_y^9 + 52\,938\,743\,355\,21 \cdot 3\,522\,223\,243\,002\,878\,074\,733\,076\,205\,267\,436\,326\,841\,766\,249\,489\,660 \cos(q_x + 2q_y) \epsilon_x \epsilon_y^8 + 1\,026\,823\,7 \right)$$

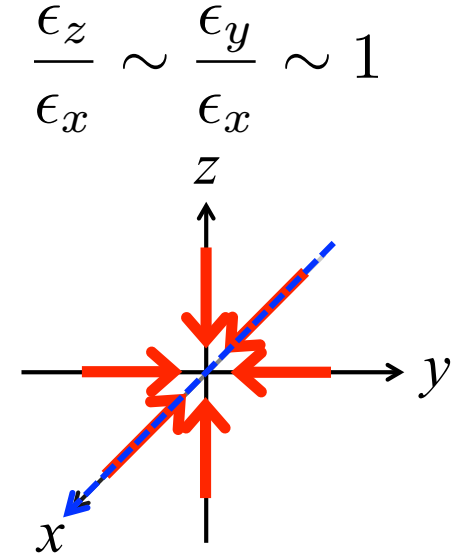
3 Classification of Initial condition



Q1D

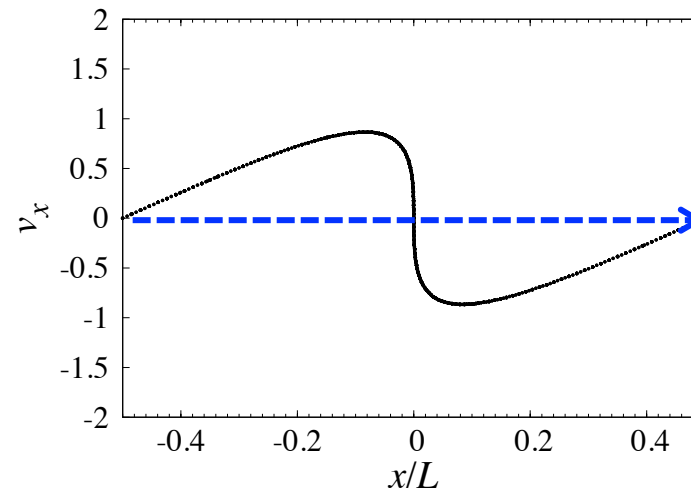


ASY

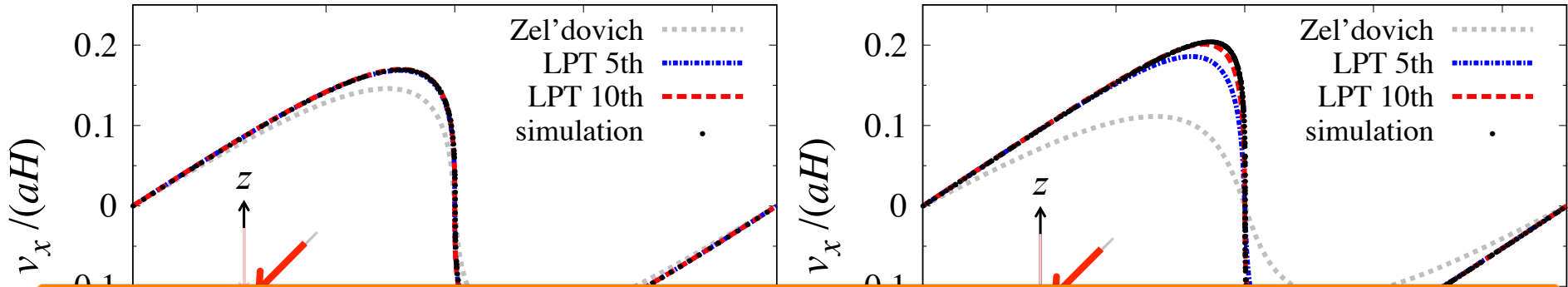


SYM

$$\Psi^{(1)}(\mathbf{q}, t) = D_+(t) \begin{pmatrix} \epsilon_x \sin q_x \\ \epsilon_y \sin q_y \\ \epsilon_z \sin q_z \end{pmatrix}$$



3.1 Result

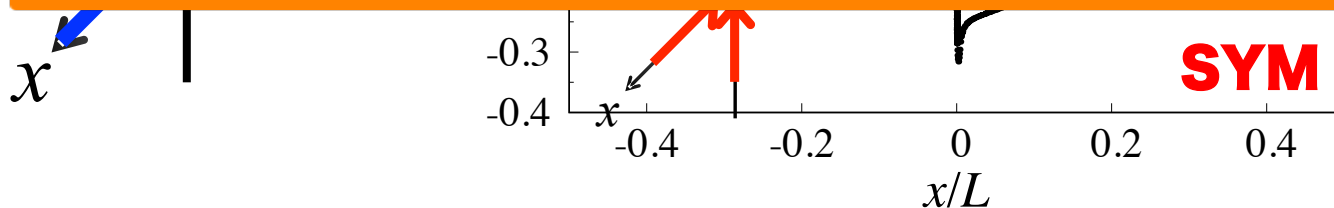


Fitting function $x_i(\mathbf{q}, t, n)$

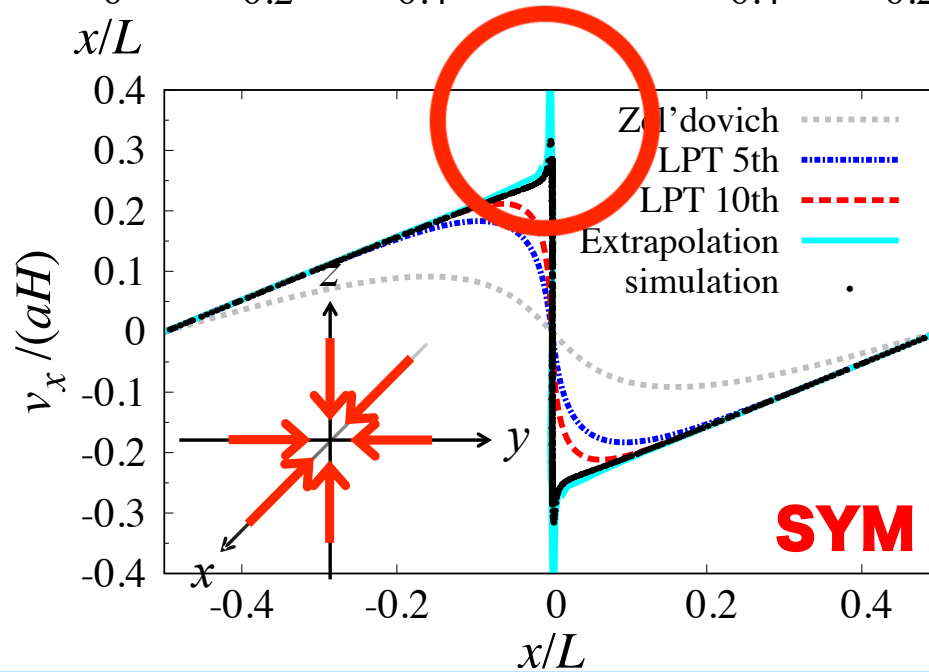
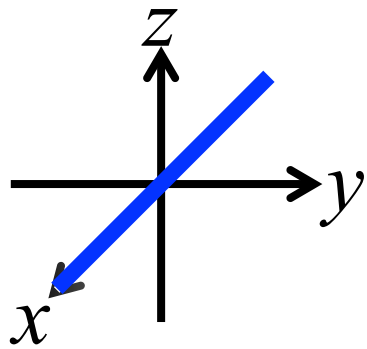
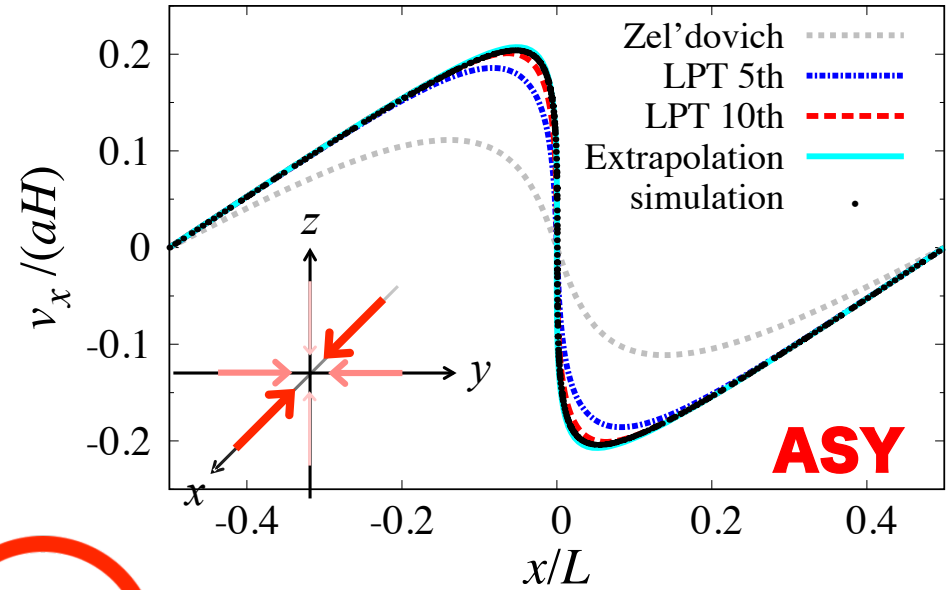
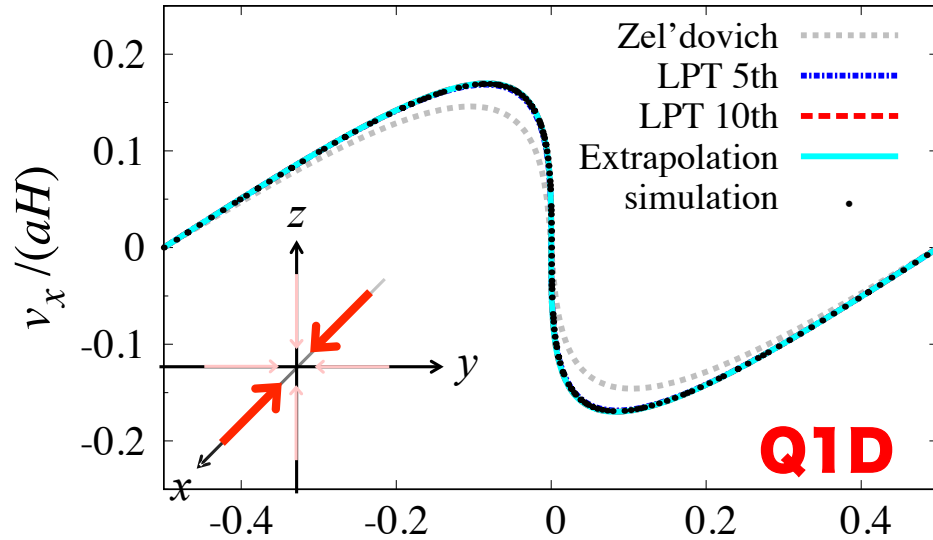
$$x_i(\mathbf{q}, t, n) = q_i + \sum_{a=1}^n \Psi_i^{(a)}(\mathbf{q}, t)$$

$$= a_i(\mathbf{q}, t) + \frac{1}{b_i(\mathbf{q}, t) + c_i(\mathbf{q}, t) \exp [d_i(\mathbf{q}, t) n^{e_i(\mathbf{q}, t)}]}$$

$n \rightarrow \infty$: “Effective” infinite order

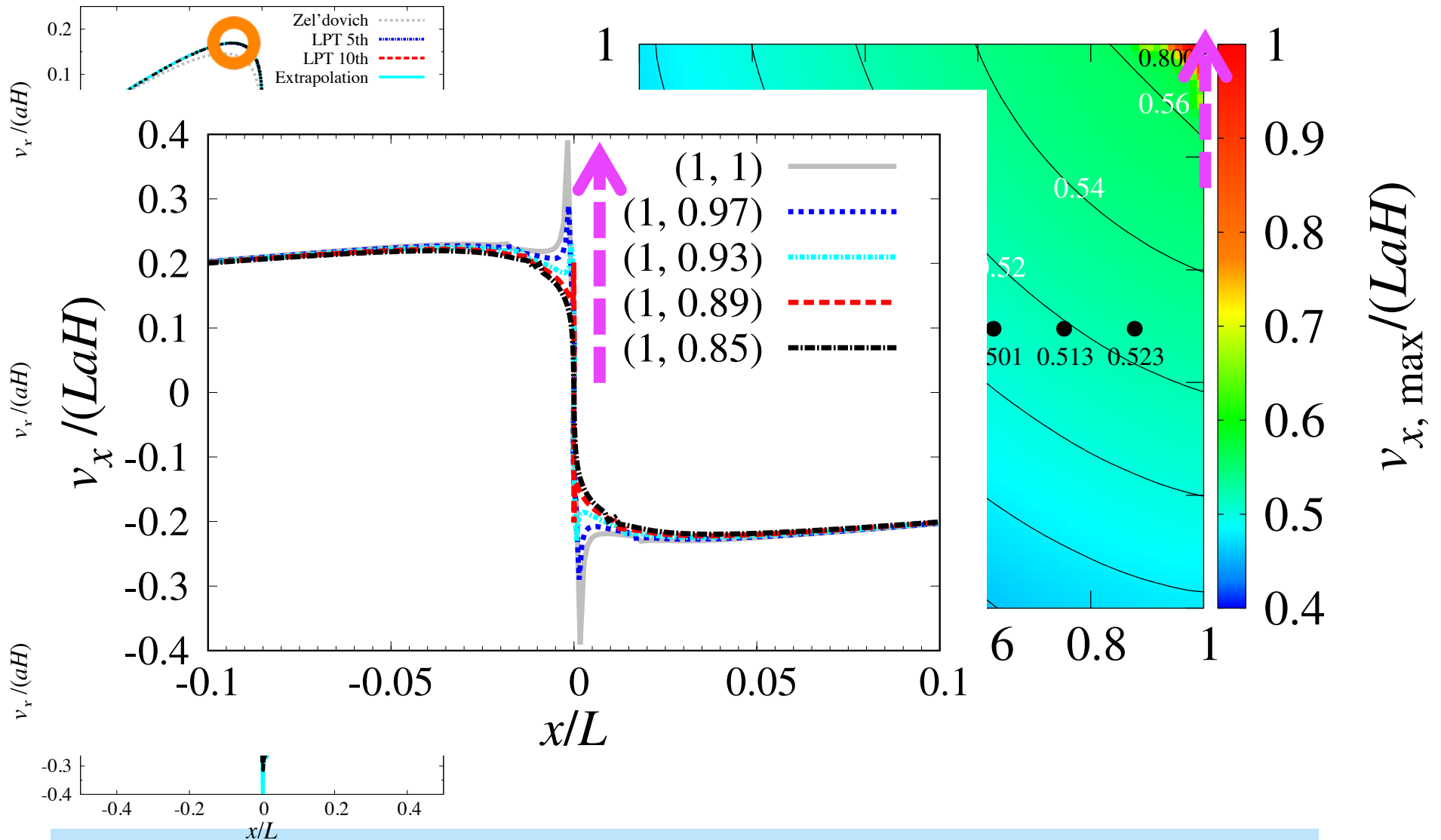


3.2 Extrapolation



3.3 Exploring parameter space

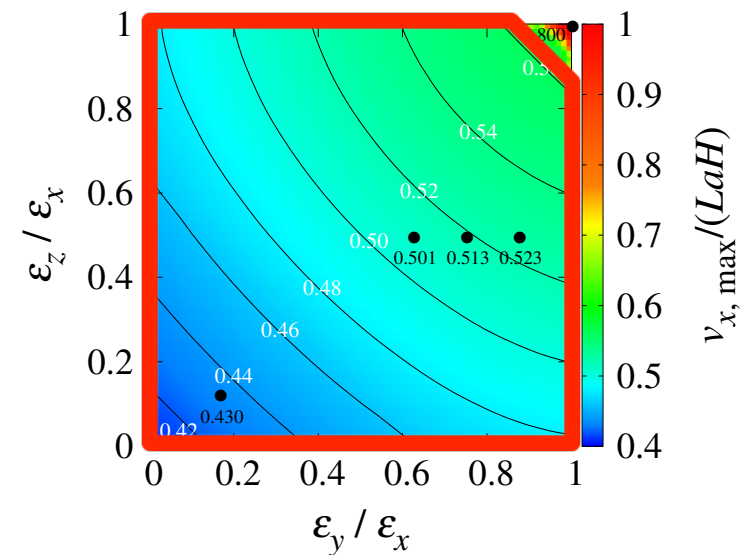
Maximum velocity at shell crossing: $(\epsilon_y/\epsilon_x, \epsilon_z/\epsilon_x)$



4. Summary

- We consider primordial dark matter halos seeded by three sine waves.
- We perform **10th order LPT and beyond**.
- Except for rare cases such as the triaxial symmetric configuration, **collapse is generally expected to produce a planar singularity.**

1D \rightarrow 2D: No transitions
 2D \rightarrow 3D: Spiky feature
 (~ spherical collapse)



Next?

- Vorticity based on high-order LPT
- Multi-stream flow in 2D/3D cosmology --- Post collapse phase
- General initial condition