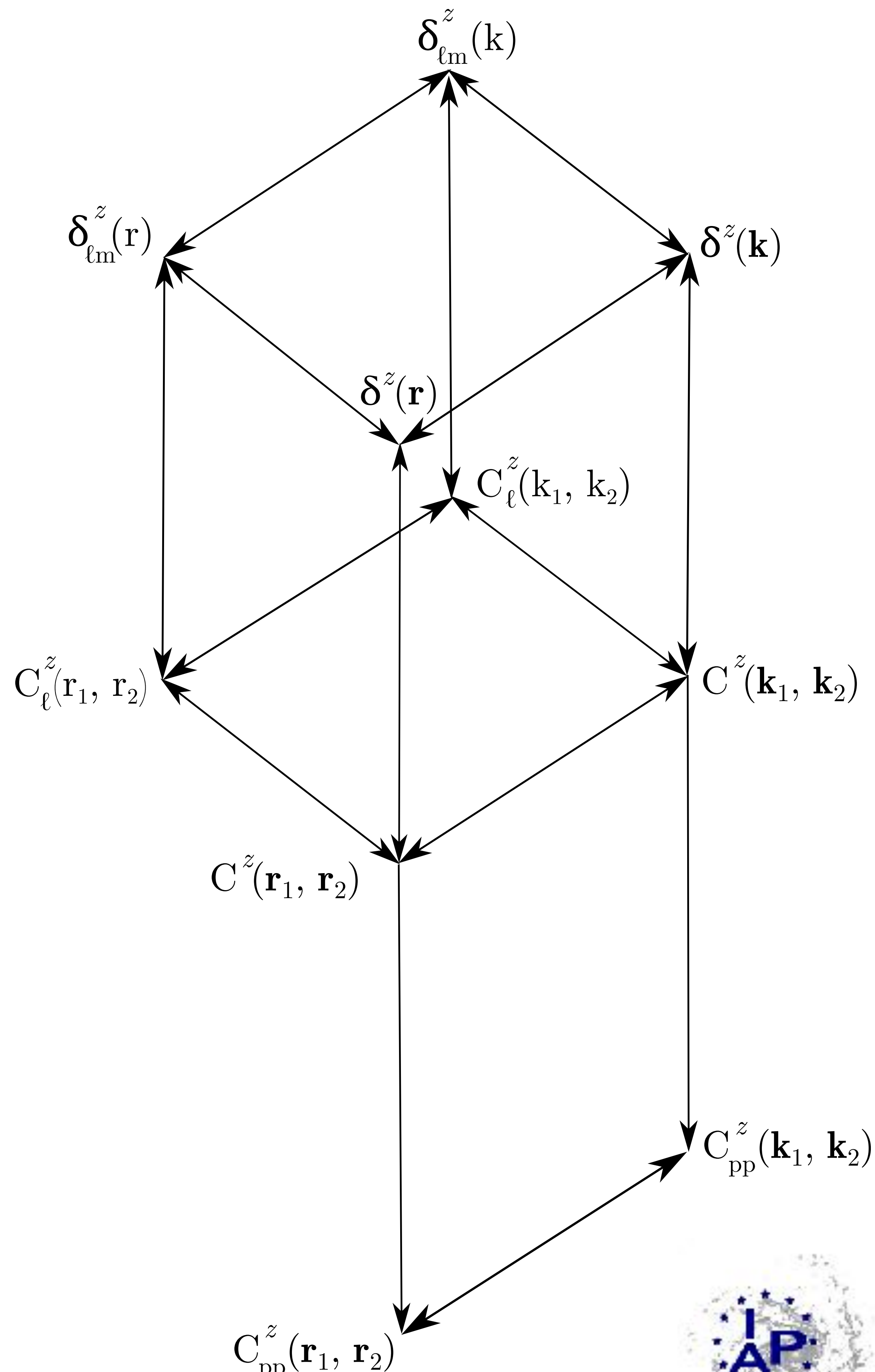
RSD and Geometry



Arxiv:1506.06596 with F. Bernardeau and C. Pitrou

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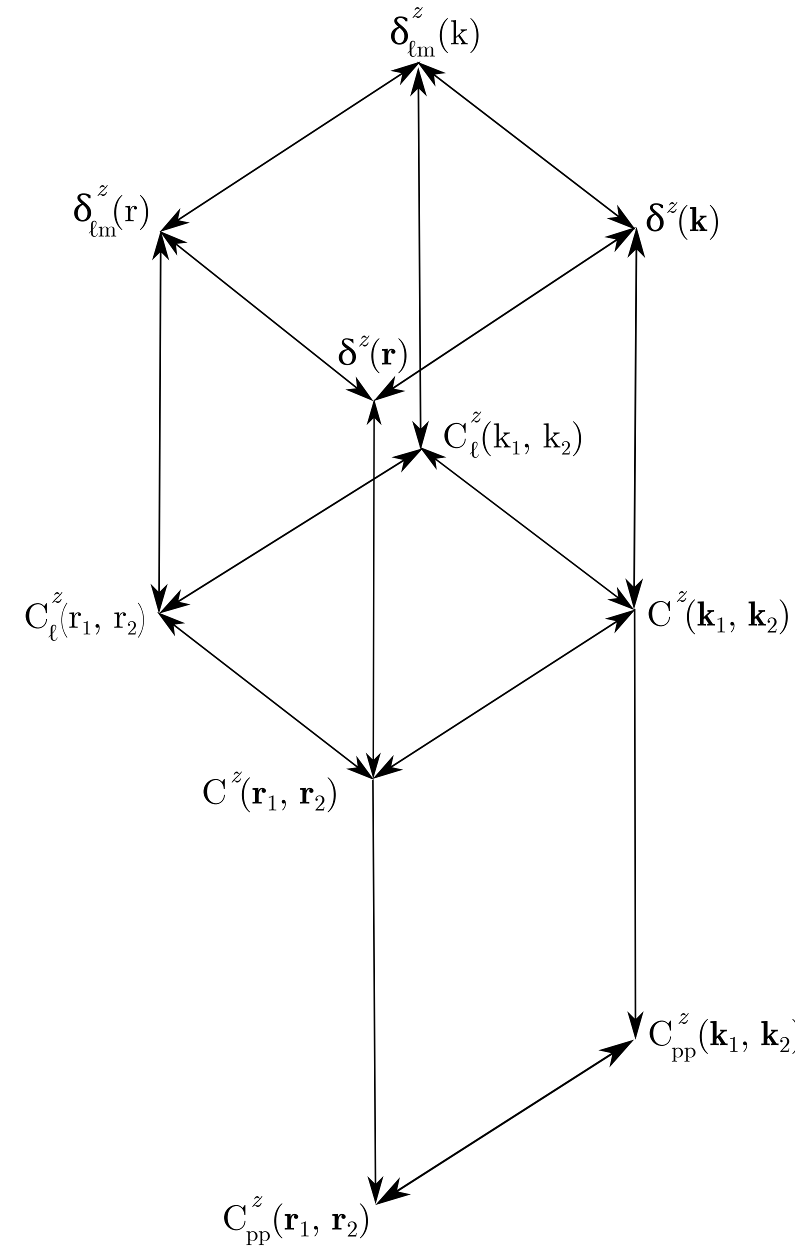
08/04

Configuration space: general formula for wide angle effects and small angle limit

Mixed space: where P(k) can be defined; Kaiser formula at the limit

Fourier space: general formula and mode coupling



Expansions around plane parallel limit *(n = 0)*	small parameter	
$\xi^z(\mathbf{d},\mathbf{r}) = \sum_{n=0}^{\infty}\left(\frac{r}{d}\right)^n \sum_{\ell=0}^{\infty} \xi_\ell^{(n)}(r) P_\ell(\mu_{\mathbf{d}\,\mathbf{r}})$	$\frac{r}{d}$	$\xi_\ell^{(n)} : \mathbb{R} \to \mathbb{R}$
$\widehat{\xi}^z(\mathbf{d},\mathbf{k}) = \widetilde{\zeta}^z(\mathbf{d},\mathbf{k}) = \sum_{n=0}^{\infty}\left(\frac{1}{kd}\right)^n \sum_{\ell=0}^{\infty} \mathcal{P}_\ell^{(n)}(k) P_\ell(\mu_{\mathbf{k}\,\mathbf{d}})$	$\frac{1}{kd}$	$\mathcal{P}_\ell^{(n)} : \mathbb{R} \to \mathbb{C}$
$\zeta^z(\Delta\mathbf{k},\mathbf{k}) = \delta_D(\Delta\mathbf{k})\zeta_0^{(0)}(k) + \frac{1}{4\pi\lvert\Delta\mathbf{k}\rvert^3} \sum_{n=0}^{\infty}\left(\frac{\lvert\Delta\mathbf{k}\rvert}{k}\right)^n \sum_{\substack{\ell=0 \\ (\ell,n)\neq(0,0)}}^{\infty} \zeta_\ell^{(n)}(k) P_\ell(\mu_{\mathbf{k}\,\boldsymbol{\Delta}})$	$\frac{\lvert\Delta\mathbf{k}\rvert}{k}$	$\zeta_\ell^{(n)} : \mathbb{R} \to \mathbb{C}$

RSD

$$\rho^z(\mathbf{s})\mathrm{d}^3\mathbf{s} = \rho(\mathbf{x})\mathrm{d}^3\mathbf{x}$$

$$\mathrm{d}^3\mathbf{s} \approx (1 + \partial_i\alpha^i)\mathrm{d}^3\mathbf{x}$$

$$s^i = x^i + \alpha^i$$

$$\delta^z = \delta - \partial_i\alpha^i$$

The volume in redshift space is affected by the divergence of the displacement field α_i, and this translates into a modification of the density with an opposite sign.

$$\partial_i\alpha_i = \frac{1}{r^2}\partial_r(r^2\alpha_r) = \frac{1}{r^2}\partial_r\left(r^2\frac{\partial_r V}{\mathcal{H}}\right) = -\frac{\beta}{r^2}\partial_r[r^2\partial_r(\triangle^{-1}\delta)]$$

$$\theta = \frac{\partial_i v^i}{\mathcal{H}} = \frac{\triangle V}{\mathcal{H}} = -\beta\delta$$

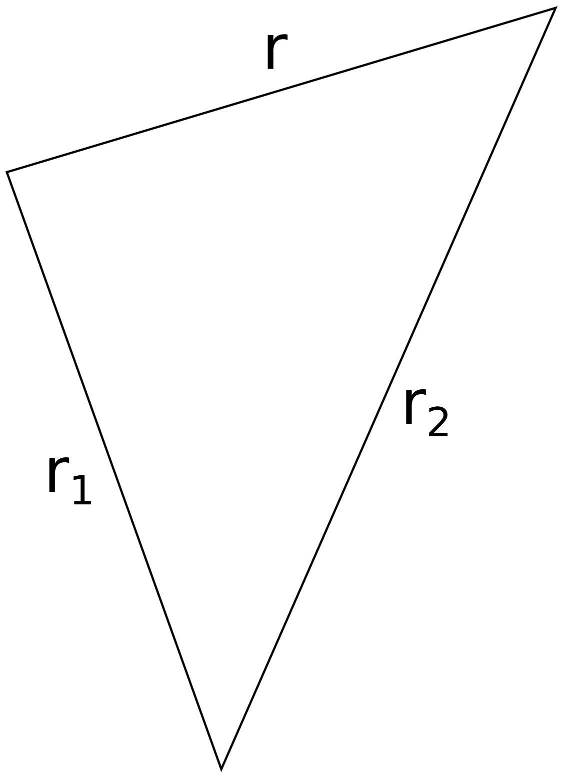
linear perturbations
no vorticity
linear bias

$$\delta^z(\mathbf{r}) = \delta(\mathbf{r}) + \beta\mathcal{O}_r(\triangle^{-1}\delta(\mathbf{r}))$$

$$\delta^z(\mathbf{r}) = \int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}}\delta(\mathbf{k})\left[1 - \frac{\beta}{k^2}\mathcal{O}_r\right]\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}$$

$$\mathcal{F}[\triangle^{-1}\delta] = -\frac{\delta(\mathbf{k})}{k^2}$$

Covariance



$$C^z(\mathbf{r}_1, \mathbf{r}_2) = \int \frac{k^2 \mathrm{d}k}{2\pi^2} \mathcal{P}(k) \left[1 - \frac{\beta}{k^2}\mathcal{O}_{r_1}\right] \left[1 - \frac{\beta}{k^2}\mathcal{O}_{r_2}\right] j_0(kr)$$

$$C(\mathbf{k}_1, \mathbf{k}_2) \equiv \langle \delta(\mathbf{k}_1)\delta^\star(\mathbf{k}_2)\rangle = \mathcal{P}(k)\delta_D(\mathbf{k}_1 - \mathbf{k}_2)$$

$$\mathrm{e}^{\mathbf{i}\mathbf{r}\cdot\mathbf{k}} = 4\pi \sum_{\ell m} \mathrm{i}^\ell j_\ell(kr) \mathrm{Y}_{\ell m}(\hat{\mathbf{r}}) \mathrm{Y}^\star_{\ell m}(\hat{\mathbf{k}}) = \sum_\ell (2\ell+1) \mathrm{i}^\ell j_\ell(kr) P_\ell(\hat{\mathbf{k}} \cdot \hat{\mathbf{r}})$$

$$j_0(kr) = \sum_\ell (2\ell+1) j_\ell(kr_1) j_\ell(kr_2) P_\ell(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_2)$$

$$C^z(\mathbf{r}_1, \mathbf{r}_2) = \int \frac{k^2 \mathrm{d}k}{2\pi^2} \mathcal{P}(k) I\left[j_0(kr)\right],$$

$$I\left[j_0(kr)\right] \equiv \left[1 + \beta + \frac{\beta \mathrm{L}^2_{\nu_{12}}}{(kr_1)^2}\right] \left[1 + \beta + \frac{\beta \mathrm{L}^2_{\nu_{12}}}{(kr_2)^2}\right] j_0(kr)$$

General formula for RSD

Triangle geometry

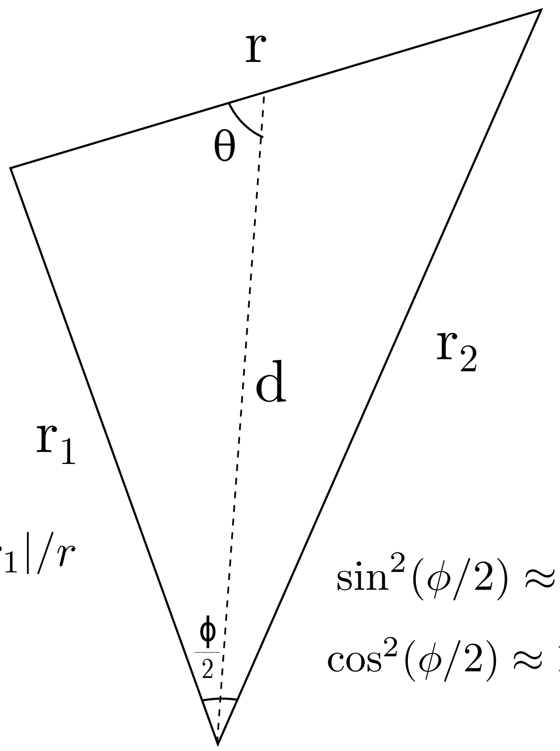
Equivalent to Papai & Szapudi (08)

Plane-parallel limit

1) configuration space

$$r^2(r_1, r_2, \phi) \equiv |\mathbf{r}_2 - \mathbf{r}_1|^2 = (r_1 + r_2)^2 \sin^2(\phi/2) + (r_2 - r_1)^2 \cos^2(\phi/2)$$

$$r^2 \approx (r_2 - r_1)^2 + \frac{1}{8}(r_1^2 + 6 r_1 r_2 + r_2^2)\phi^2$$



$$\cos\theta \approx |r_2 - r_1|/r$$

$$d\phi/r \approx \sin\theta$$

$$\sin^2(\phi/2) \approx \phi^2/4$$

$$\cos^2(\phi/2) \approx 1 - \phi^2/8$$

$$\mathrm{d}r/\mathrm{d}\phi \approx (r_1^2 + 6 r_1 r_2 + r_2^2)\phi/(8r) \approx d^2\phi/r$$

$$\mathrm{L}^2_{\nu_{12}} \approx \frac{1}{\phi}\partial_\phi \phi \partial_\phi$$

$$\frac{2}{d^2}\left(\frac{1}{\phi}\partial_\phi \phi \partial_\phi\right) j_0(kr(\phi)) = -\frac{4}{3}k^2 \left[j_0(kr)P_0(\mu_{\mathbf{d}\,\mathbf{r}}) + j_2(kr)P_2(\mu_{\mathbf{d}\,\mathbf{r}})\right]$$

$$\frac{1}{d^4}\left(\frac{1}{\phi}\partial_\phi \phi \partial_\phi\right)^2 j_0(kr(\phi)) = k^4 \left[\frac{8}{15}j_0(kr)P_0(\mu_{\mathbf{d}\,\mathbf{r}}) - \frac{16}{21}j_2(kr)P_2(\mu_{\mathbf{d}\,\mathbf{r}}) + \frac{8}{35}j_4(kr)P_4(\mu_{\mathbf{d}\,\mathbf{r}})\right]$$

$$C^z_{\mathrm{pp}}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{\ell=0,2,4} \xi^{(0)}_\ell(r) P_\ell(\mu_{\mathbf{d}\,\mathbf{r}})$$

$$\xi^{(0)}_0(r) = \left(1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2\right)\Xi^0_0(r) \quad \xi^{(0)}_4(r) = \frac{8}{35}\beta^2 \Xi^0_4(r)$$

$$\xi^{(0)}_2(r) = -\left(\frac{4}{3}\beta + \frac{4}{7}\beta^2\right)\Xi^0_2(r) \qquad \Xi^m_\ell(r) \equiv \int \frac{k^2 \mathrm{d}k}{2\pi^2}(kr)^{-m} j_\ell(kr)\mathcal{P}(k)$$

Hamilton 92

Fourier space: kernel $\mathcal{K}$

$$\delta^z(\mathbf{k}) = (1+\beta)\delta(\mathbf{k}) + \beta \int \frac{\mathrm{d}^3\mathbf{p}}{p^2}\delta(\mathbf{p})\mathrm{L}^2_{\hat{\mathbf{p}}}\left(\int \frac{\mathrm{d}^3\mathbf{r}}{(2\pi)^3}\frac{\mathrm{e}^{\mathrm{i}(\mathbf{p}-\mathbf{k})\cdot\mathbf{r}}}{r^2}\right)$$

$$\int \frac{\mathrm{d}^3\mathbf{r}}{(2\pi)^3}\frac{\mathrm{e}^{\mathrm{i}(\mathbf{p}-\mathbf{k})\cdot\mathbf{r}}}{r^2} = \frac{1}{4\pi|\mathbf{k}-\mathbf{p}|}$$

$$\frac{1}{|\mathbf{k}-\mathbf{k}'|} = \sum_\ell \frac{k_<^\ell}{k_>^{\ell+1}} P_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')$$

$$\mathcal{K}(\mathbf{k},\mathbf{k}') = \mathrm{L}^2_{\nu_{kk'}}\left(\frac{1}{4\pi|\mathbf{k}-\mathbf{k}'|}\right) = \frac{1}{4\pi}\sum_\ell \frac{k_<^\ell}{k_>^{\ell+1}}\mathrm{L}^2_{\nu_{kk'}} P_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')$$

$$= -\frac{1}{4\pi}\sum_\ell \ell(\ell+1)\frac{k_<^\ell}{k_>^{\ell+1}} P_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')$$

$$C^z(\mathbf{k}_1,\mathbf{k}_2) = (1+\beta)^2\mathcal{P}(k_1)\delta_D^3(\mathbf{k}_1-\mathbf{k}_2) + \beta(1+\beta)\left[\frac{\mathcal{P}(k_1)}{k_1^2}\mathcal{K}(\mathbf{k}_1,\mathbf{k}_2) + \frac{\mathcal{P}(k_2)}{k_2^2}\mathcal{K}(\mathbf{k}_2,\mathbf{k}_1)\right]$$
$$+\beta^2\int \mathrm{d}^3\mathbf{p}\frac{\mathcal{P}(p)}{p^4}\mathcal{K}(\mathbf{k}_1,\mathbf{p})\mathcal{K}(\mathbf{p},\mathbf{k}_2)\,.$$

Fourier space: kernel $\mathcal{N}$

$$\Delta\left(\frac{-1}{4\pi|\mathbf{r}-\mathbf{r}'|}\right)=\delta_D(\mathbf{r}-\mathbf{r}')$$

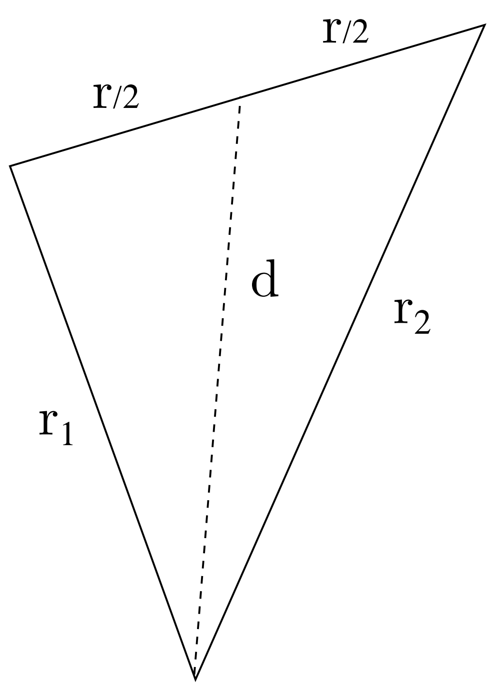
$$\mathcal{N}(\mathbf{k}_1,\mathbf{k}_2)\equiv -k_1^2\mathcal{O}_{k_1}\left(\frac{1}{4\pi|\mathbf{k}_1-\mathbf{k}_2|}\right)=\mathcal{K}(\mathbf{k}_1,\mathbf{k}_2)+k_1^2\delta_D(\mathbf{k_1}-\mathbf{k}_2)$$

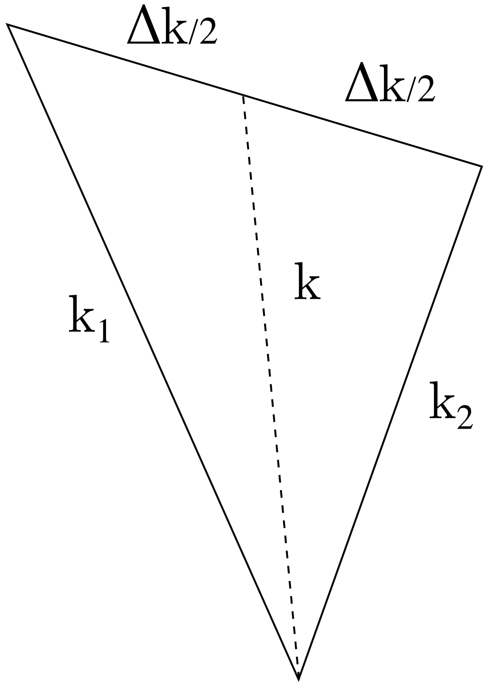
$$C^z(\mathbf{k}_1,\mathbf{k_2})=P(k_1)\delta_D^3(\mathbf{k}_1-\mathbf{k}_2)+\beta\left[\frac{P(k_1)}{k_1^2}\mathcal{N}(\mathbf{k}_1,\mathbf{k}_2)+\frac{P(k_2)}{k_2^2}\mathcal{N}(\mathbf{k}_2,\mathbf{k}_1)\right]+\beta^2\int \mathrm{d}^3\mathbf{p}\frac{P(p)}{p^4}\mathcal{N}(\mathbf{k}_1,\mathbf{p})\mathcal{N}(\mathbf{p},\mathbf{k}_2)$$

Always mode coupling: Zaroubi & Hoffman (1993)

Plane-parallel limit

2) Fourier space





$$\mathrm{e}^{\mathrm{i}\mathbf{k}_1\cdot\mathbf{r}_1}\mathrm{e}^{-\mathrm{i}\mathbf{k}_2\cdot\mathbf{r}_2} = \mathrm{e}^{\mathrm{i}\mathbf{d}\cdot\Delta\mathbf{k}}\mathrm{e}^{-\mathrm{i}\mathbf{r}\cdot\mathbf{k}}$$

with:

$$\Delta\mathbf{k} \equiv \mathbf{k}_1 - \mathbf{k}_2 \qquad \mathbf{k} \equiv (\mathbf{k}_1 + \mathbf{k}_2)/2$$

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 \qquad \mathbf{d} = (\mathbf{r}_1 + \mathbf{r}_2)/2$$

$$\mathcal{O}_k \frac{1}{4\pi|\mathbf{k}-\mathbf{k}'|} = \mathcal{O}_k^{\mathrm{PV}} \frac{1}{4\pi|\mathbf{k}-\mathbf{k}'|} - \frac{1}{3}\delta(\mathbf{k}-\mathbf{k}') \xrightarrow{\frac{|\Delta\mathbf{k}|}{k} \ll 1} 4\pi\overline{\mathcal{N}}(\mathbf{k}_1,\mathbf{k}_2) = -\frac{2k^2}{|\Delta\mathbf{k}|^3}P_2(\mu_{\mathbf{k}\,\Delta}) + \frac{4\pi k^2}{3}\delta_D(\Delta\mathbf{k})$$

Blanchet et al. (04)

$$C^z_{\mathrm{pp}}(\mathbf{k}_1,\mathbf{k}_2) = \delta_D(\Delta\mathbf{k})\zeta^{(0)}_\ell(k) + \frac{1}{4\pi|\Delta\mathbf{k}|^3}\sum_{\ell=2,4}\zeta^{(0)}_\ell(k)P_\ell(\mu_{k\Delta})$$

$$\zeta_0^{(0)}(k) = \left(1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2\right)\mathcal{P}(k)$$

$$\zeta_2^{(0)}(k) = \left(-4\beta - \frac{12}{7}\beta^2\right)\mathcal{P}(k)$$

$$\zeta_4^{(0)}(k) = \frac{12}{7}\beta^2\mathcal{P}(k)$$

Plane-parallel limit

3) Mixed space

"Power spectrum *at* a given **d**"

$$\zeta^z_{\rm pp}(\Delta\mathbf{k},\mathbf{k}) = \mathcal{P}(k)\delta_D(\Delta\mathbf{k}) + 2\beta\frac{\mathcal{P}(k)}{k^2}\overline{\mathcal{N}}_{\mathbf{k}}(\Delta\mathbf{k}) + (2\pi)^{3/2}\beta^2\frac{\mathcal{P}(k)}{k^4}\int\frac{\mathrm{d}^3\Delta\mathbf{p}}{(2\pi)^{3/2}}\overline{\mathcal{N}}_{\mathbf{k}}(\Delta\mathbf{p})\overline{\mathcal{N}}_{\mathbf{k}}(\Delta\mathbf{k}-\Delta\mathbf{p})$$

$\uparrow \mathcal{F}^{-1}$ (convolution becomes product)

$$\widetilde{\overline{\mathcal{N}}}_{\mathbf{k}}(\mathbf{d}) \equiv \int\frac{\mathrm{d}^3\Delta\mathbf{p}}{(2\pi)^{3/2}}\bar{\mathcal{N}}_{\mathbf{k}}(\Delta\mathbf{p})\mathrm{e}^{\mathrm{i}\Delta\mathbf{p}\cdot\mathbf{d}} = \frac{1}{(2\pi)^{3/2}}\left(\frac{2}{3}k^2P_2(\mu_{\mathbf{k}\,\mathbf{d}}) + \frac{1}{3}k^2\right) = \frac{k^2\,\mu^2_{\mathbf{k}\,\mathbf{d}}}{(2\pi)^{3/2}}$$

$$[\mathcal{P}_{\mathbf{d}}(\mathbf{k}) =]\ \widetilde{\zeta}^z_{\rm pp}(\mathbf{d},\mathbf{k}) \equiv \int\frac{\mathrm{d}^3\Delta\mathbf{k}}{(2\pi)^{3/2}}\zeta^z_{\rm pp}(\Delta\mathbf{k},\mathbf{k})\mathrm{e}^{\mathrm{i}\Delta\mathbf{k}\cdot\mathbf{d}} = \frac{\mathcal{P}(k)}{(2\pi)^{3/2}}(1+\beta\mu^2_{\mathbf{k}\,\mathbf{d}})^2\,.$$

$$\widetilde{\zeta}^z_{\rm pp}(\mathbf{d},\mathbf{k}) = \sum_{\ell=0,2,4}\mathcal{P}^{(0)}_\ell(k)P_\ell(\mu_{\mathbf{k}\,\mathbf{d}})$$

$$\mathcal{P}^{(0)}_0(k) = \left(1+\frac{2}{3}\beta+\frac{1}{5}\beta^2\right)\frac{\mathcal{P}(k)}{(2\pi)^{3/2}}$$

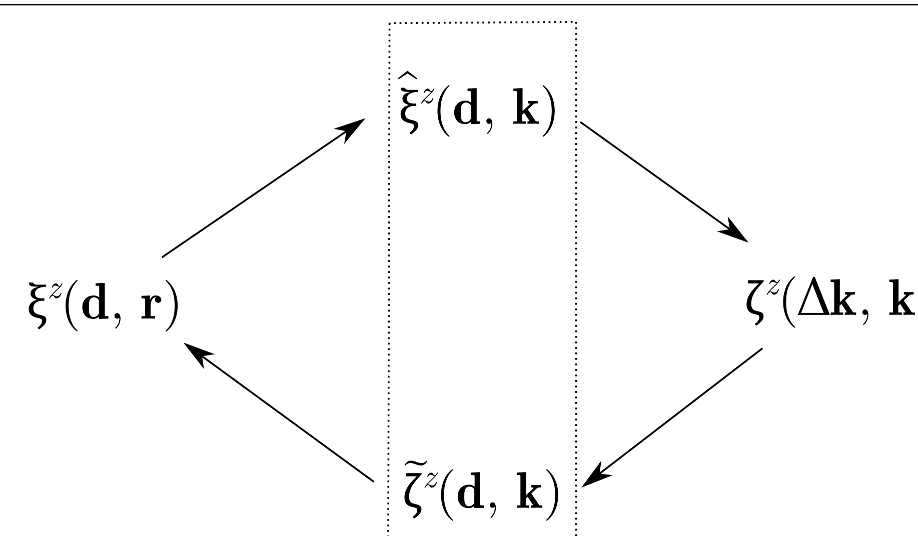
$$\mathcal{P}^{(0)}_2(k) = \left(\frac{4}{3}\beta+\frac{4}{7}\beta^2\right)\frac{\mathcal{P}(k)}{(2\pi)^{3/2}}$$

$$\mathcal{P}^{(0)}_4(k) = \frac{8}{35}\beta^2\frac{\mathcal{P}(k)}{(2\pi)^{3/2}}$$

$$\widehat{\xi}^z_{\rm pp}(\mathbf{d},\mathbf{k}) \equiv \int\frac{\mathrm{d}^3\mathbf{r}}{(2\pi)^{3/2}}\xi^z_{\rm pp}(\mathbf{d},\mathbf{r})\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}}$$

$$\widehat{\xi}^z_{\rm pp}(\mathbf{d},\mathbf{k}) = \sum_{\ell=0,2,4}\left[\sqrt{\frac{2}{\pi}}(-i)^\ell\int\mathrm{d}r r^2 j_\ell(kr)\xi^{(0)}_\ell(r)\right]P_\ell(\mu_{\mathbf{k}\,\mathbf{d}})$$

(Hankel transform)



$$\widehat{\xi}^z_{\rm pp}(\mathbf{d},\mathbf{k}) = \widetilde{\zeta}^z_{\rm pp}(\mathbf{d},\mathbf{k})$$

Plane-parallel limit

4) Kaiser formula

$$\widetilde{\zeta}^z_{\rm pp}(\mathbf{d},\mathbf{k}) \;\equiv\; \int \frac{\mathrm{d}^3\Delta\mathbf{k}}{(2\pi)^{3/2}}\zeta^z_{\rm pp}(\Delta\mathbf{k},\mathbf{k})\mathrm{e}^{\mathrm{i}\Delta\mathbf{k}\cdot\mathbf{d}} \;=\; \frac{\mathcal{P}(k)}{(2\pi)^{3/2}}(1+\beta\mu^2_{\mathbf{k}\,\mathbf{d}})^2\,.$$

Fourier transform in **d** after the replacement:

$$\mu_{\mathbf{k}\,\mathbf{d}} \to \mu_{\mathbf{k}\,\mathbf{z}}$$ → makes sense in mixed space

$$\zeta^z_{\rm pp}(\Delta\mathbf{k},\mathbf{k}) \approx \zeta^z_{\rm Kaiser}(\Delta\mathbf{k},\mathbf{k}) = \delta_D(\Delta\mathbf{k})\mathcal{P}(k)(1+\beta\mu^2_{\mathbf{k}\,\mathbf{z}})^2 = \delta_D(\Delta\mathbf{k})\sum_{\ell=0,2,4}\mathcal{P}^{(0)}_\ell(k)P_\ell(\mu_{\mathbf{k}\,\mathbf{z}})$$

Kaiser (87)

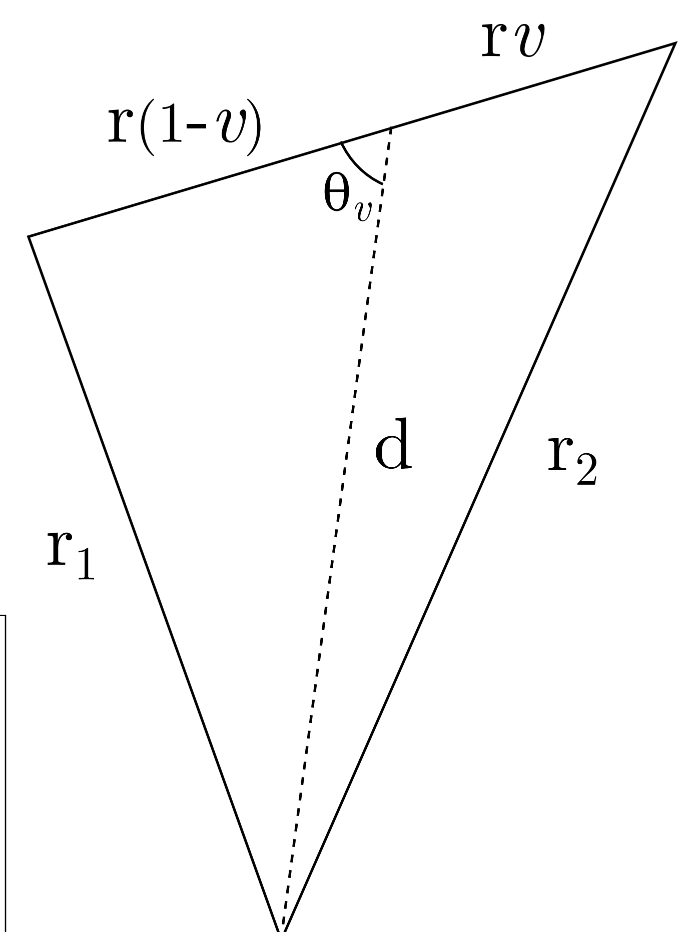
↓ not well defined in Fourier space

Only the power-spectrum for a given **d** is well defined, and it has by definition a mixed dependence on variables in configuration and Fourier spaces.

$$C^z(\mathbf{r}_1,\mathbf{r}_2) = \int \frac{k^2 \mathrm{d}k}{2\pi^2}\mathcal{P}(k) I\left[j_0(kr)\right] \,, \qquad I\left[j_0(kr)\right] \equiv \left[1+\beta+\frac{\beta \mathrm{L}^2_{\nu_{12}}}{(kr_1)^2}\right]\left[1+\beta+\frac{\beta \mathrm{L}^2_{\nu_{12}}}{(kr_2)^2}\right] j_0(kr)$$

Expansions around plane parallel limit $(n = 0)$	small parameter	
$\xi^z(\mathbf{d},\mathbf{r}) = \sum_{n=0}^{\infty}\left(\frac{r}{d}\right)^n \sum_{\ell=0}^{\infty} \xi_\ell^{(n)}(r) P_\ell(\mu_{\mathbf{d}\,\mathbf{r}})$	$\frac{r}{d}$	$\xi_\ell^{(n)} : \mathbb{R} \to \mathbb{R}$
$\widehat{\xi}^z(\mathbf{d},\mathbf{k}) = \widetilde{\zeta}^z(\mathbf{d},\mathbf{k}) = \sum_{n=0}^{\infty}\left(\frac{1}{kd}\right)^n \sum_{\ell=0}^{\infty} \mathcal{P}_\ell^{(n)}(k) P_\ell(\mu_{\mathbf{k}\,\mathbf{d}})$	$\frac{1}{kd}$	$\mathcal{P}_\ell^{(n)} : \mathbb{R} \to \mathbb{C}$

(full Fourier coefficients are proportional to those in mixed space)



$v = 0$	n	ℓ
	0	0, 2, 4
	1	1, 3, 5
	2	0, 2, 4, 6

$v = 1/2$	n	ℓ
	0	0, 2, 4
	1	X
	2	0, 2, 4, 6

bisector	n	ℓ
	0	0, 2, 4
	1	X
	2	0, 2, 4

natural extension of plane parallel limit

$(\ell_{max} = n+2,\ n > 0)$

parity rule: n odd $\Rightarrow \ell$ odd
n even $\Rightarrow \ell$ even

Configuration space

$$\xi^z(\mathbf{d},\mathbf{r}) = \sum_{n=0}^{\infty}\left(\frac{r}{d}\right)^n \sum_{\ell=0}^{\infty}\xi_\ell^{(n)}(r)P_\ell(\mu_{\mathbf{d}\,\mathbf{r}})$$

Order zero (plane parallel):

$$\xi_0^{(0)}(r) = \left(1+\frac{2}{3}\beta+\frac{1}{5}\beta^2\right)\Xi_0^0(r)$$

$$\xi_2^{(0)}(r) = -\left(\frac{4}{3}\beta+\frac{4}{7}\beta^2\right)\Xi_2^0(r)$$

$$\xi_4^{(0)}(r) = \frac{8}{35}\beta^2\Xi_4^0(r)$$

Second order (first correction to plane parallel):

$$\xi_0^{(2)}(r) \;=\; \left(\frac{2}{9}\beta-\frac{14}{15}\beta^2\right)\Xi_0^0(r)+\left(\frac{1}{45}\beta-\frac{11}{45}\beta^2\right)\Xi_2^0(r)+\frac{4}{3}\beta^2\Xi_0^2(r)\,,$$

$$\xi_2^{(2)}(r) \;=\; -\left(\frac{8}{9}\beta+\frac{4}{15}\beta^2\right)\Xi_0^0(r)-\left(\frac{29}{63}\beta+\frac{29}{147}\beta^2\right)\Xi_2^0(r)-\frac{4}{245}\beta^2\Xi_4^0(r)\,,$$

$$\xi_4^{(2)}(r) \;=\; -\left(\frac{8}{35}\beta+\frac{24}{248}\beta^2\right)\Xi_2^0(r)+\frac{4}{245}\beta^2\Xi_4^0(r)\,.$$

$$\Xi_\ell^0(r) = \int\frac{\mathrm{d}kk^2}{2\pi^2}\mathcal{P}(k)j_\ell(kr)\quad\text{with}\quad \ell=0,2,4,\quad\text{and}\quad \Xi_0^2(r) = \int\frac{dk}{2\pi^2r^2}\mathcal{P}(k)j_0(kr)$$

Mixed space

$$\widehat{\xi}^z(\mathbf{d},\mathbf{k}) \equiv \int \frac{\mathrm{d}^3\mathbf{r}}{(2\pi)^{3/2}} \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}} \xi^z(\mathbf{d},\mathbf{r}) \equiv \sum_{n=0}^{\infty} \frac{1}{(kd)^n} \widehat{\xi}^{z(n)}(k,\mu_{\mathbf{k}\,\mathbf{d}})$$

$$\widehat{\xi}^{z(n)}(k,\mu_{\mathbf{k}\,\mathbf{d}}) \equiv \sum_{\ell=0}^{\infty} \mathcal{P}_\ell^{(n)}(k) P_\ell(\mu_{\mathbf{k}\,\mathbf{d}})\,, \qquad \mathcal{P}_\ell^{(n)}(k) = \left[\sqrt{\frac{2}{\pi}}(-\mathrm{i})^\ell \int r^2 \mathrm{d}r (kr)^n j_\ell(kr) \xi_\ell^{(n)}(r)\right]$$

Order zero (plane parallel):

$$\begin{aligned}
\widehat{\xi}^{z(0)}(k,\mu_k) &= \frac{1}{(2\pi)^{3/2}} \left\{ \left[1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2\right] P_0(\mu_{\mathbf{k}\,\mathbf{d}}) + \left[\frac{4}{3}\beta + \frac{4}{7}\beta^2\right] P_2(\mu_{\mathbf{k}\,\mathbf{d}}) + \beta^2 \frac{8}{35} P_4(\mu_{\mathbf{k}\,\mathbf{d}}) \right\} \mathcal{P}(k) \\
&= \frac{\mathcal{P}(k)}{(2\pi)^{3/2}} (1 + \beta\mu_{\mathbf{k}\,\mathbf{d}}^2)^2\,,
\end{aligned}$$

Second order (first correction to plane parallel):

$$\begin{aligned}
\widehat{\xi}^{z(2)}(k,\mu_{\mathbf{k}\,\mathbf{d}}) = \frac{1}{(2\pi)^{3/2}} \Bigg\{ &\mathcal{P}(k) \left[\left(-\frac{64}{35}\beta - \frac{16}{35}\beta^2\right) P_4(\mu_{\mathbf{k}\,\mathbf{d}}) + \left(\frac{58}{21}\beta + \frac{82}{49}\beta^2\right) P_2(\mu_{\mathbf{k}\,\mathbf{d}}) + \frac{1}{15}\beta + \frac{3}{5}\beta^2\right] \\
&+ k\mathcal{P}'(k) \left[\left(\frac{8}{7}\beta + \frac{16}{35}\beta^2\right) P_4(\mu_{\mathbf{k}\,\mathbf{d}}) + \left(-\frac{38}{21}\beta - \frac{18}{49}\beta^2\right) P_2(\mu_{\mathbf{k}\,\mathbf{d}}) - \frac{1}{3}\beta - \frac{3}{5}\beta^2\right] \\
&+ k^2\mathcal{P}''(k) \left[\left(-\frac{8}{35}\beta - \frac{4}{35}\beta^2\right) P_4(\mu_{\mathbf{k}\,\mathbf{d}}) + \left(\frac{3}{7}\beta + \frac{5}{49}\beta^2\right) P_2(\mu_{\mathbf{k}\,\mathbf{d}}) - \frac{1}{5}\beta + \frac{1}{15}\beta^2\right] \Bigg\}
\end{aligned}$$

Conclusions

RSD induces mode coupling and only the power spectrum at a given distance can be defined (in a mixed space)

Wide angle effects can be incorporated as an expansion around the plane parallel limit. The expansion depends on the geometry chosen, and the bisector angle parametrization is optimal.