Forecasting special events in cosmic history

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IAP / KIAS

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A 15-year long project initiated by S. Colombi

Statistics of Merging Peaks of Random Gaussian Fluctuations: Skeleton Tree Formalism

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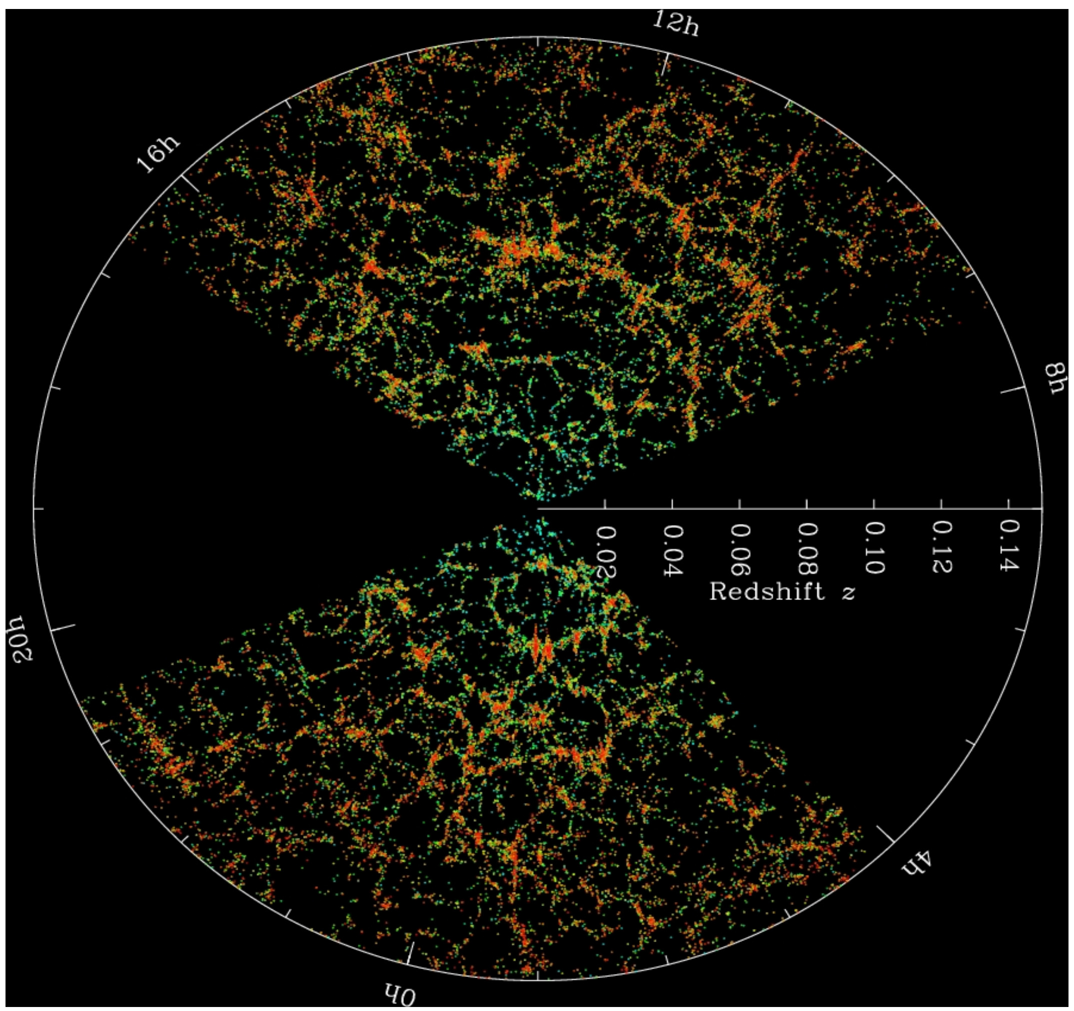
horizon-AGN

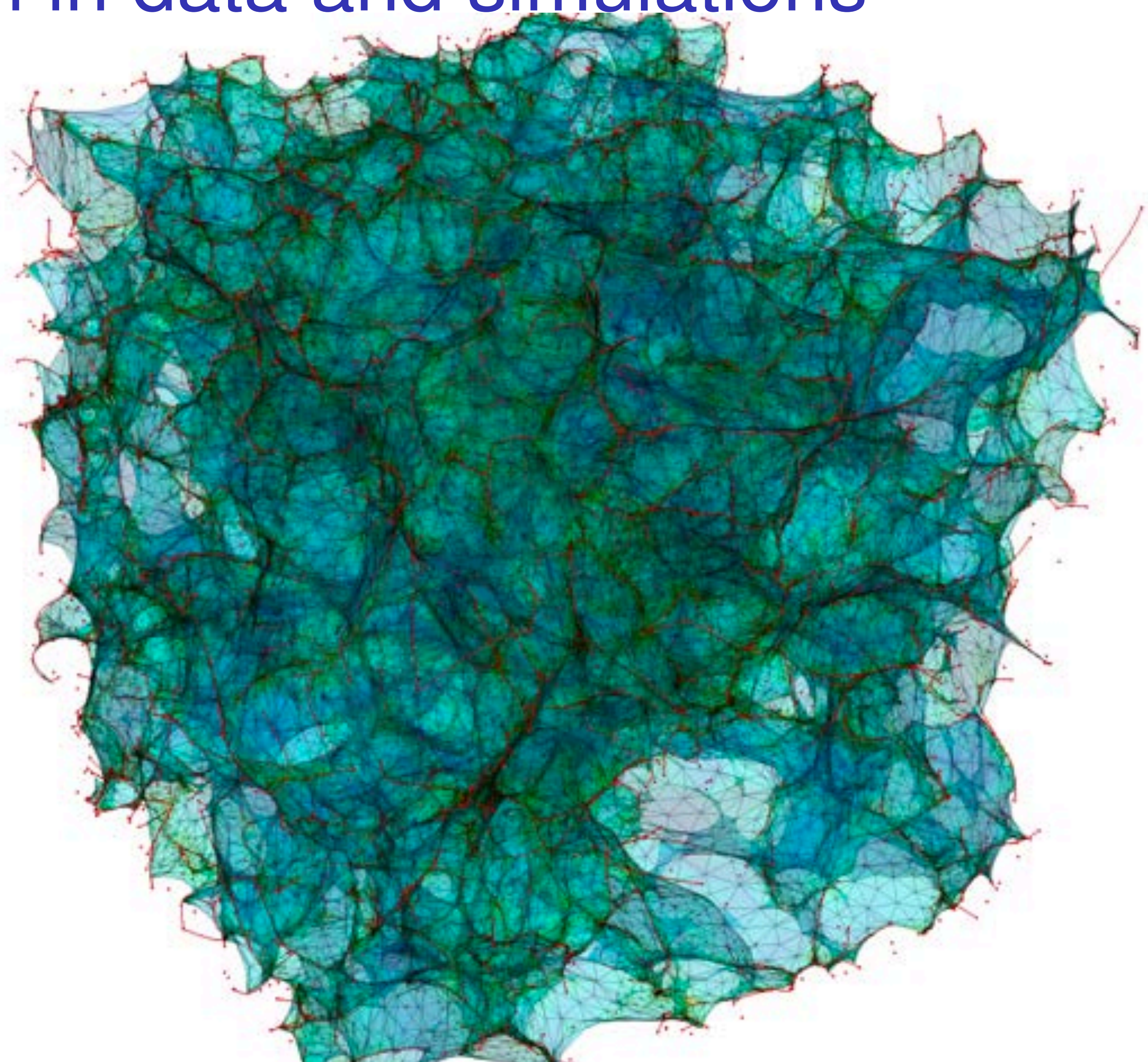
12/04/2019 Kyoto

Context: geometry & Topology

Large scale filamentary structure connecting clusters of galaxies is evident both in data and simulations



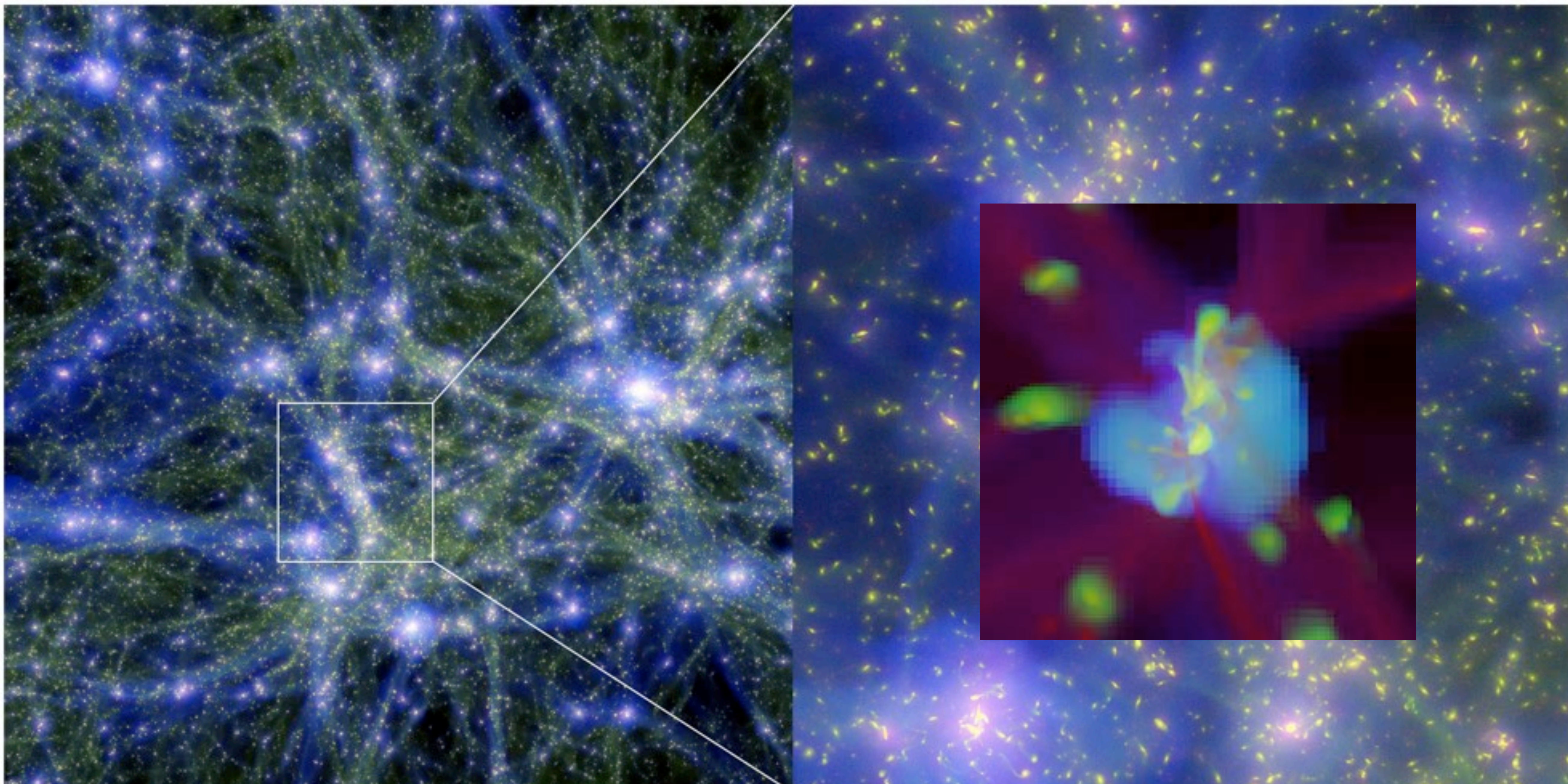
Sloan DS Survey



Skeleton & Walls of Horizon-AGN

Context: feeding galaxies via cold flows?

Large scale filamentary structure connecting clusters of galaxies is evident both in data and simulations



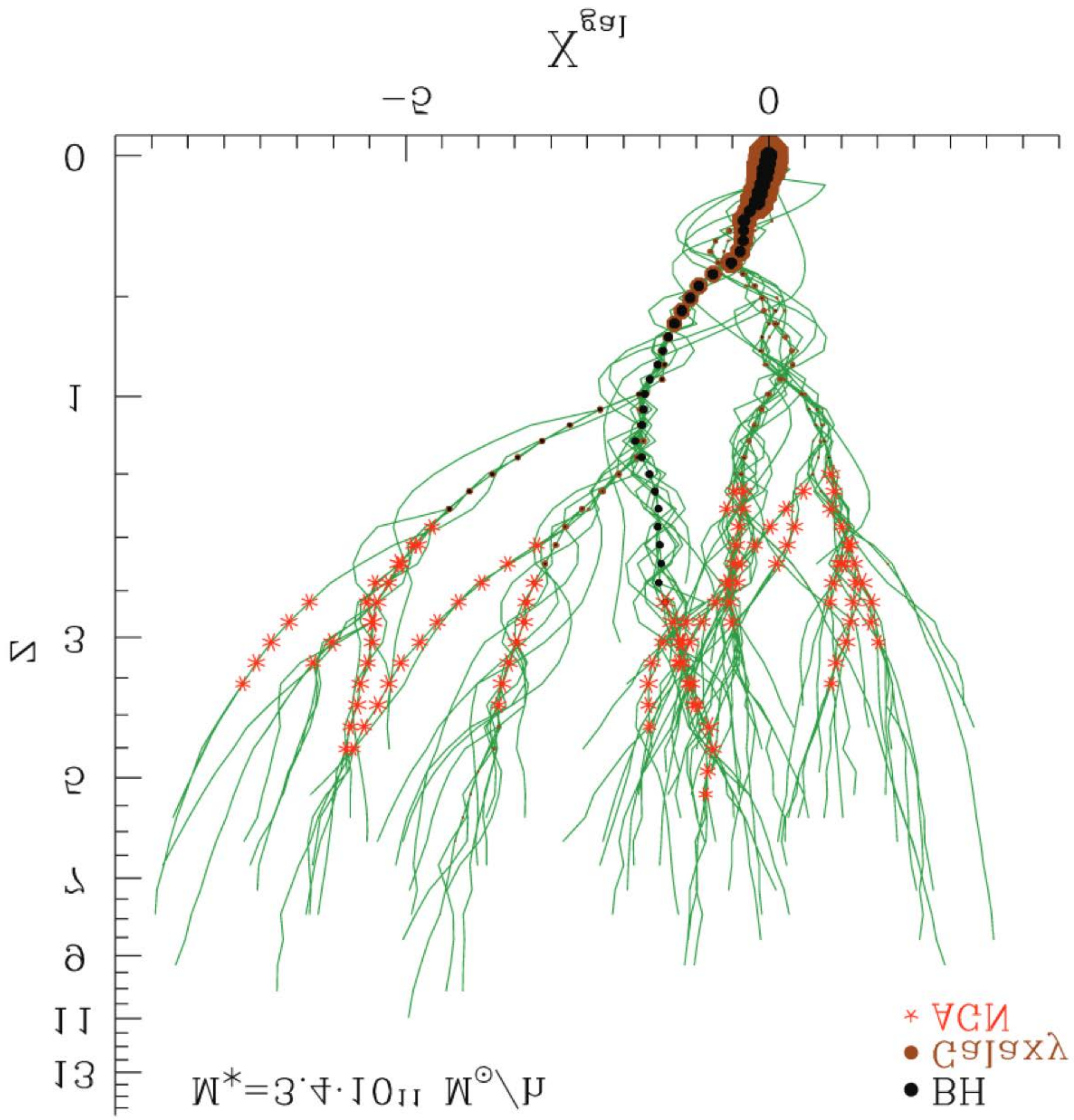
Filamentary accretion regulating stellar formation / AGN feedback/AM acquisition

Context: skeleton tree

The skeleton tree formalism

Can we build a merger-tree like structure from the initial conditions?

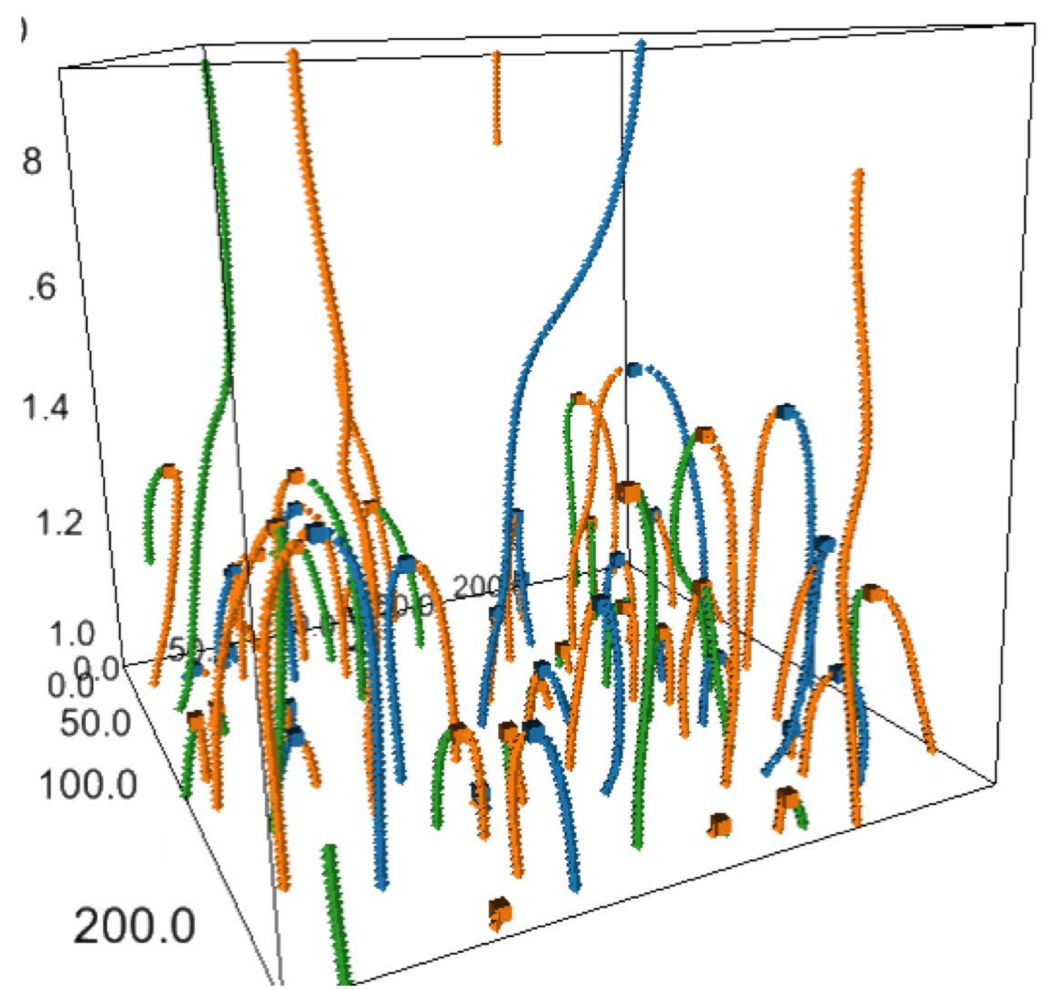
⇒ Yes! Study the topological structure of the ICs at different scales (Hanami 2001)



Marulli+2009

Time

Smoothing scale



Cadiou+ in prep

Extend Hanami '01 to *other* critical events

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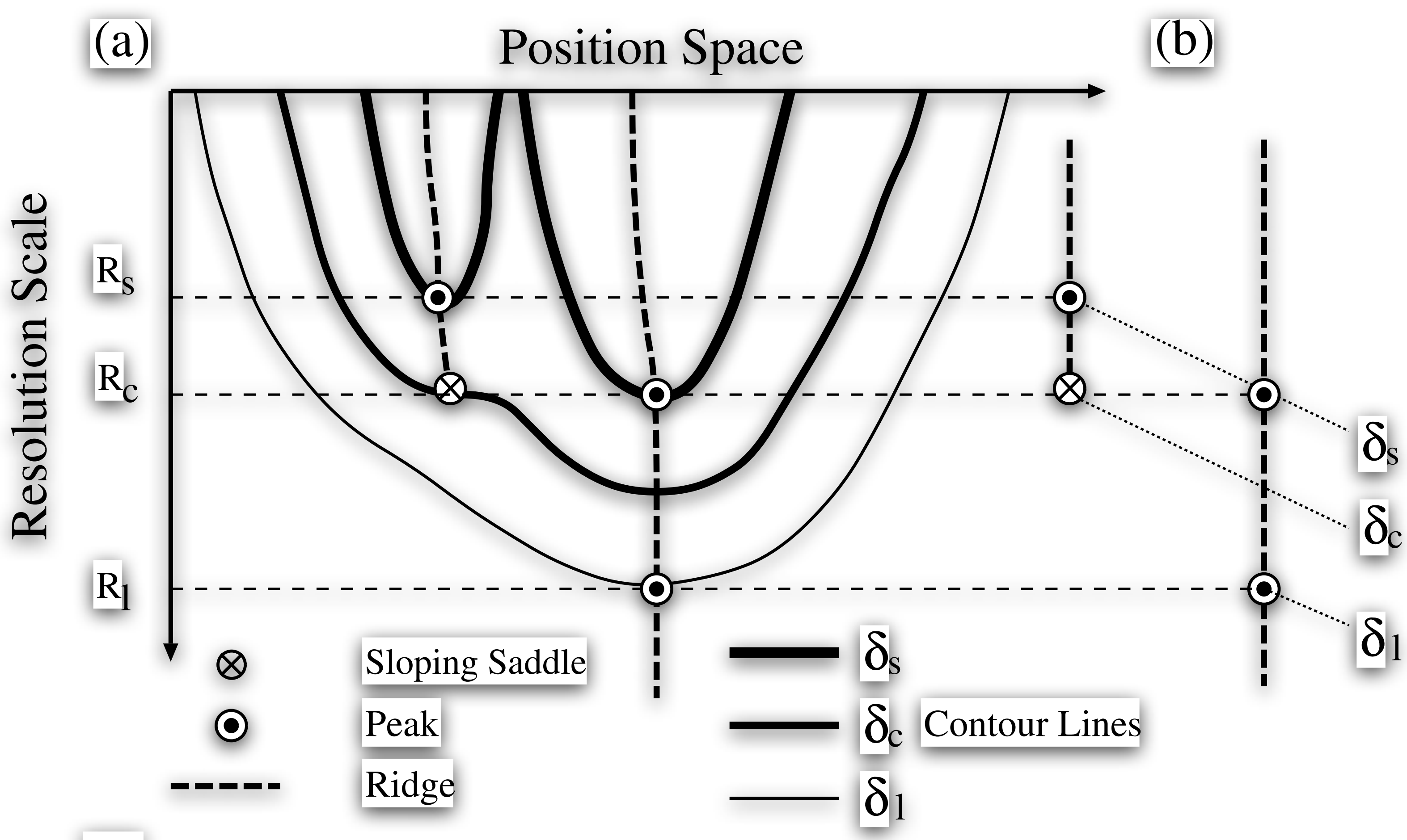

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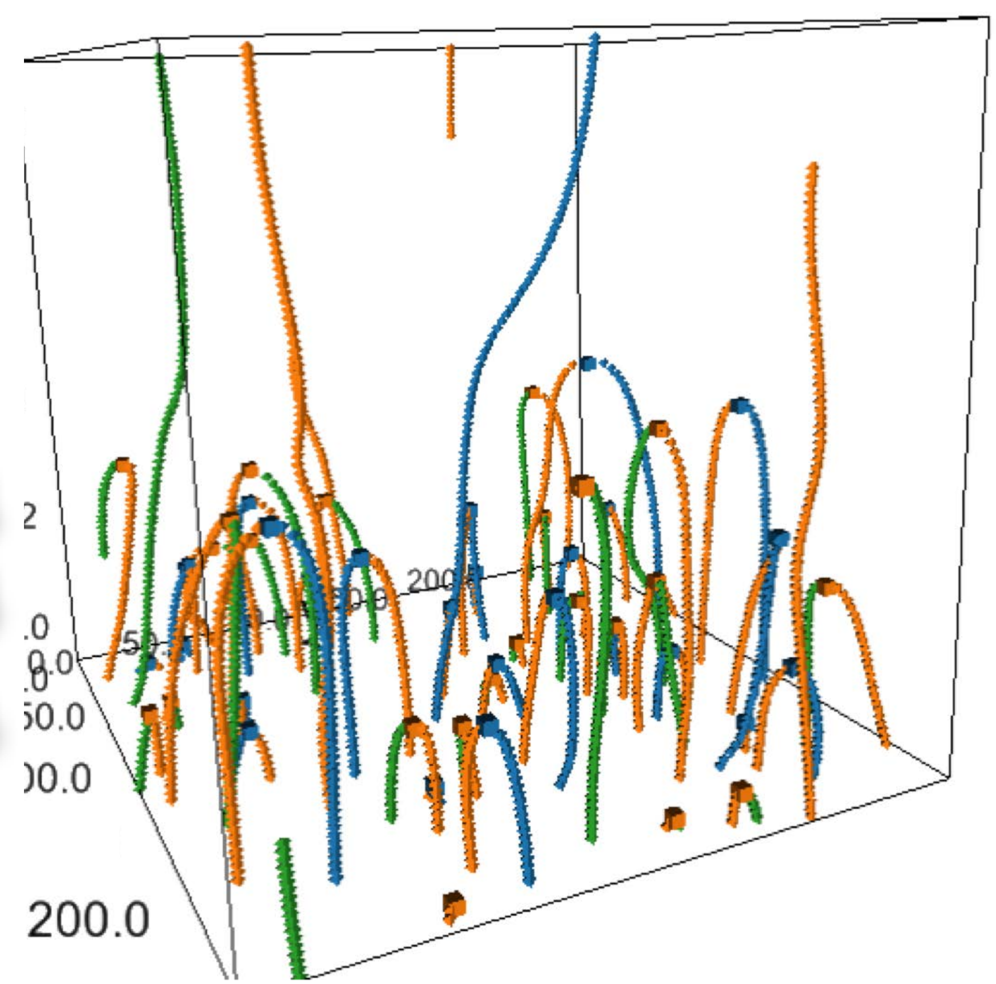
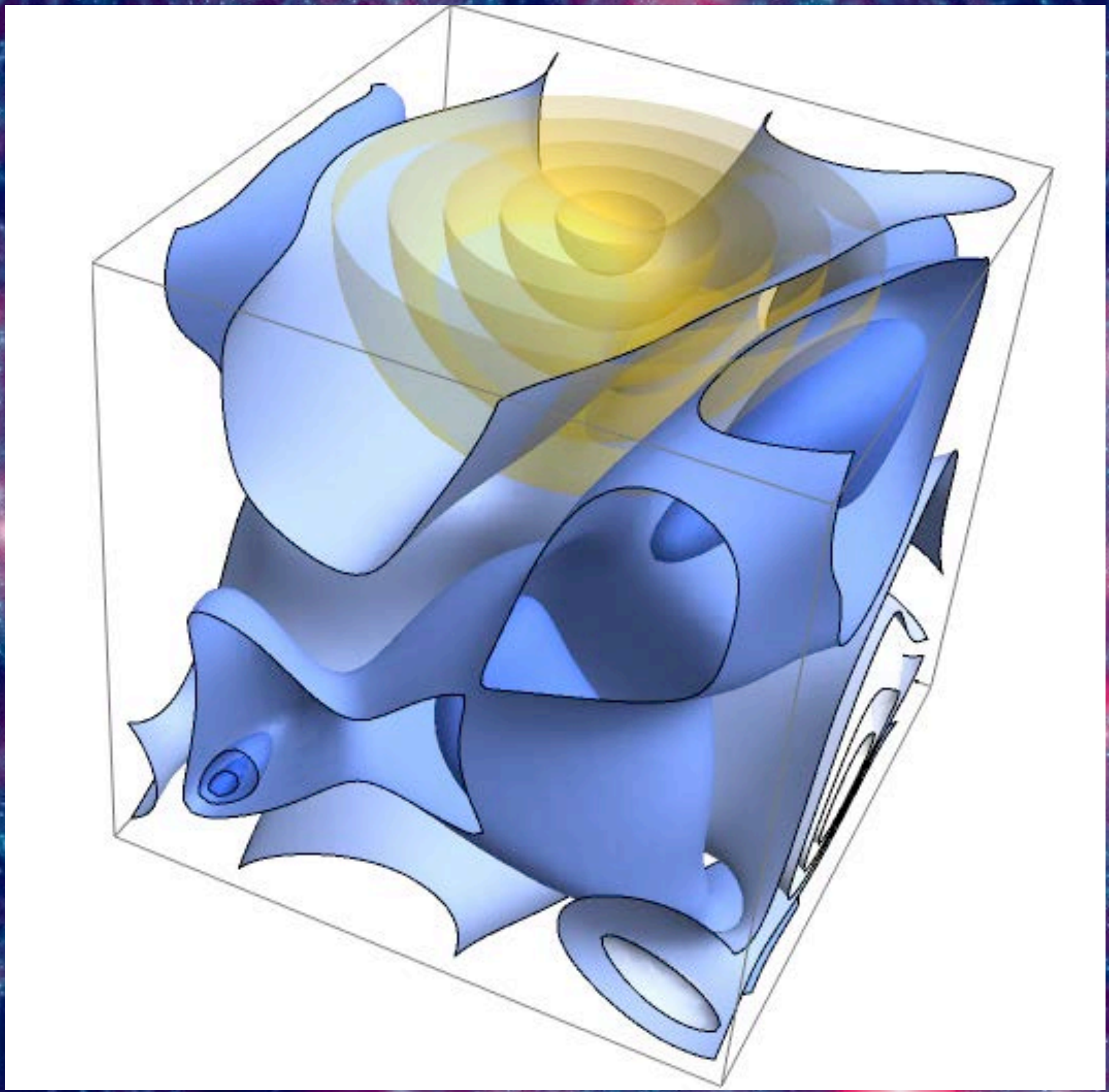


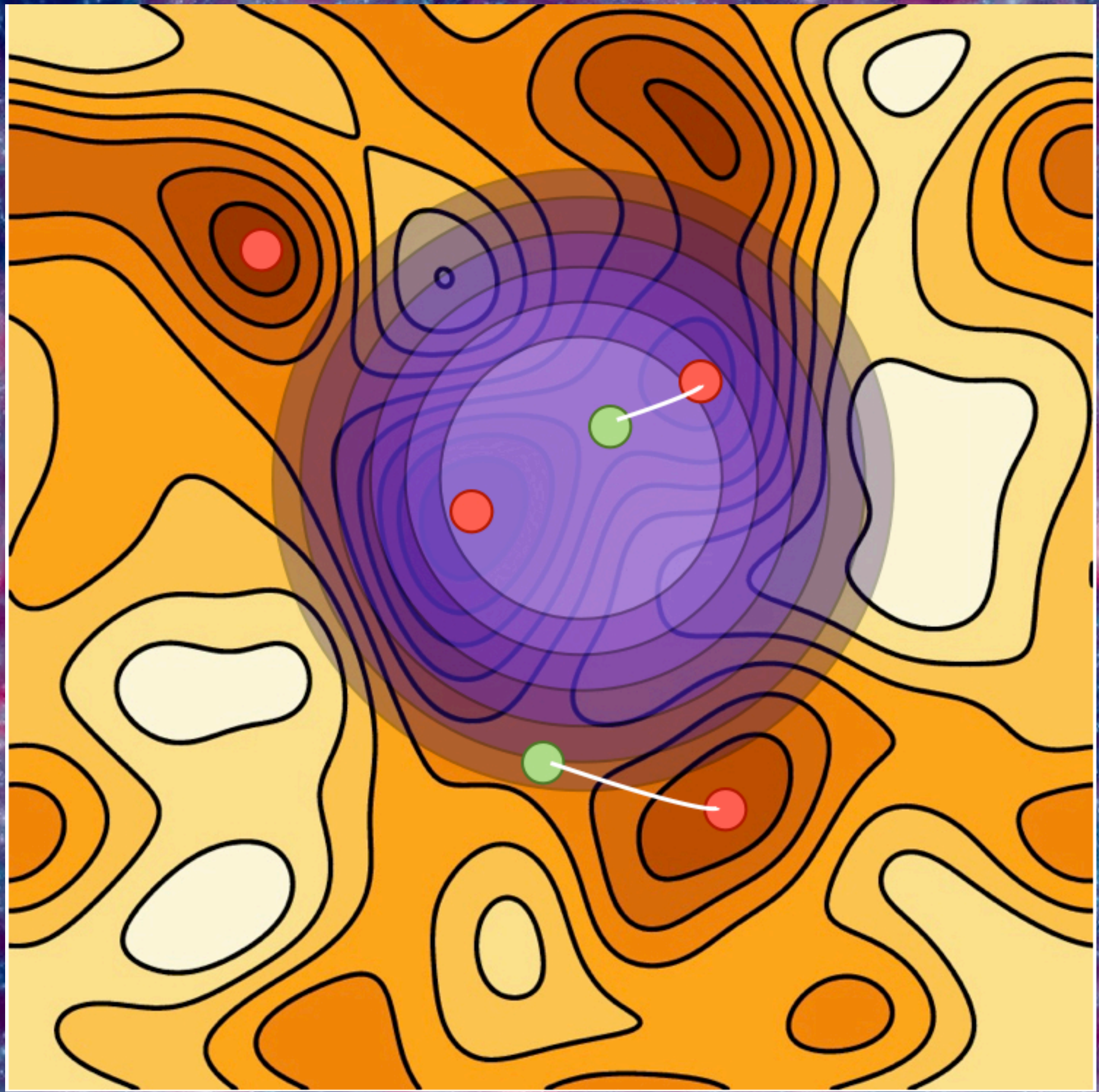
Cadiou+ in prep

Extend Hanami '01 to *other* critical events

Context

Spherical collapse: time-smoothing duality

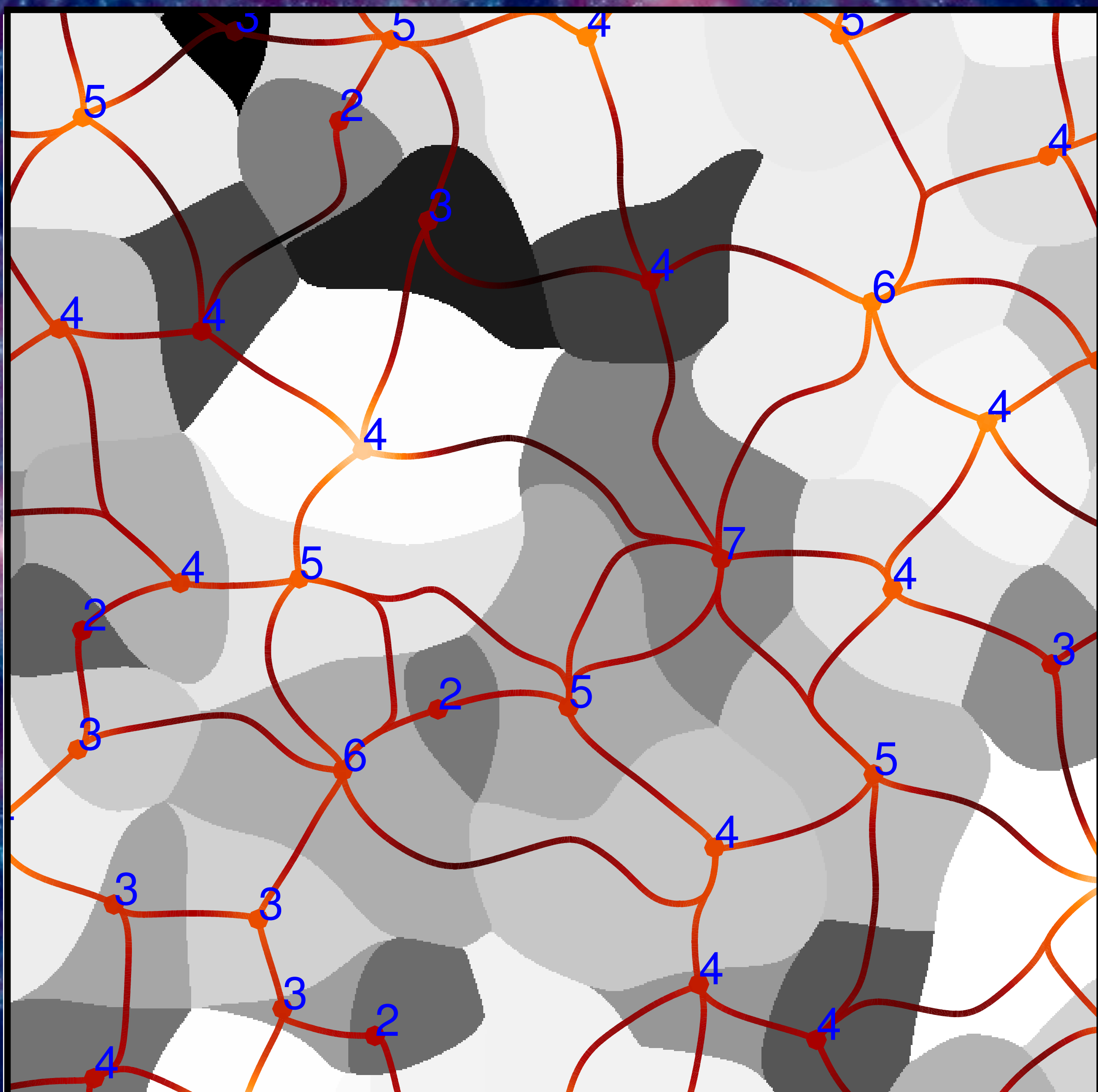


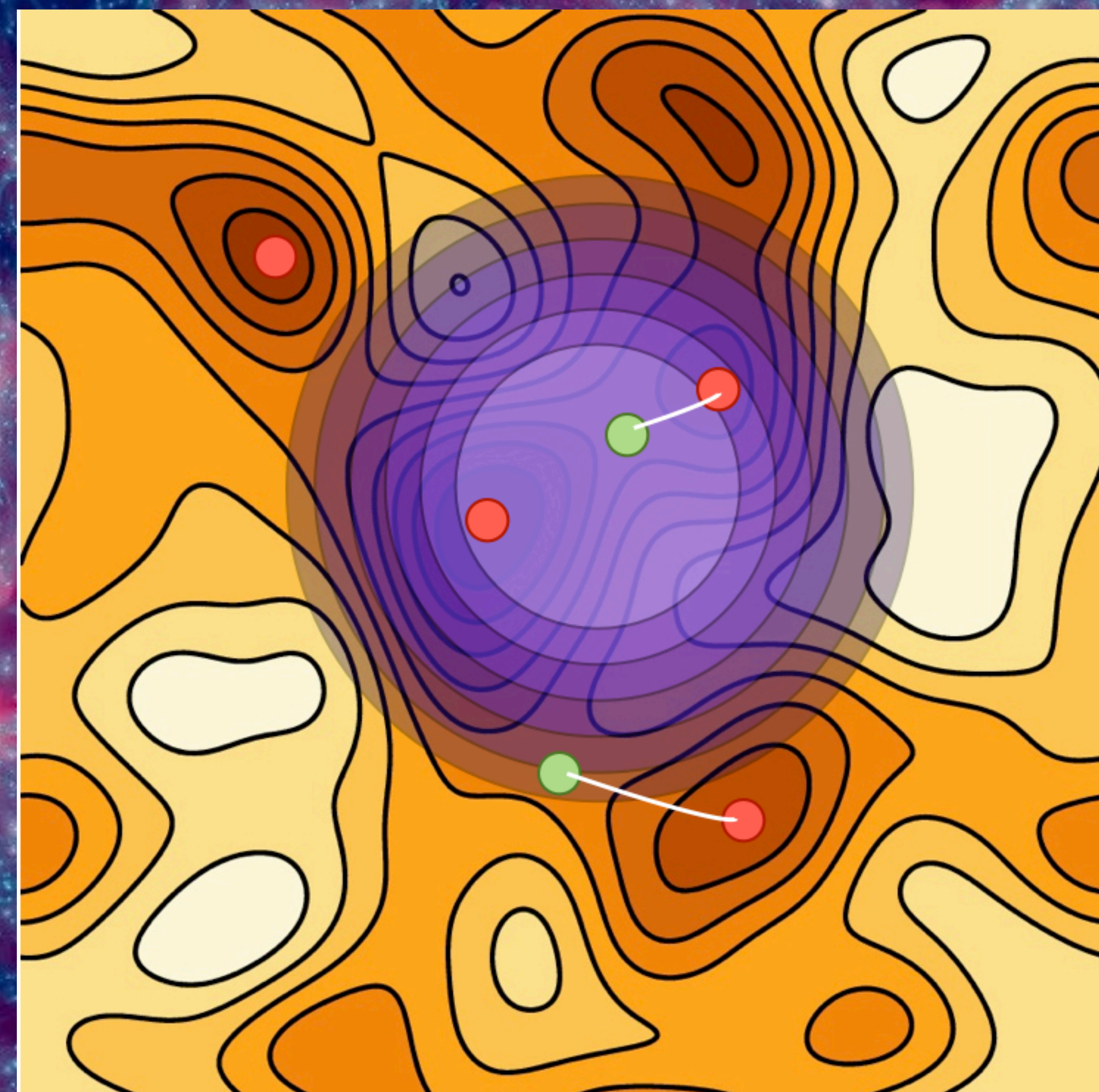


Understand special events in evolution of cosmic web

Context

Spherical collapse: time-smoothing duality





Understand special events in evolution of cosmic web

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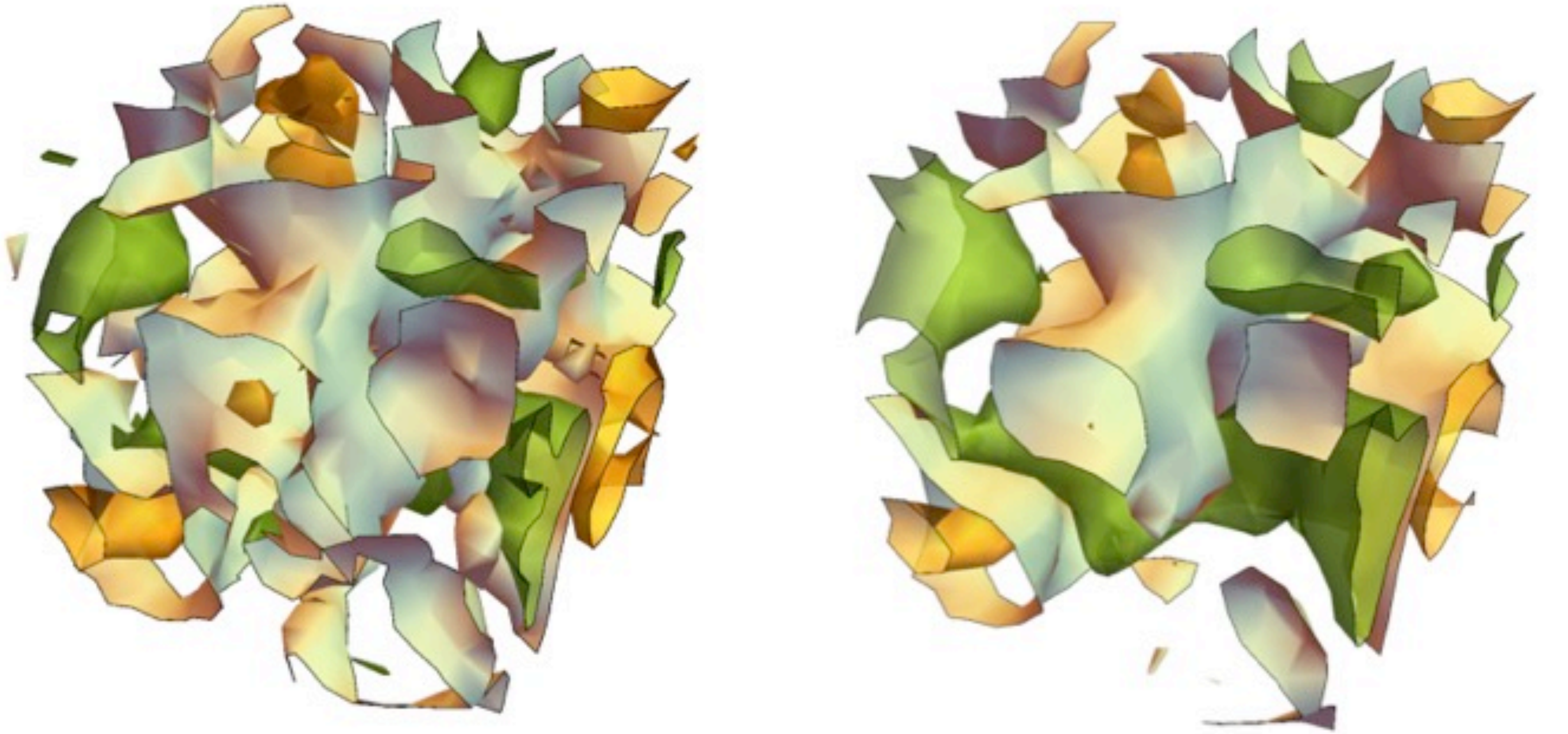
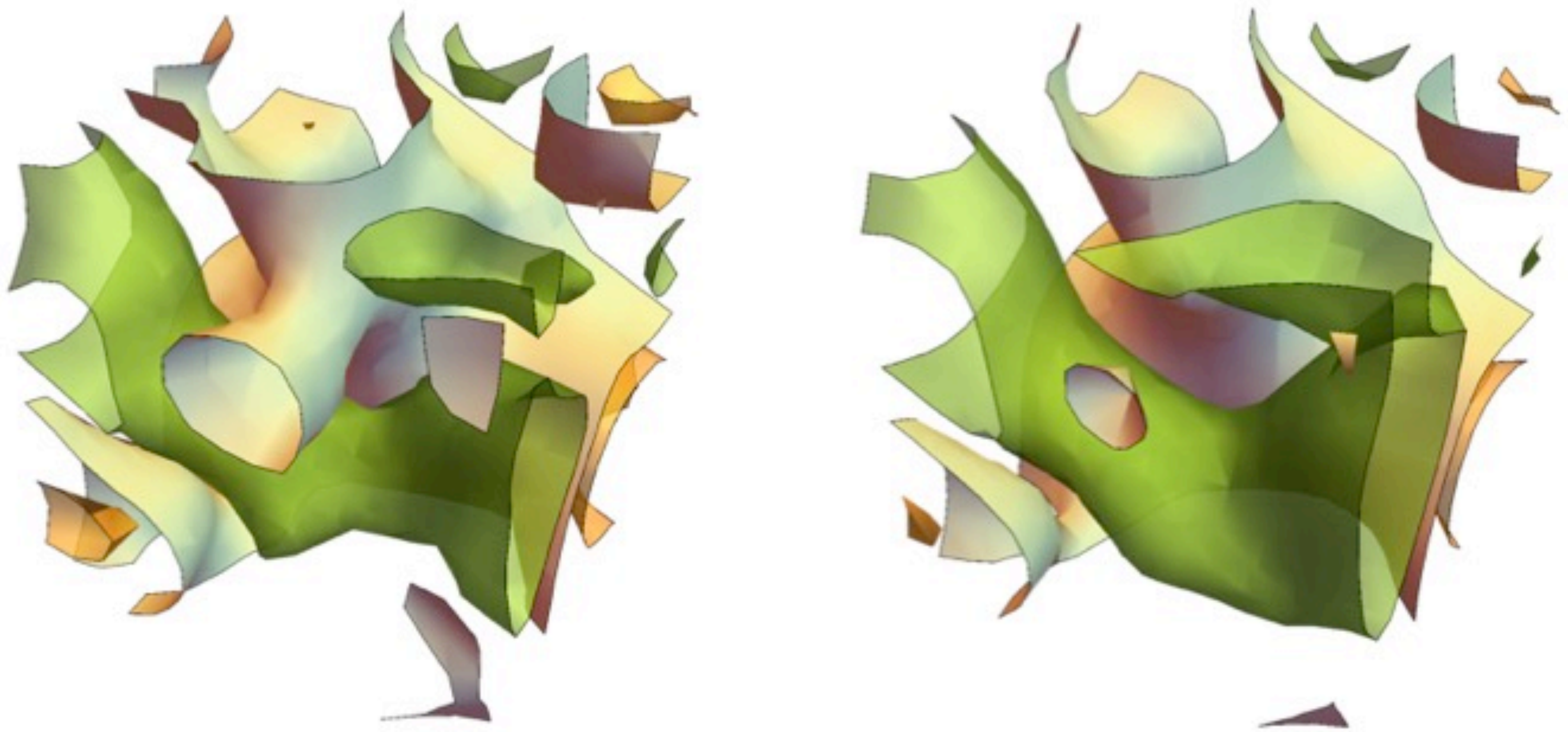
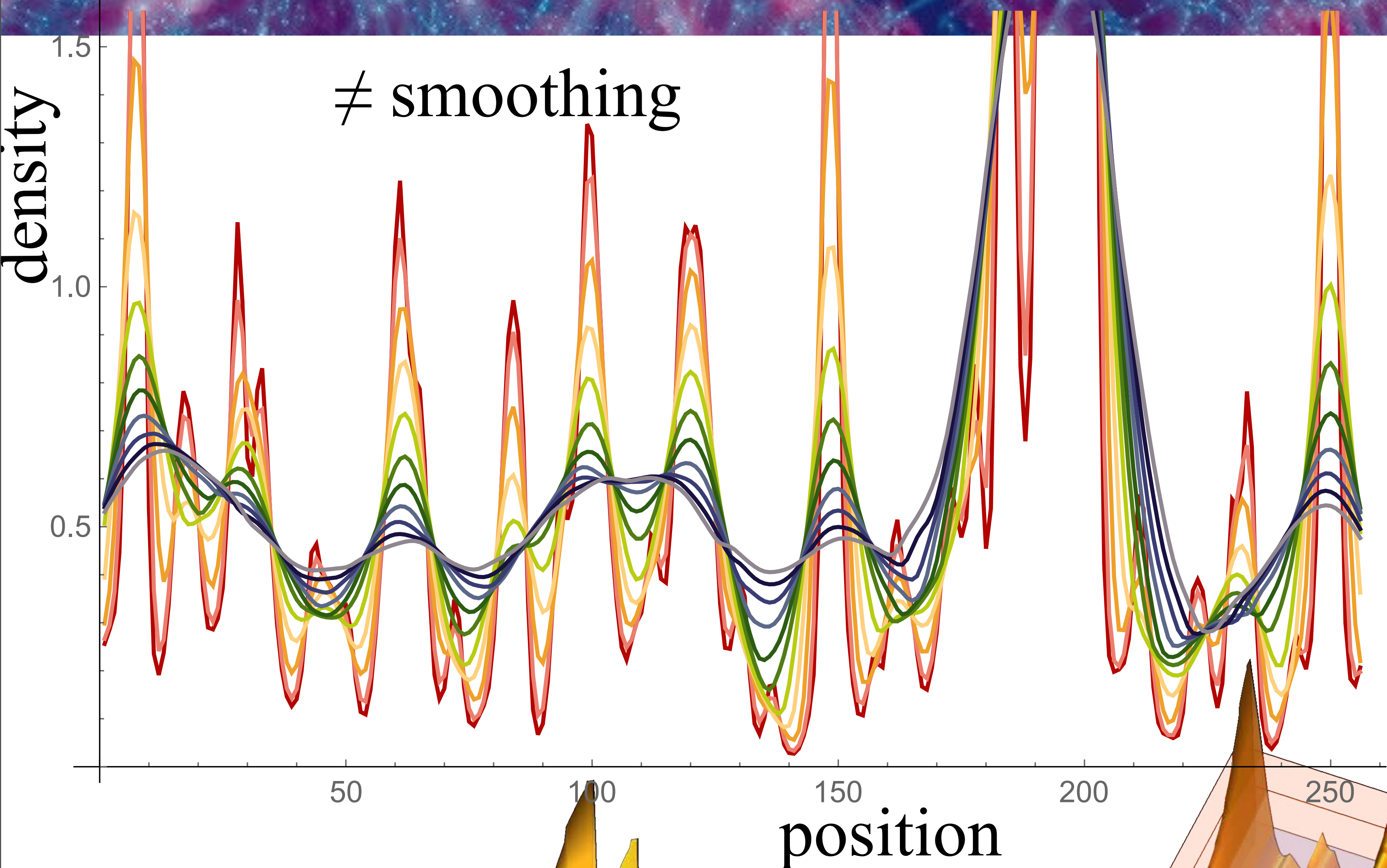
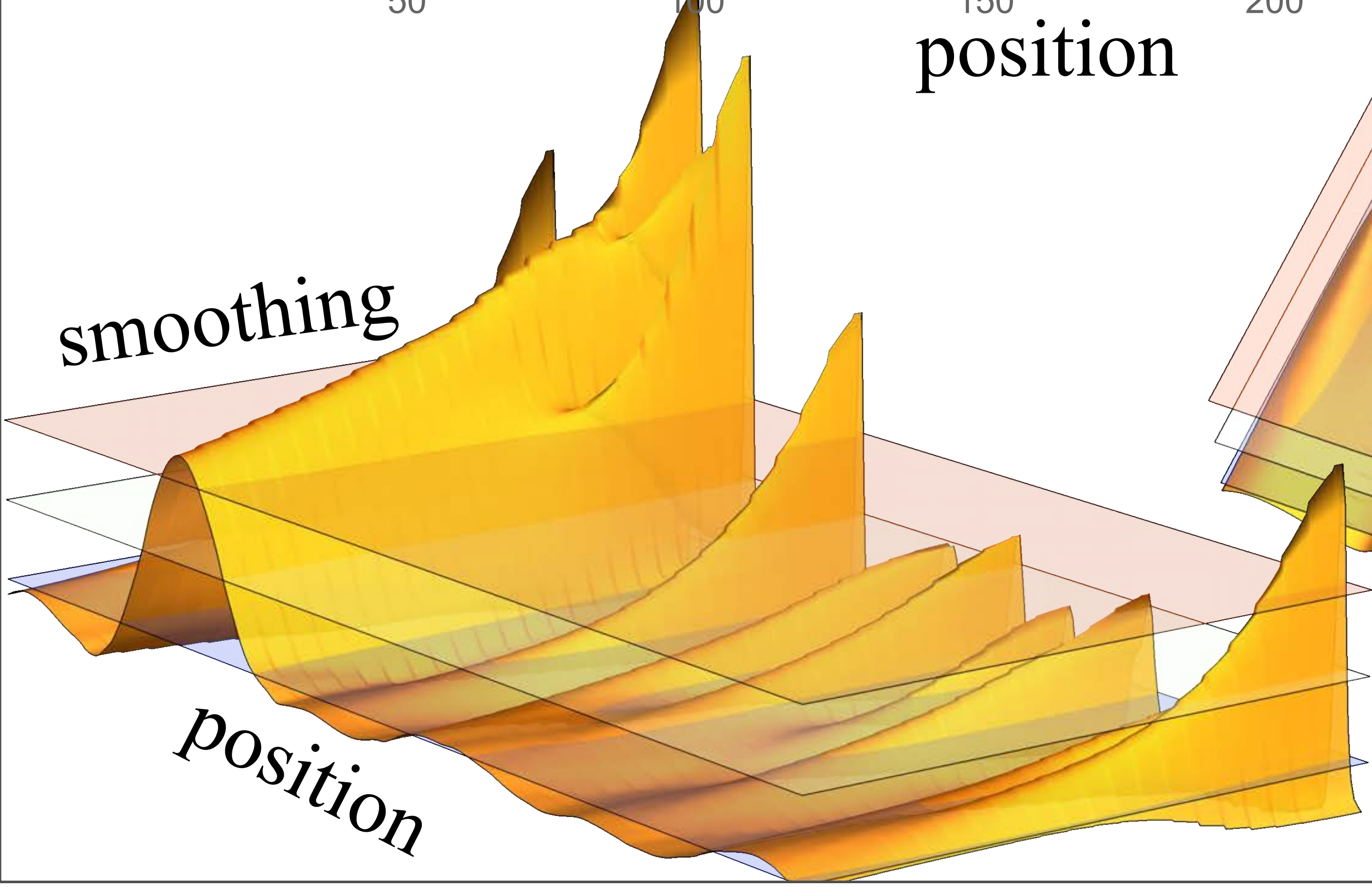


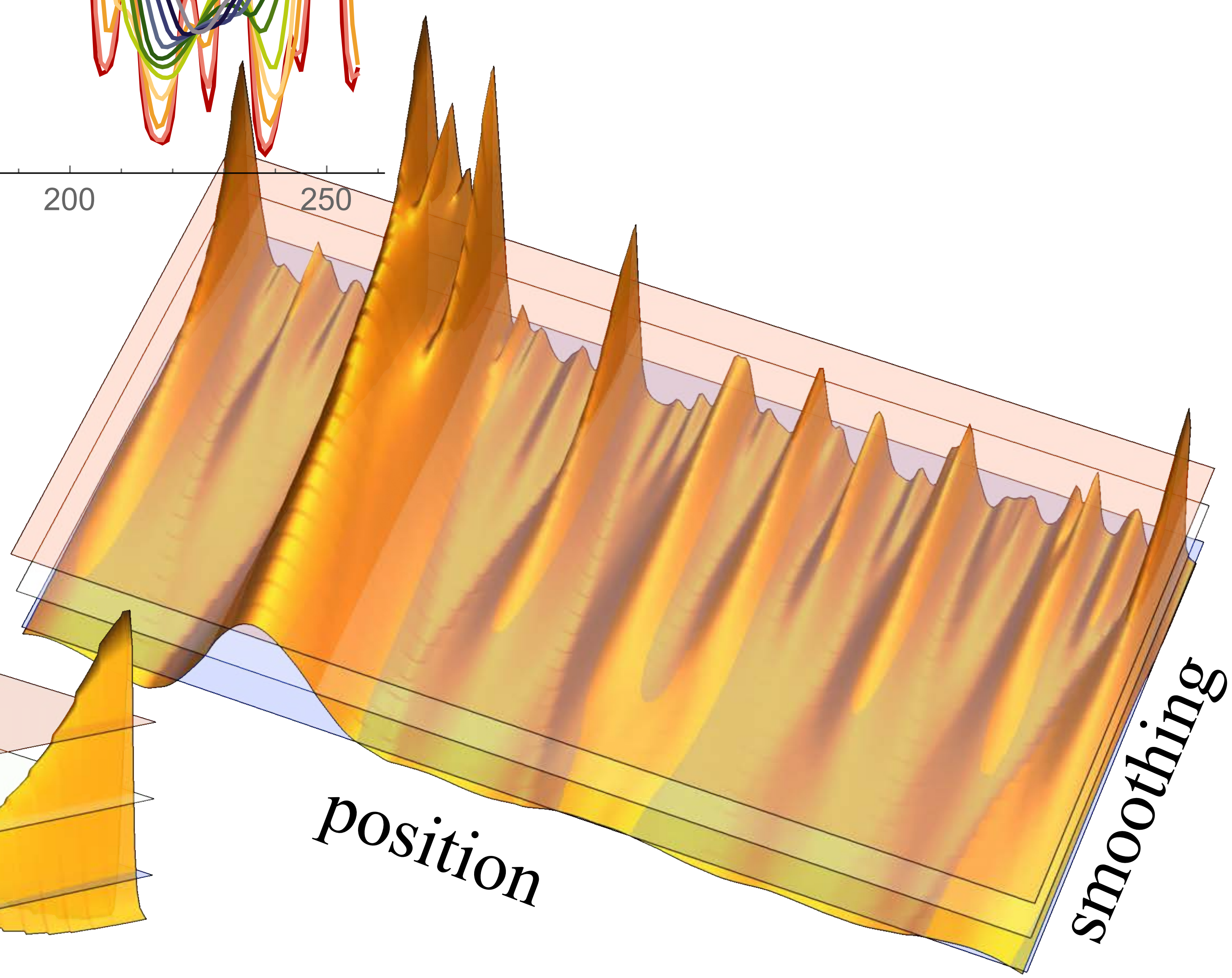
Change in excursion's topology impacts galaxy formation



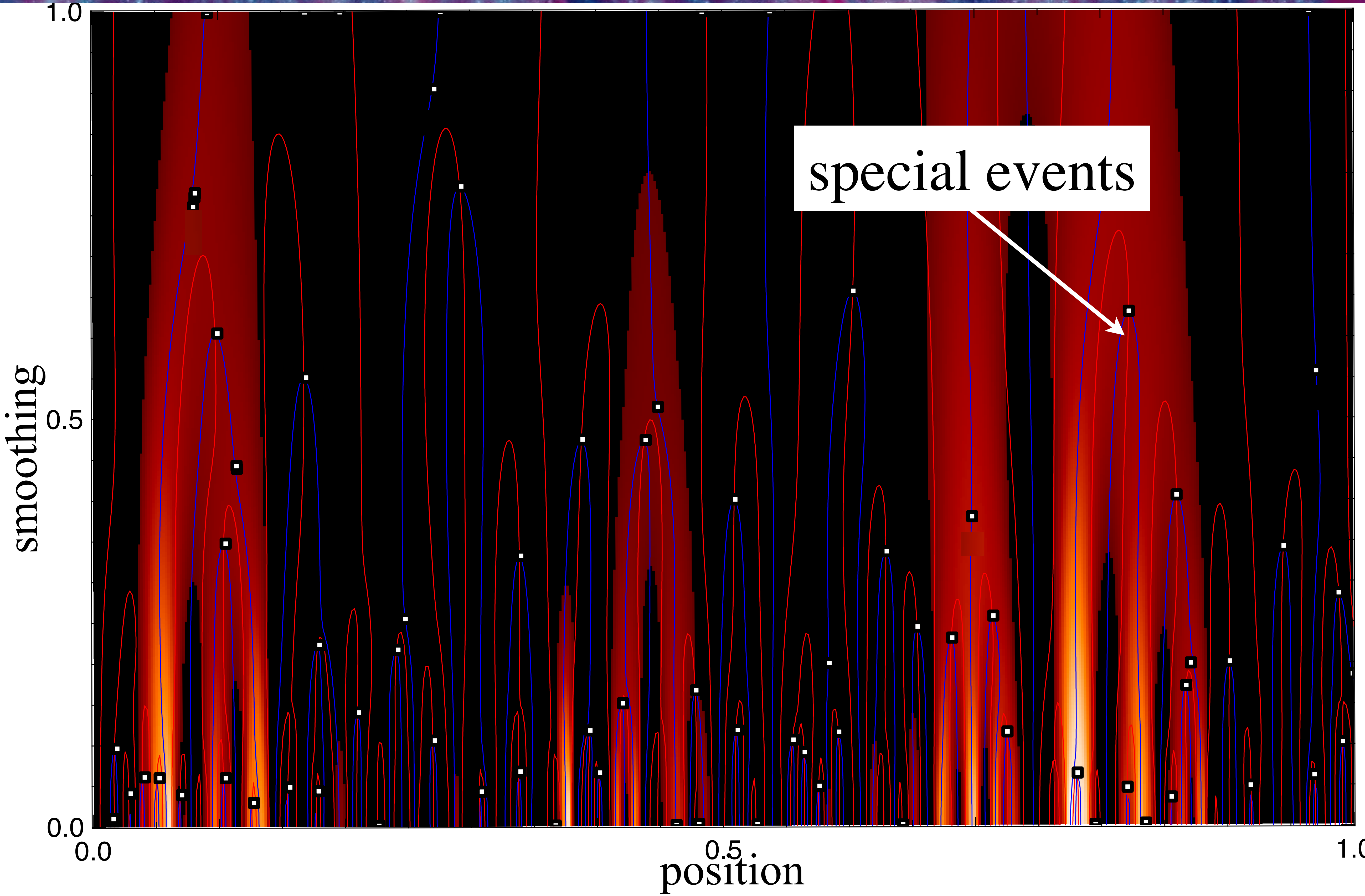
1D outlook: ridges in position-smoothing landscape





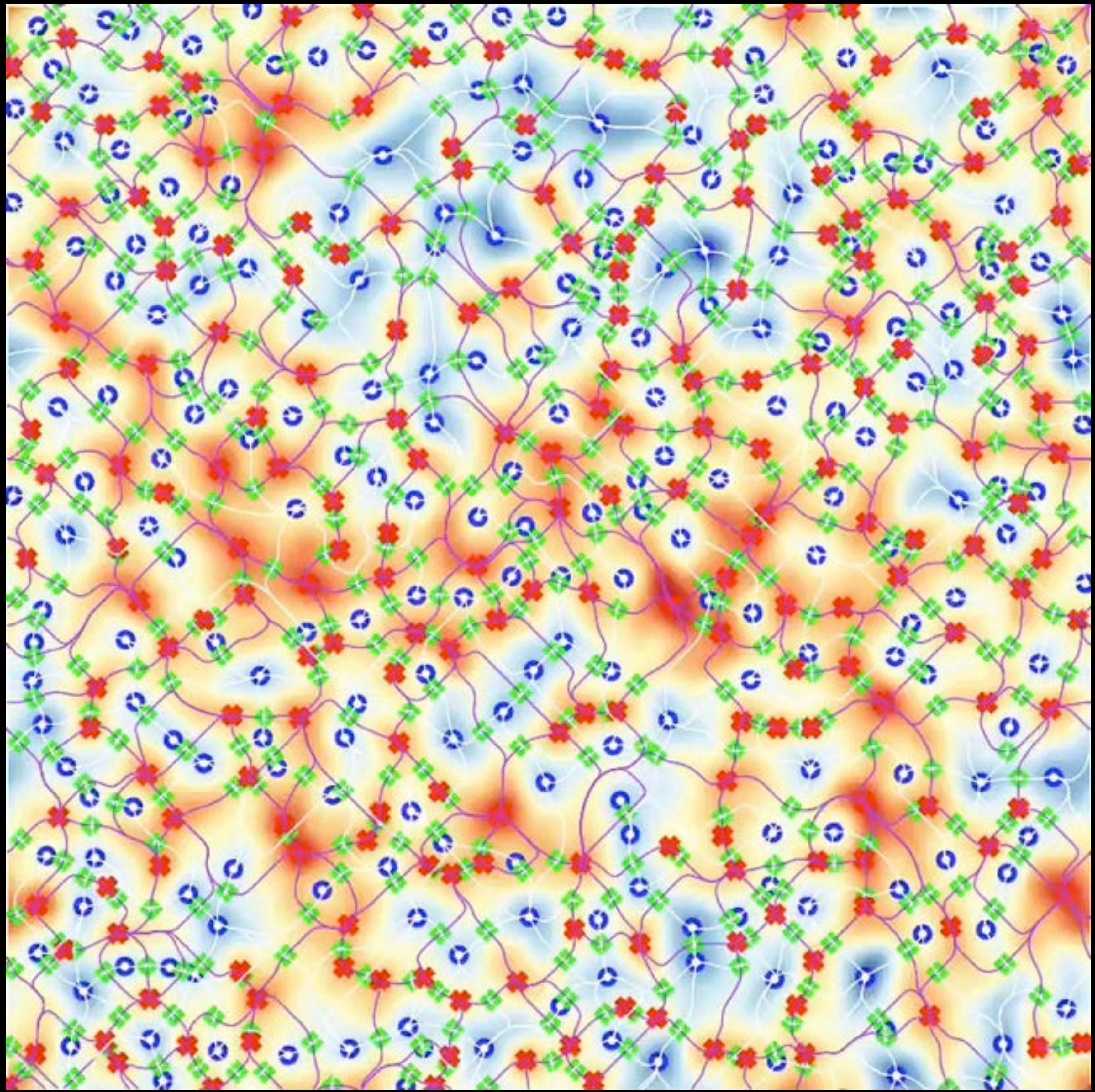


1D outlook



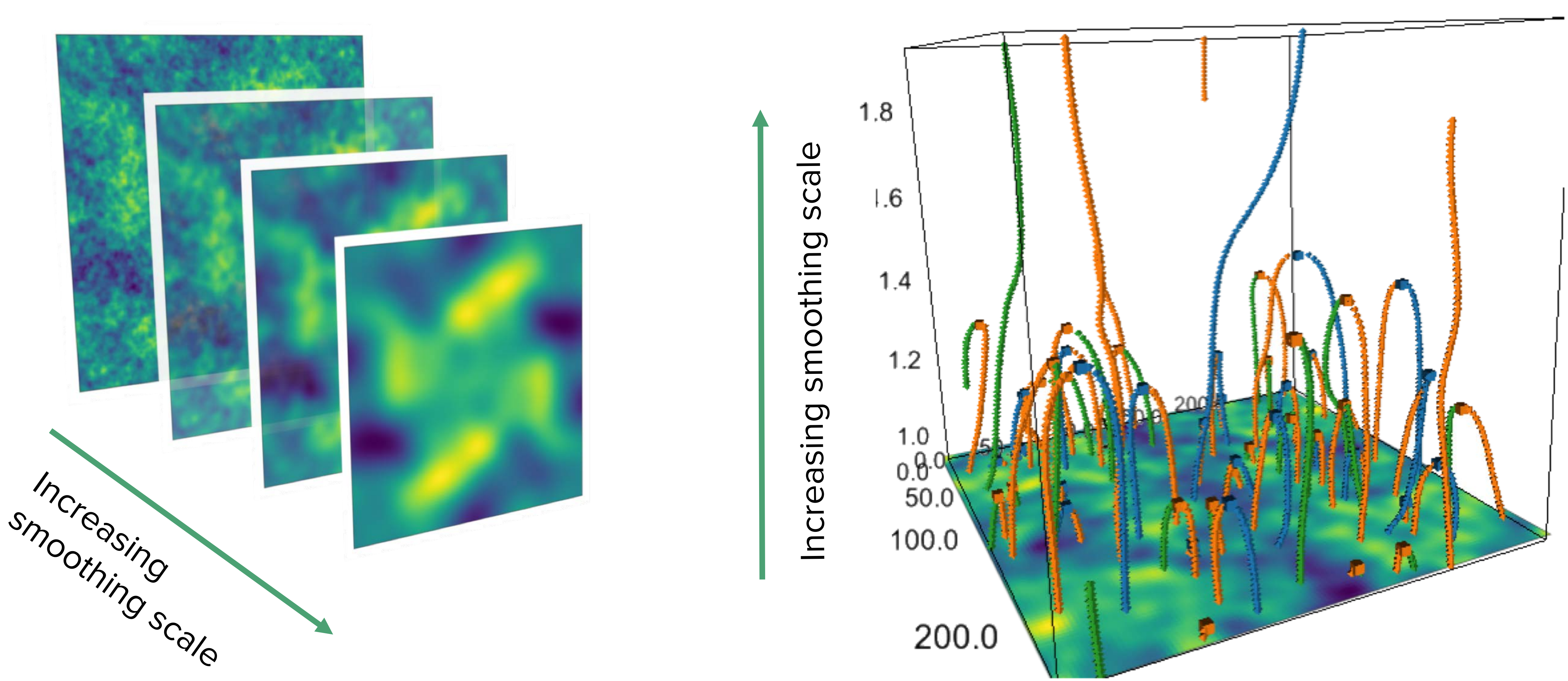
Special event : topology of underlying field changes

2D outlook

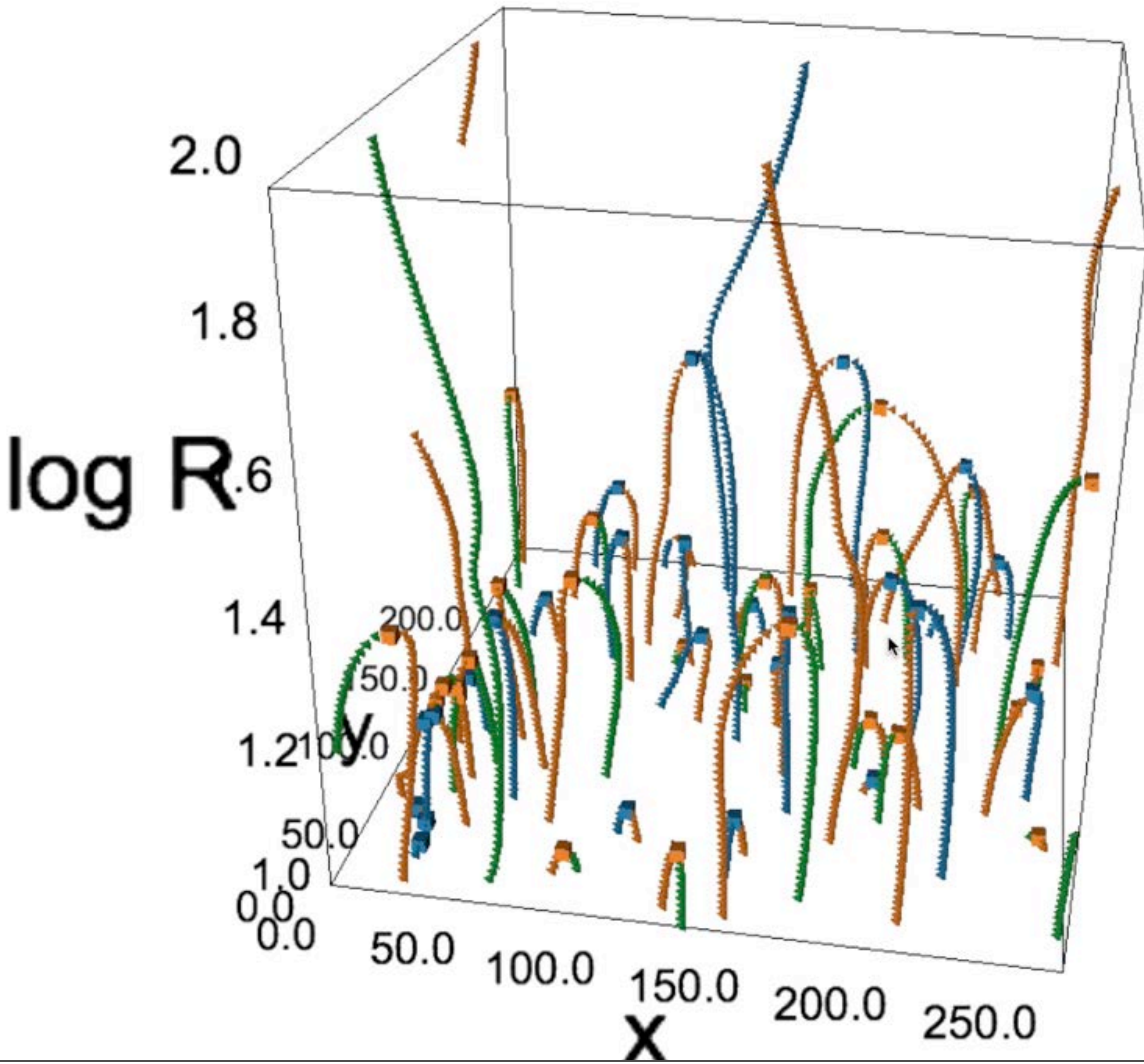


2D outlook

Building the skeleton tree

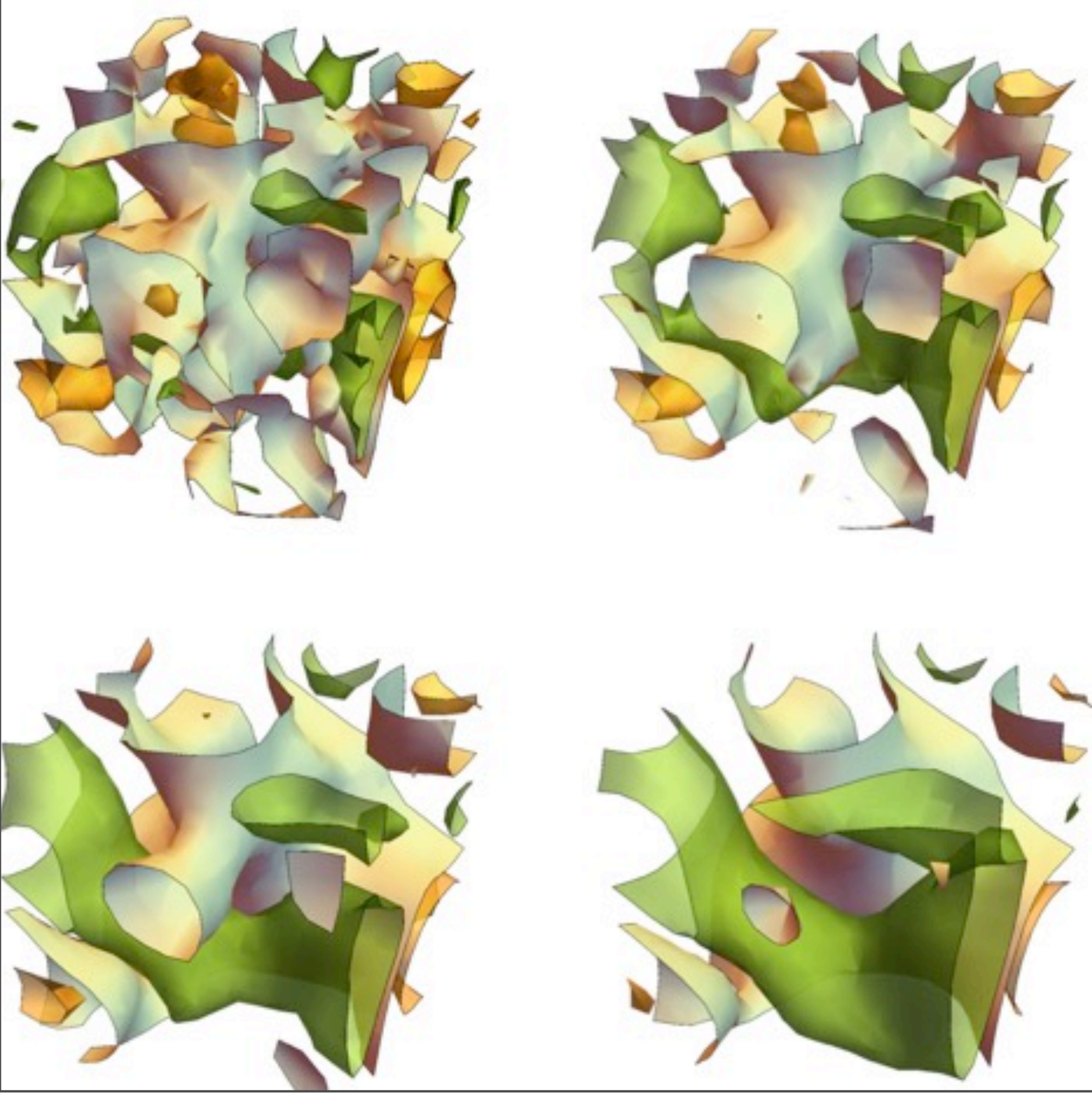


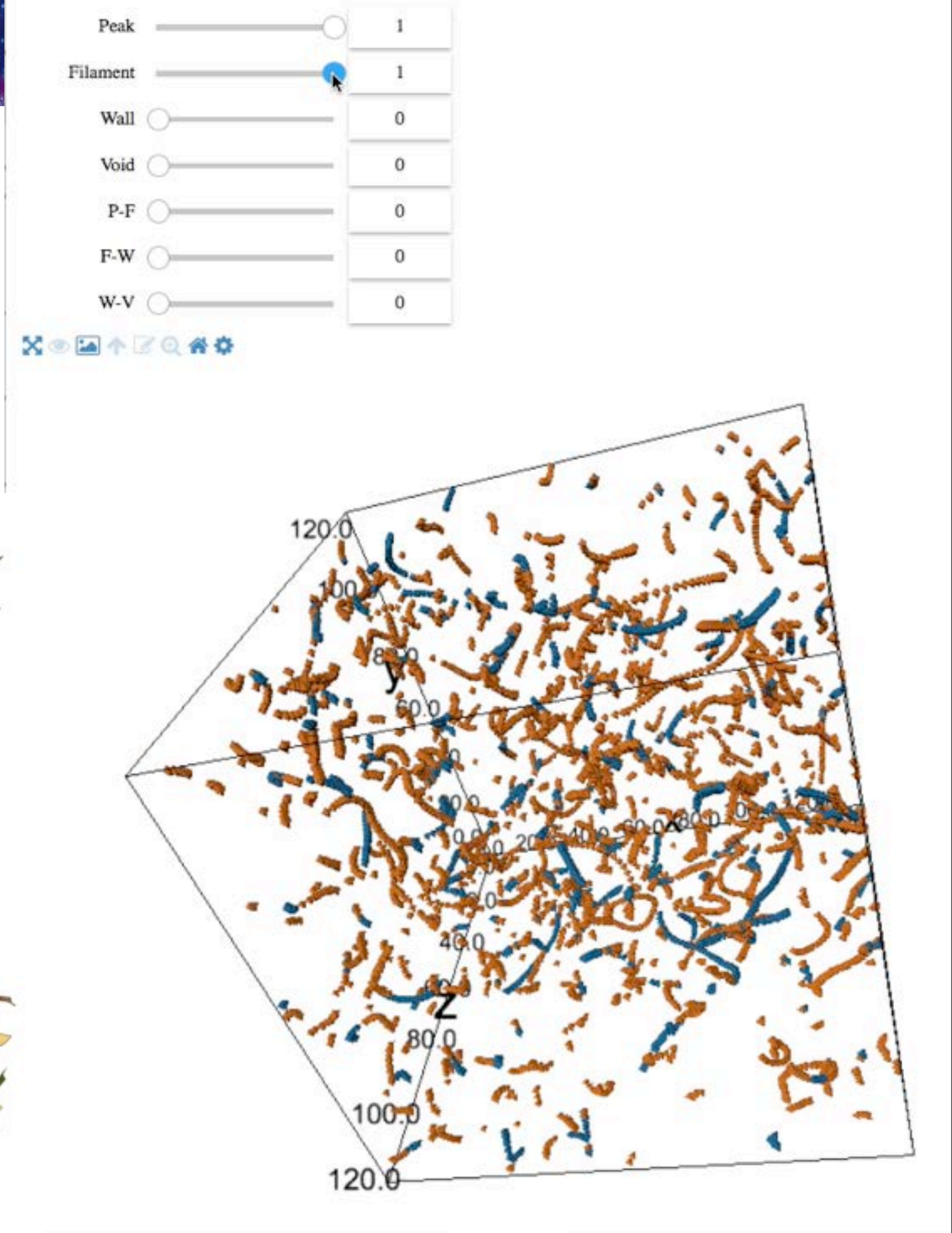
2D outlook


log R
2.0
1.8
1.6
1.4
1.2
1.0
0.0
y
200.0
150.0
100.0
50.0
0.0
50.0
100.0
150.0
200.0
250.0
x

3D outlook

- GRF smoothed at ≠ scales
- identify critical points
- build skeleton tree
- find critical events





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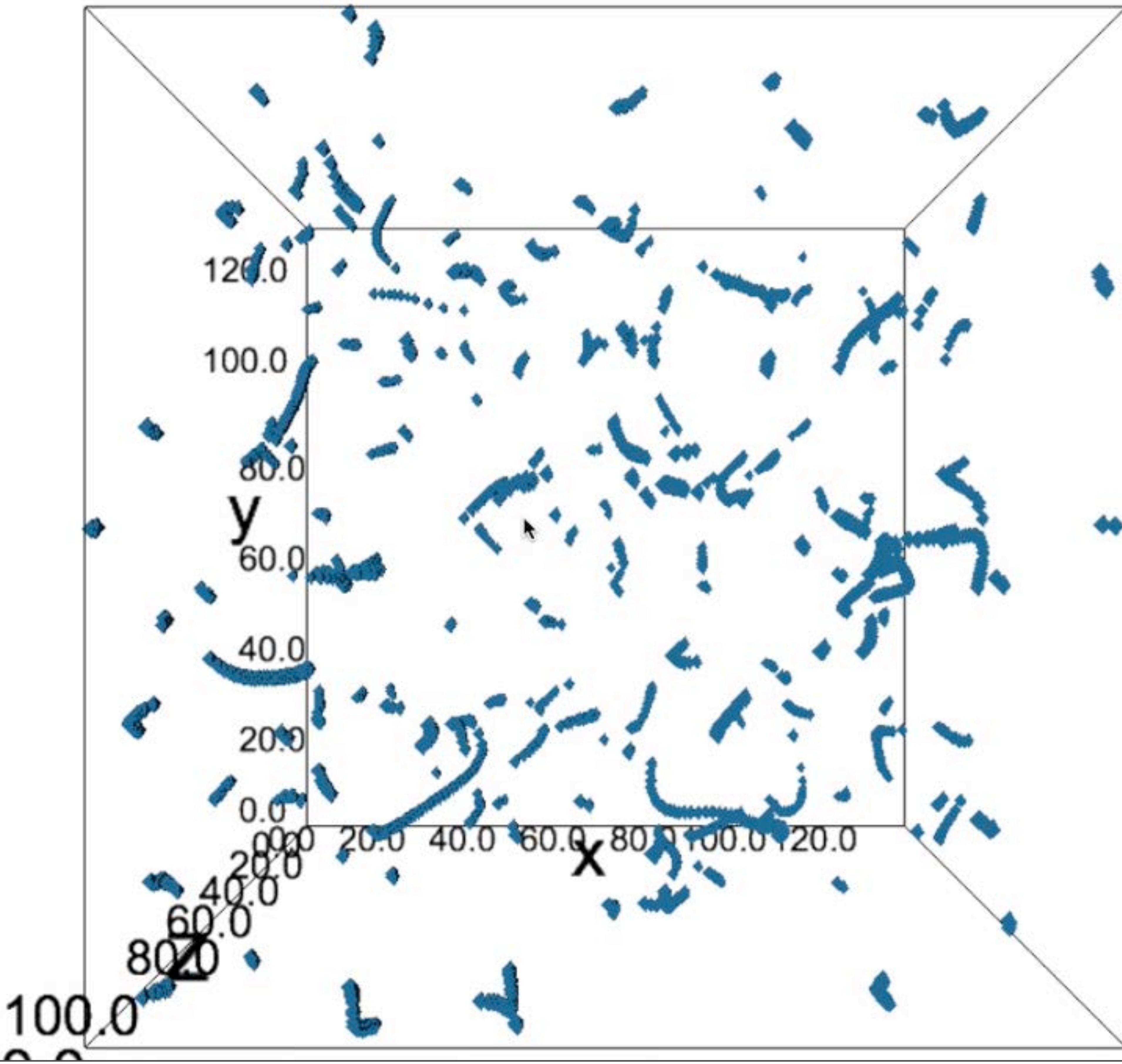
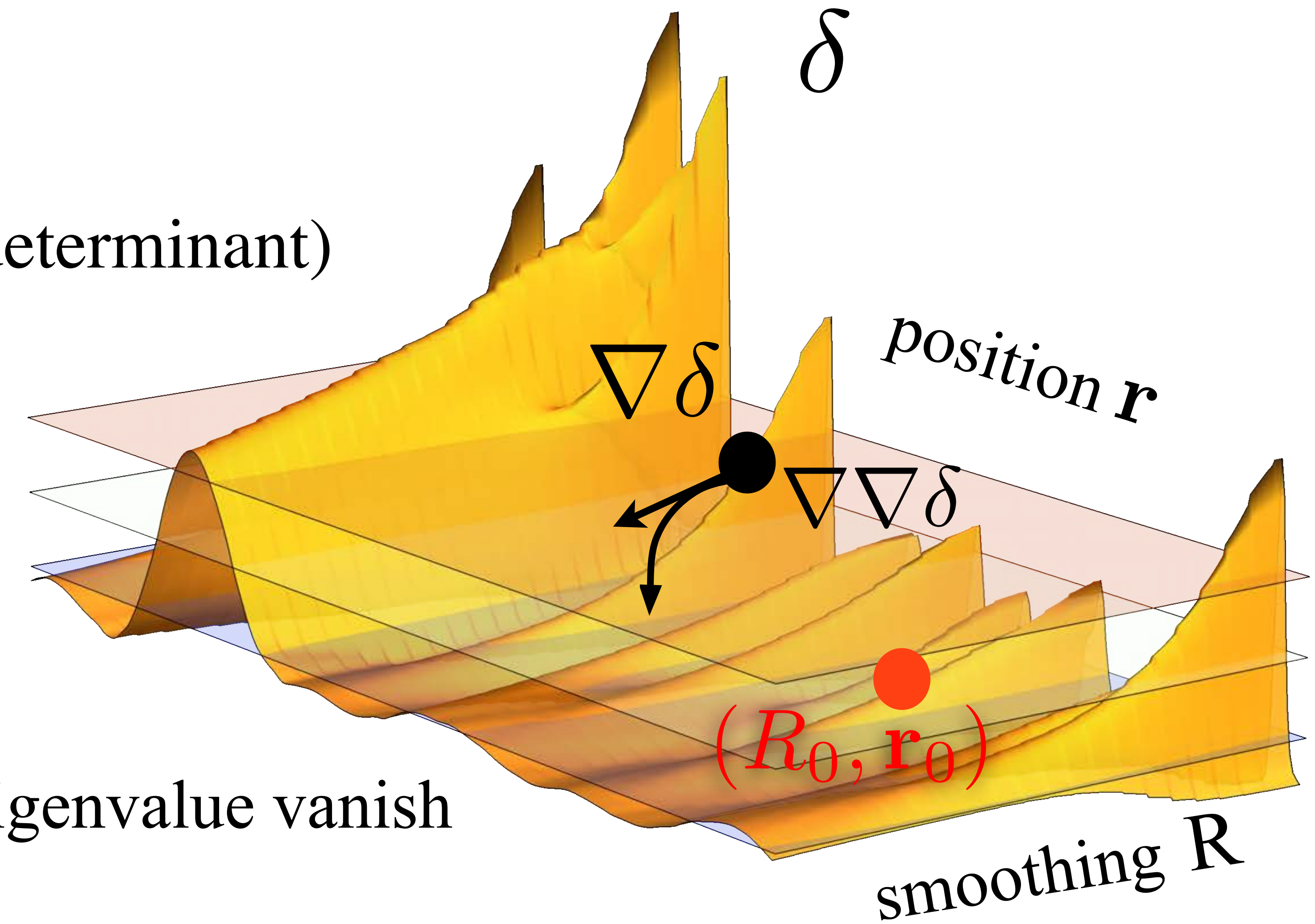

120.0
100.0
80.0
y
60.0
40.0
20.0
0.0
0.0 20.0 40.0 60.0 80.0 100.0 120.0
x
20.0
40.0
60.0
80.0
z
100.0

Critical event PDF

$$\boxed{\frac{\partial^2 \mathcal{N}}{\partial r^3 \partial R} \equiv \langle \delta_{\rm D}^{(3)}(\mathbf{r}-\mathbf{r}_0)\delta_{\rm D}(R-R_0)\rangle \,,}$$

where $\mathbf{r}_0$ is a (double) critical point in real space and R_0 the scale at which the two critical points merge.

✔ Invoque ergodicity
✔ Change variable to (gradient, determinant)



At $(R_0, \mathbf{r}_0)$ gradient and one eigenvalue vanish

Critical event PDF

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$$d(\delta) \equiv \det(\nabla\nabla\delta) = \lambda_1\lambda_2\lambda_3$$

$$\boxed{\frac{\partial^2 \mathcal{N}}{\partial r^3 \partial R} = \left\langle J\, \delta_{\rm D}^{(3)}(\nabla\delta)\delta_{\rm D}(d) \right\rangle}$$

$$J(d,\delta) = \begin{vmatrix} \partial_R d & \vec{\nabla} d \\ \partial_R \vec{\nabla}\delta^T & \vec{\nabla}\vec{\nabla}\delta \end{vmatrix}$$

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for a Gaussian filter

$$\partial_R \delta = -R\Delta\delta$$

Critical event PDF

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at which the two critical poi

$$d(\delta) \equiv \det(\nabla\nabla\delta)$$

$$\frac{J(d,\delta)}{\sigma_1\sigma_2^4\sigma_3} = |x_{11}x_{22}| \begin{vmatrix} \partial_R x_{33} & x_{33i} \\ \partial_R x_i & x_{ij} \end{vmatrix},$$

$$= |x_{11}x_{22}| \begin{vmatrix} \partial_R x_{33} & x_{133} & x_{233} & x_{333} \\ \partial_R x_1 & x_{11} & 0 & 0 \\ \partial_R x_2 & 0 & x_{22} & 0 \\ \partial_R x_3 & 0 & 0 & 0 \end{vmatrix}$$

$$= |x_{11}x_{22}|^2 |\partial_R x_3||x_{333}|,$$

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$$x \equiv \frac{\delta}{\sigma_0},\ x_k \equiv \frac{\nabla_k \delta}{\sigma_1},\ x_{kl} \equiv \frac{\nabla_k\nabla_l\delta}{\sigma_2},\ x_{klm} \equiv \frac{\nabla_m\nabla_l\nabla_k\delta}{\sigma_3}$$

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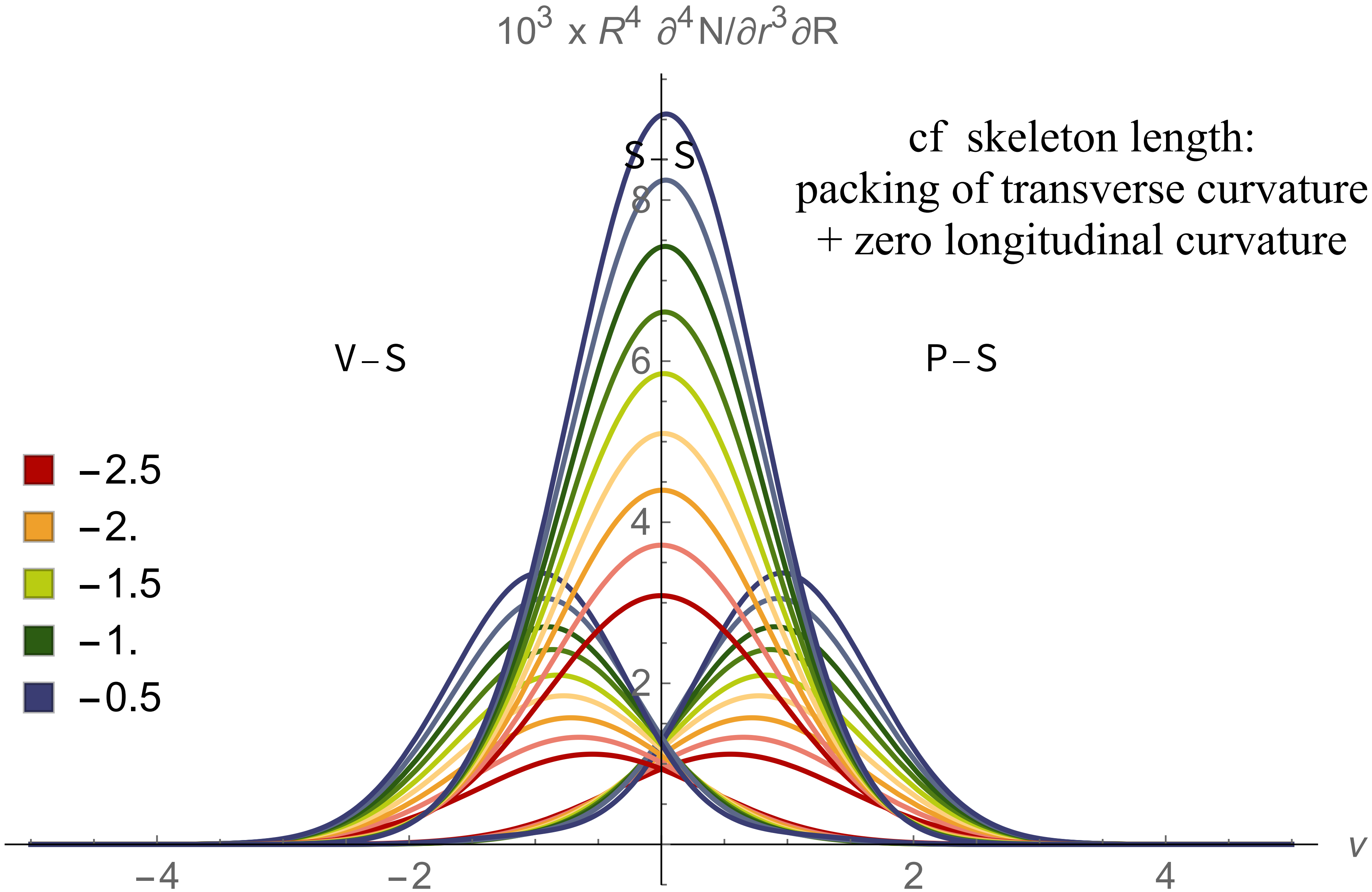
Derivation

$$\frac{\partial^2 n}{\partial R \partial \nu} = \frac{\partial^5 \mathcal{N}}{\partial r^3 \partial R \partial \nu},$$

$$= \frac{R}{\tilde{R}} \frac{\left\langle |x_{11} x_{22}| |x_{3ii}| |x_{333}| \delta_{\rm D}^{(3)}(x_i) \delta_{\rm D}(x_{33}) \delta_{\rm D}(x-\nu) \right\rangle}{R_*^3 \tilde{R}},$$



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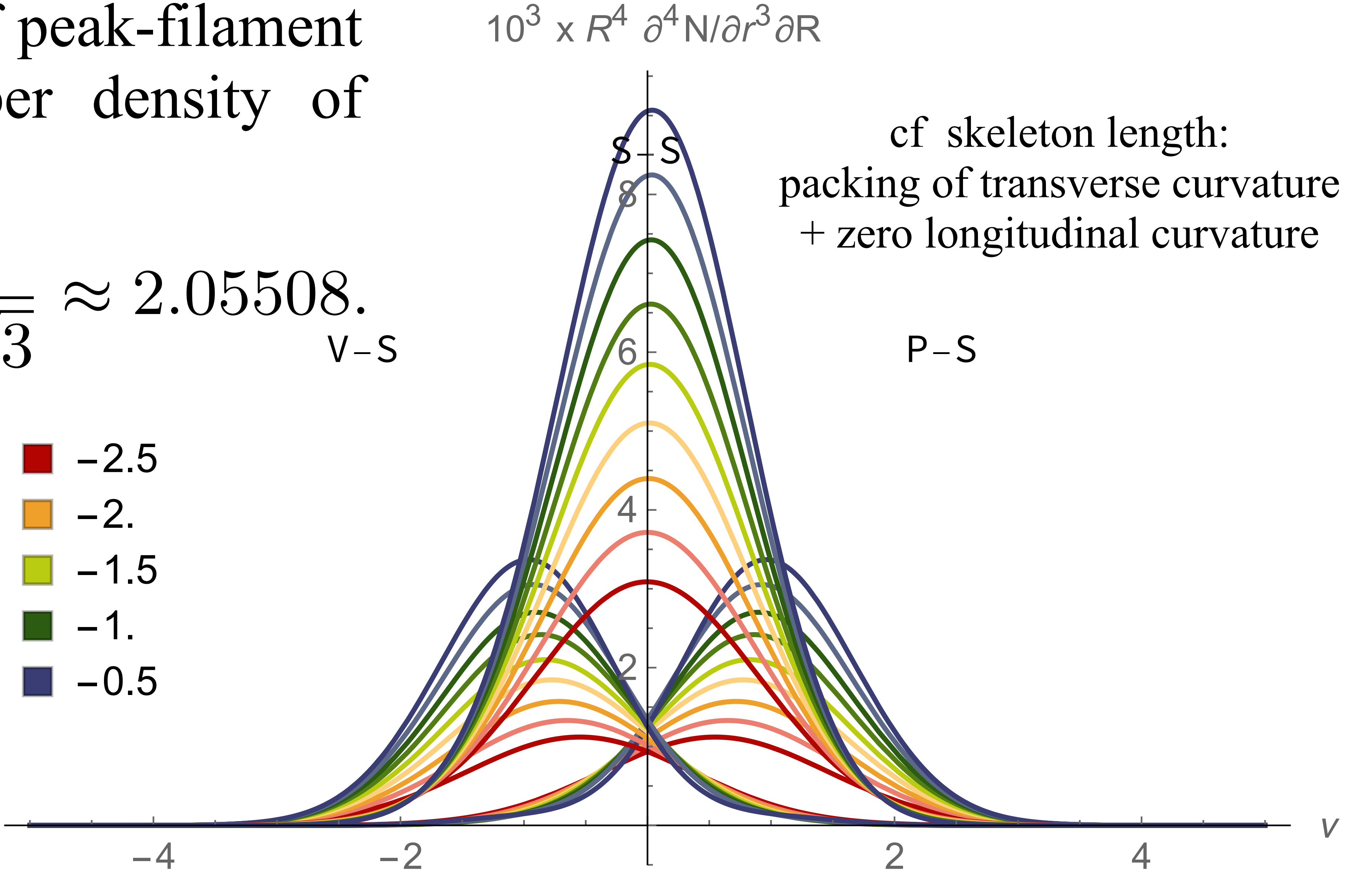
E.g. number density of peak-filament mergers to the number density of filament-wall mergers

$$r_{2/1} = \frac{24\sqrt{3}}{29\sqrt{2} - 12\sqrt{3}} \approx 2.05508.$$



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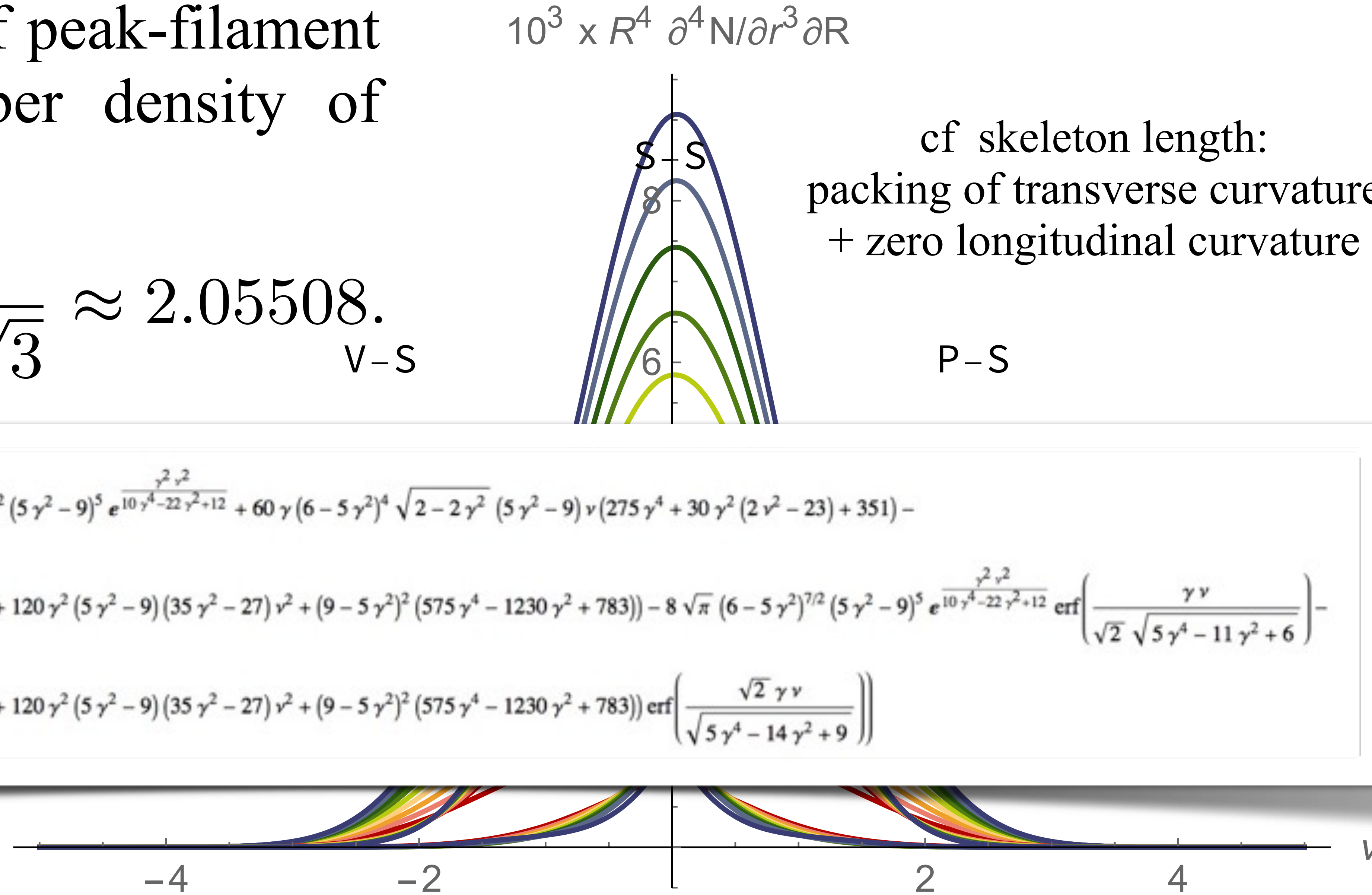


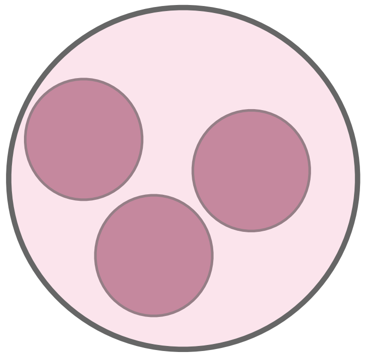
cf skeleton length:
packing of transverse curvature
+ zero longitudinal curvature

$$\frac{1}{8\sqrt{5}\,\pi^{3/2}(6-5\gamma^2)^4(5\gamma^2-9)^5} e^{\frac{\nu^2}{2(\gamma^2-1)}} \Bigg(-8\sqrt{\pi}(6-5\gamma^2)^{7/2}(5\gamma^2-9)^5 e^{\frac{\gamma^2\nu^2}{10\gamma^4-22\gamma^2+12}} + 60\gamma(6-5\gamma^2)^4\sqrt{2-2\gamma^2}(5\gamma^2-9)\nu(275\gamma^4+30\gamma^2(2\nu^2-23)+351) -$$
$$2\sqrt{\pi}(6-5\gamma^2)^4\sqrt{9-5\gamma^2}\, e^{\frac{2\gamma^2\nu^2}{5\gamma^4-14\gamma^2+9}}(3600\gamma^4\nu^4+120\gamma^2(5\gamma^2-9)(35\gamma^2-27)\nu^2+(9-5\gamma^2)^2(575\gamma^4-1230\gamma^2+783)) - 8\sqrt{\pi}(6-5\gamma^2)^{7/2}(5\gamma^2-9)^5 e^{\frac{\gamma^2\nu^2}{10\gamma^4-22\gamma^2+12}} \operatorname{erf}\left(\frac{\gamma\nu}{\sqrt{2}\sqrt{5\gamma^4-11\gamma^2+6}}\right) -$$
$$2\sqrt{\pi}(6-5\gamma^2)^4\sqrt{9-5\gamma^2}\, e^{\frac{2\gamma^2\nu^2}{5\gamma^4-14\gamma^2+9}}(3600\gamma^4\nu^4+120\gamma^2(5\gamma^2-9)(35\gamma^2-27)\nu^2+(9-5\gamma^2)^2(575\gamma^4-1230\gamma^2+783)) \operatorname{erf}\left(\frac{\sqrt{2}\gamma\nu}{\sqrt{5\gamma^4-14\gamma^2+9}}\right)\Bigg)$$

In (invariant) Hessian frame:

extrema

BBKS



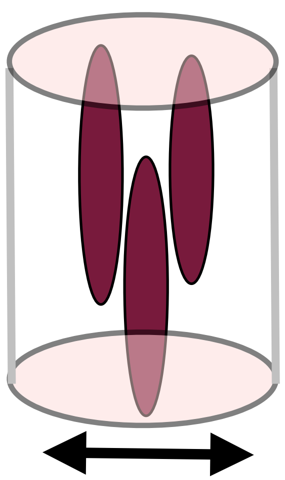
$$n_{\rm ext} = \int \mathcal{P}(\mathbf{x}) \prod_{1\leq i\leq 3} \delta_{\rm D}(x_i)\lambda_i d\mathbf{x}$$

$\text{volume} \propto 1/\lambda_1\lambda_2\lambda_3$

packing sphere problem: curvature.

skeleton

SPCP



$$n_{skl} = \int P(\mathbf{x}) \prod_{1< i\leq 3} \delta_{\rm D}(x_i)\lambda_i d\mathbf{x}$$

$\text{section} \propto 1/\lambda_2\lambda_3$

packing tube problem: transverse curvature.

Peak theory: Gaussian predictions

If the field is Gaussian (large scales/early times), the total number density of critical points then reads

2D

$$\langle n_{\rm max}\rangle = \langle n_{\rm min}\rangle = \frac{1}{8\sqrt{3}\pi R_\star^2}$$

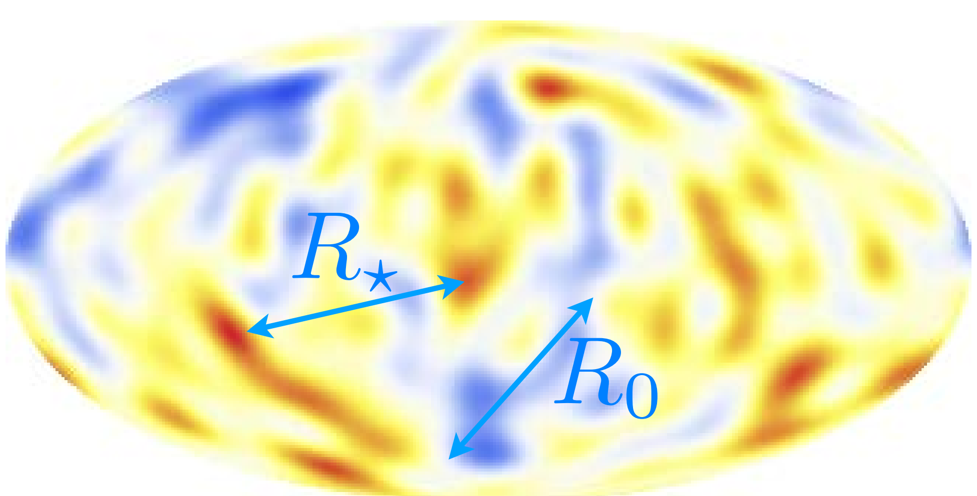
$$\langle n_{\rm sad}\rangle = \frac{1}{4\sqrt{3}\pi R_\star^2}$$

3D

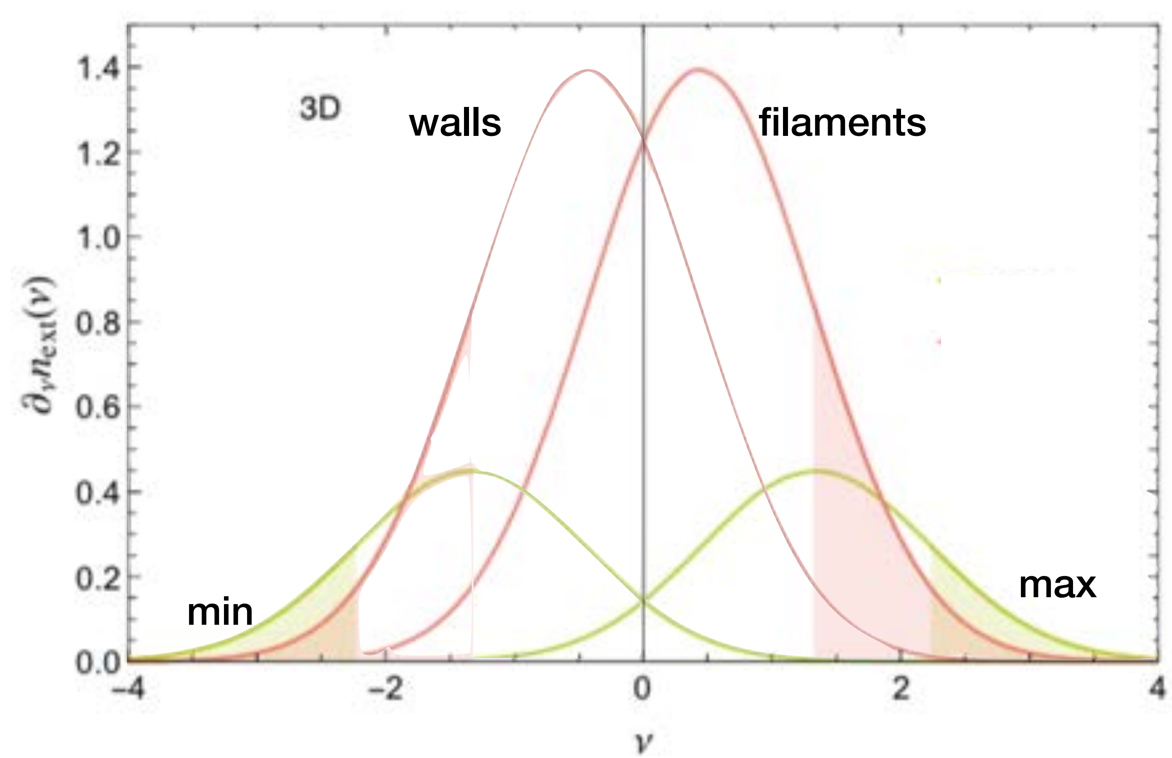
$$\langle n_{\rm max}\rangle = \langle n_{\rm min}\rangle = \frac{29\sqrt{15}-18\sqrt{10}}{1800\pi^2 R_\star^3}$$

$$\langle n_{\rm sadf}\rangle = \langle n_{\rm sadw}\rangle = \frac{29\sqrt{15}+18\sqrt{10}}{1800\pi^2 R_\star^3},$$

And as a function of peak height (analytical in 2D, not in 3D) :



$R_\star = \sigma_1/\sigma_2 = $ distance between peaks

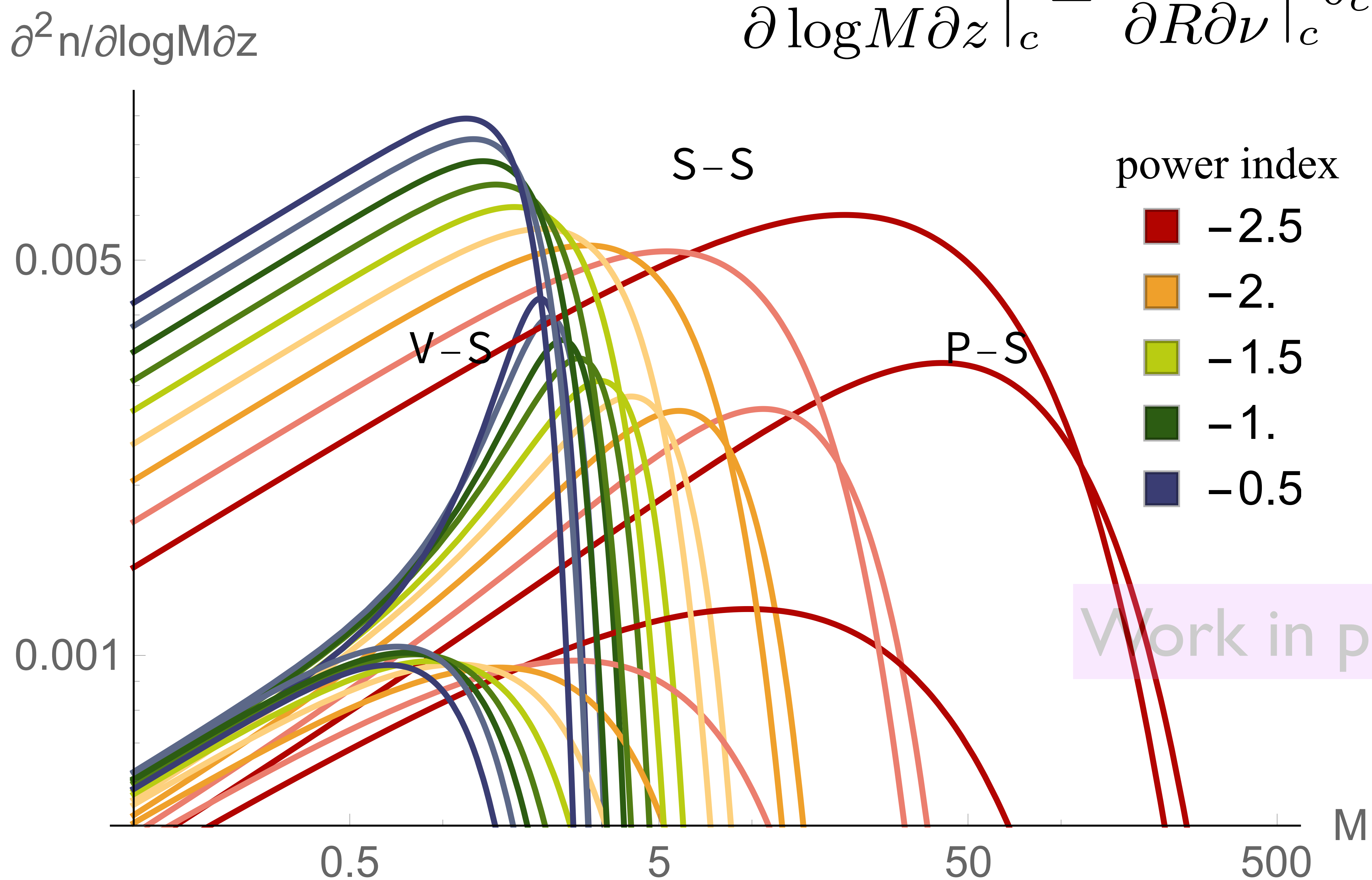


Application: merger rates

$$\nu\,\sigma_0 = \delta_c D(z) \quad \text{where} \quad \delta_c = \frac{3}{20}(12\pi)^{2/3}$$

Map event count to (z,M)

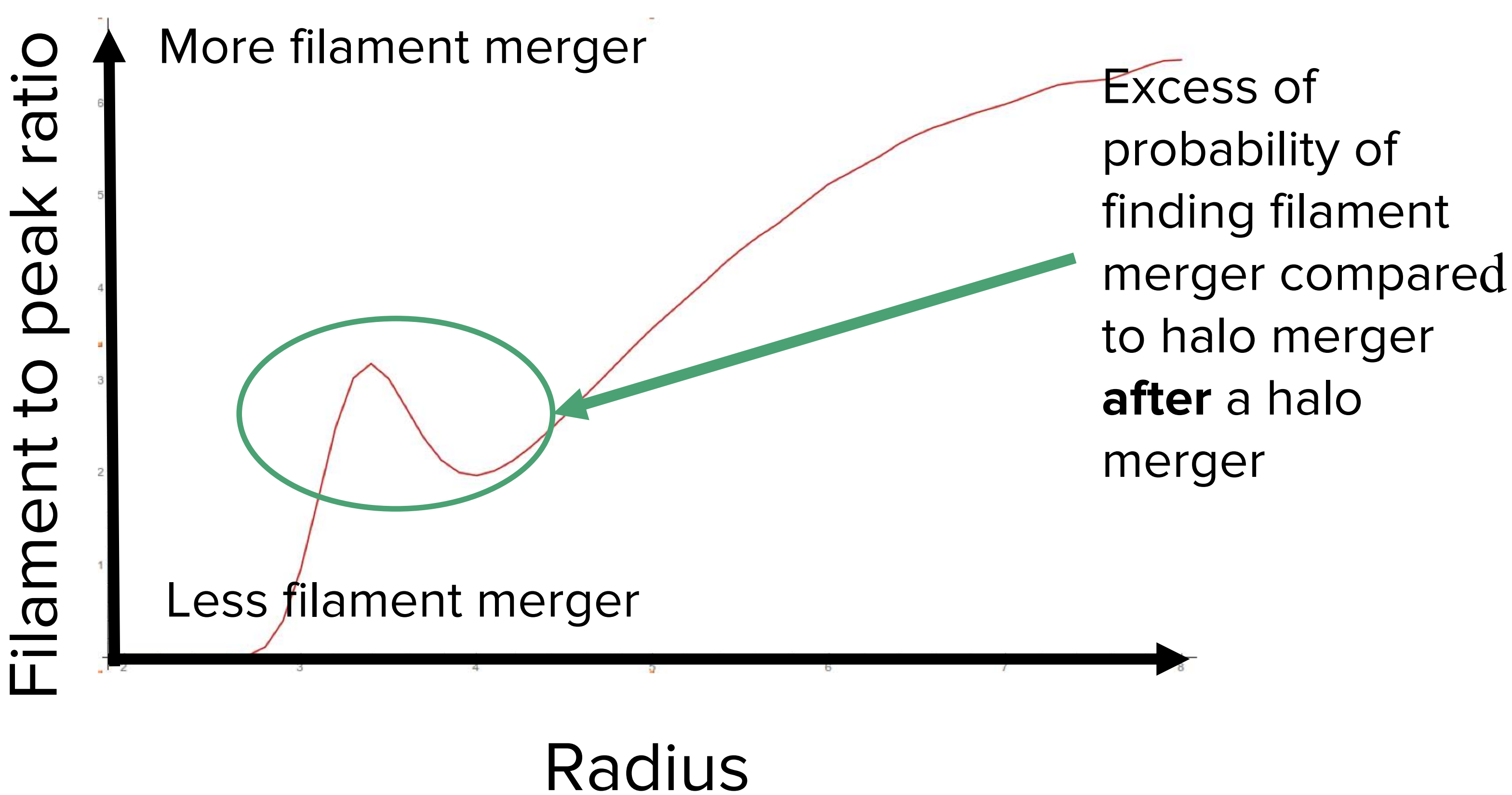
$$\left.\frac{\partial^2 n}{\partial \log M \partial z}\right|_c = \left.\frac{\partial^2 n}{\partial R \partial \nu}\right|_c \delta_c \frac{dD}{dz}\frac{R}{3}$$

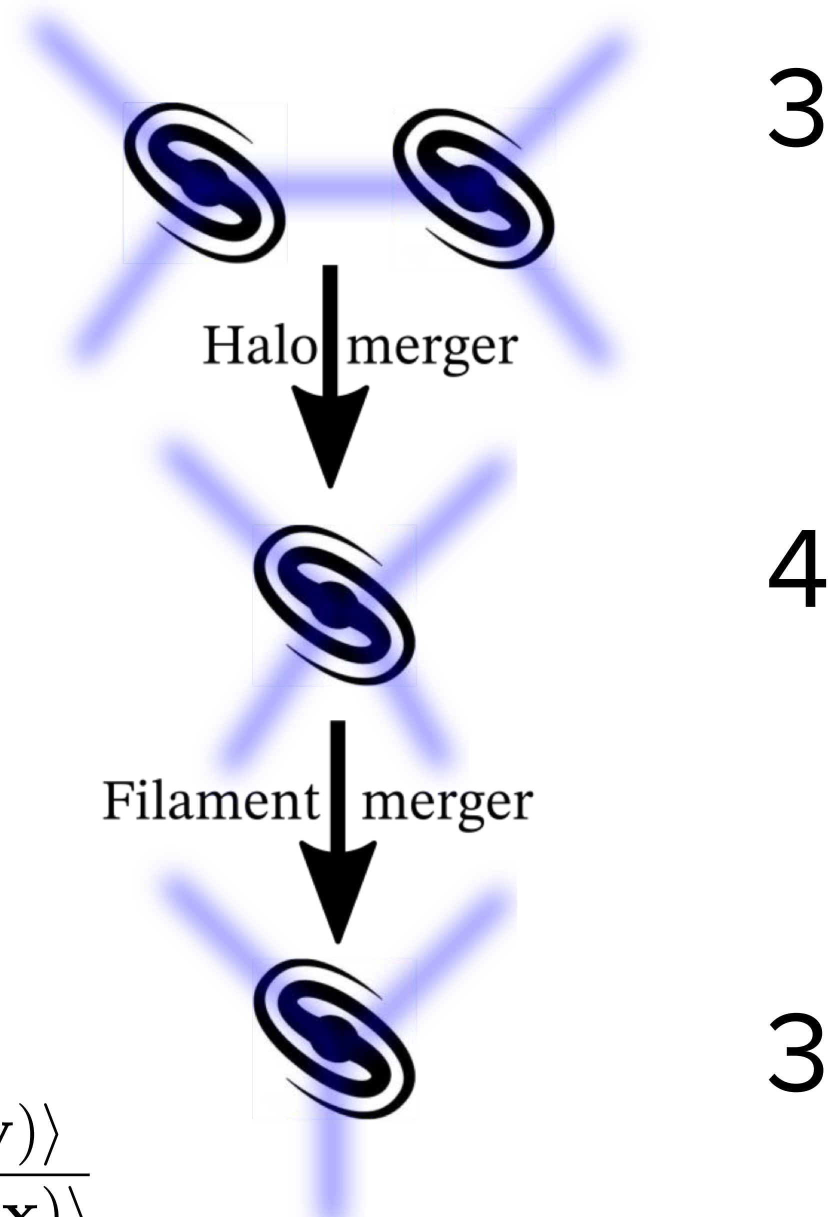


Application: preserving cosmic connectivity

On the connectivity of halos

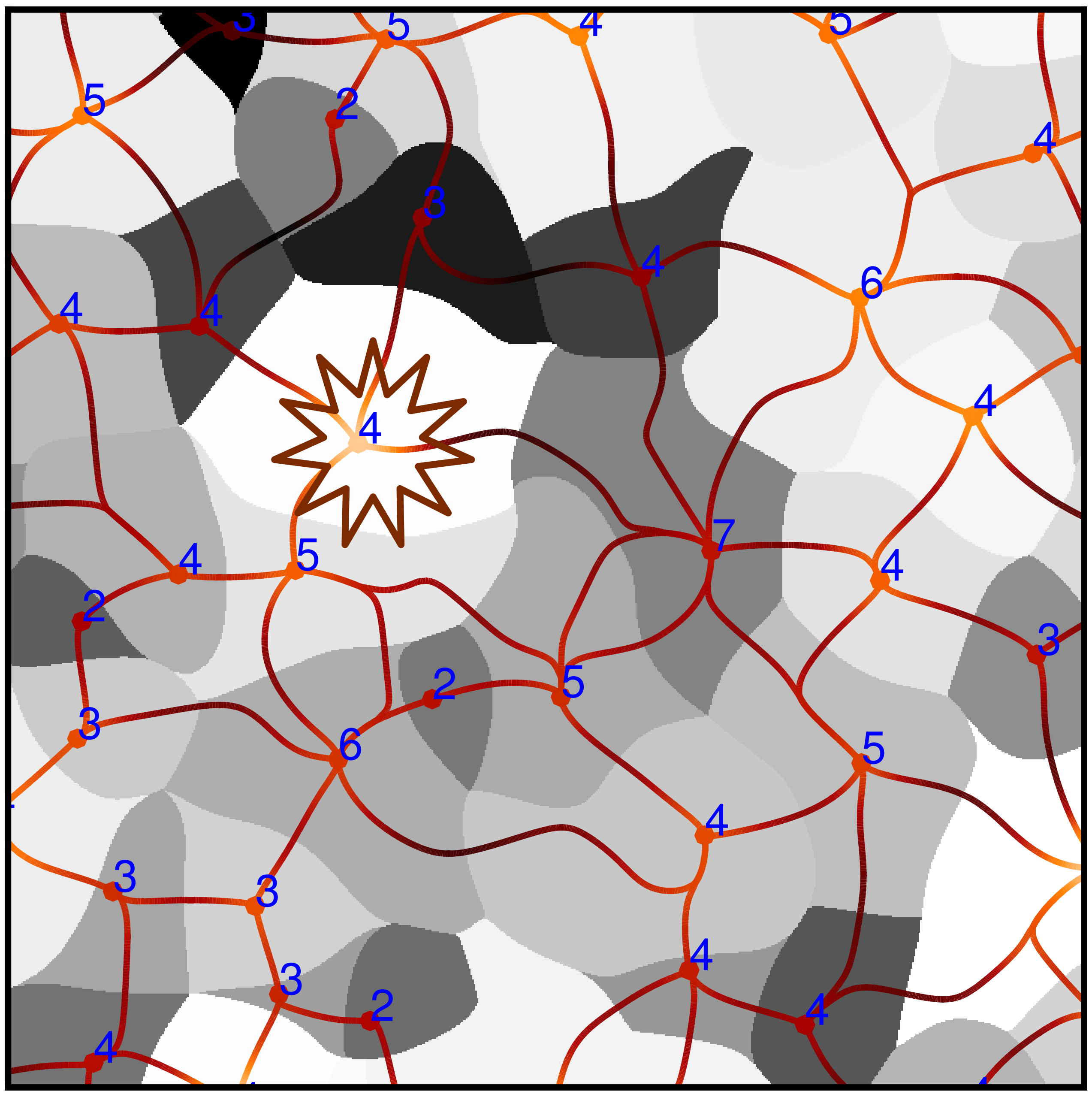
Compute frequency of filament merger compared to halo merger in the vicinity of a halo merger event $\xi_{\rm hf}(r)/\xi_{\rm hh}(r)$.





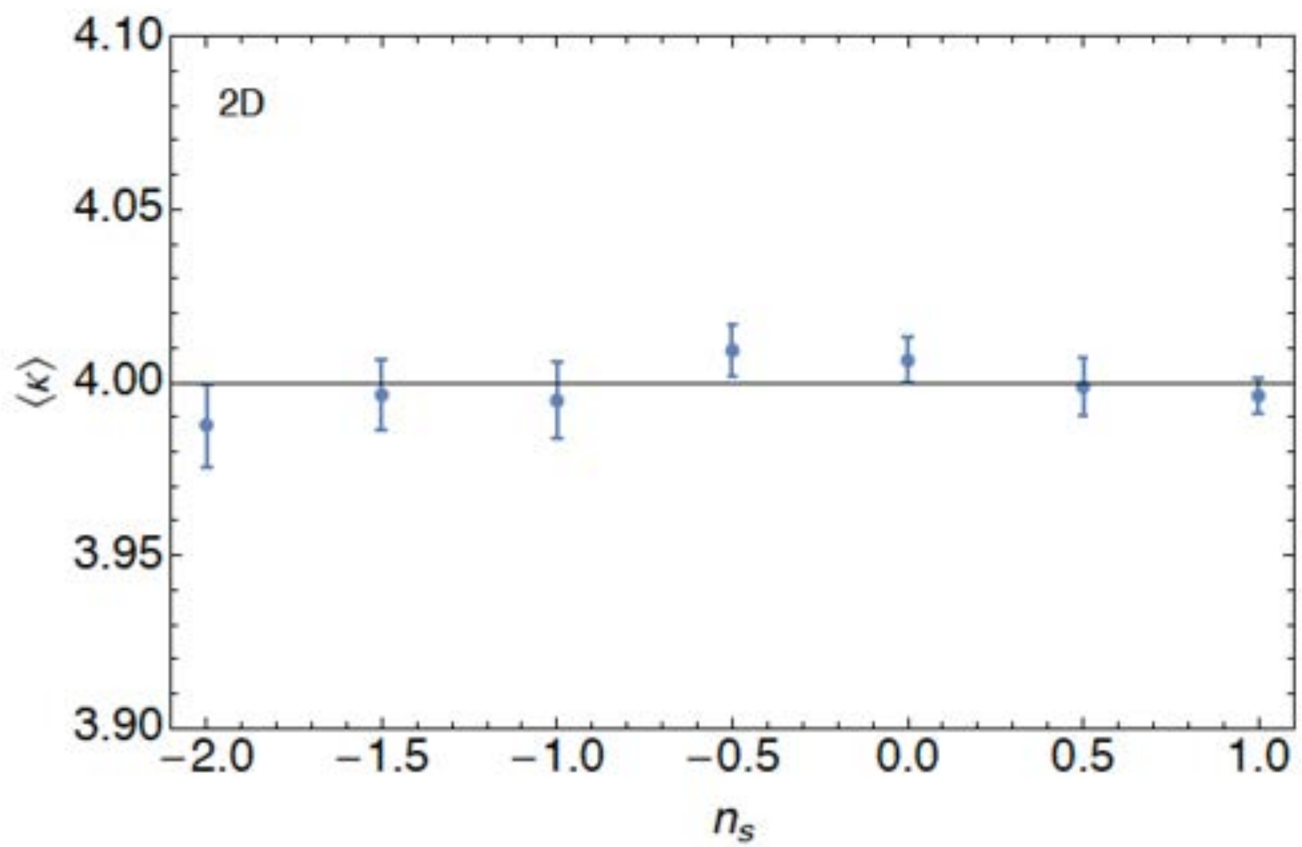
$$1+\xi_p = \frac{\langle \mathrm{cond}_p(\mathbf{x})\mathrm{cond}_p(\mathbf{y})\rangle}{\langle \mathrm{cond}_p(\mathbf{x})\rangle^2},\ 1+\xi_f = \frac{\langle \mathrm{cond}_f(\mathbf{x})\mathrm{cond}_p(\mathbf{y})\rangle}{\langle \mathrm{cond}_f(\mathbf{x})\rangle\langle \mathrm{cond}_p(\mathbf{x})\rangle}$$

Global connectivity for GRF



How many filaments connect to a node?

Number of connected saddles are measured using the **Disperse** skeleton algorithm (Sousbie +11) in GRF realisations.



Can we predict the mean connectivity?

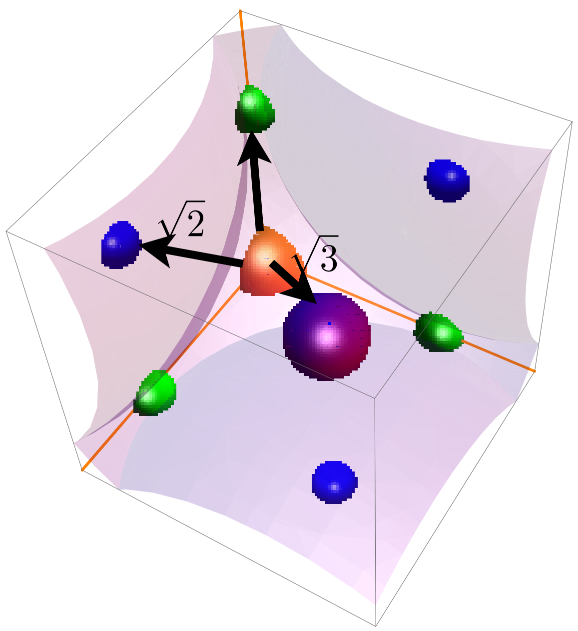
Global connectivity for GRF: theory

Because each filament goes through one and only one saddle pt, on average:

$$\langle \kappa \rangle = \frac{2\bar{n}_{\rm sad}}{\bar{n}_{\rm max}}$$

$= 4$ in 2D GRF

$$= \frac{2\left(1057 + 348\sqrt{6}\right)}{625} \approx 6.11 \text{ in 3D GRF}$$



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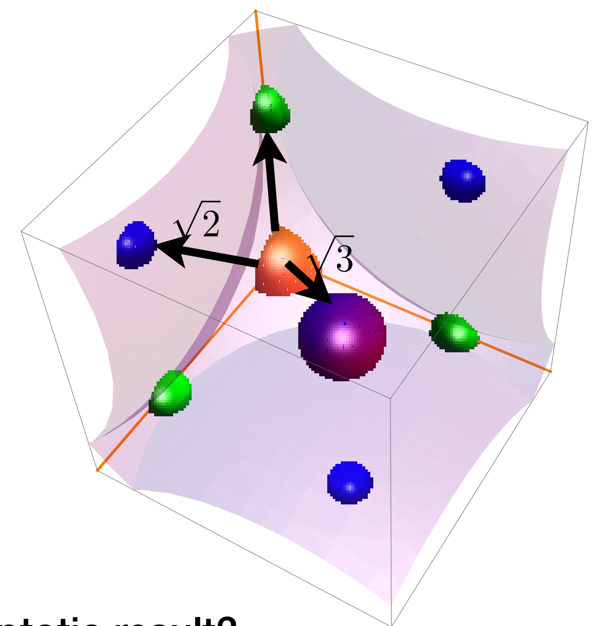
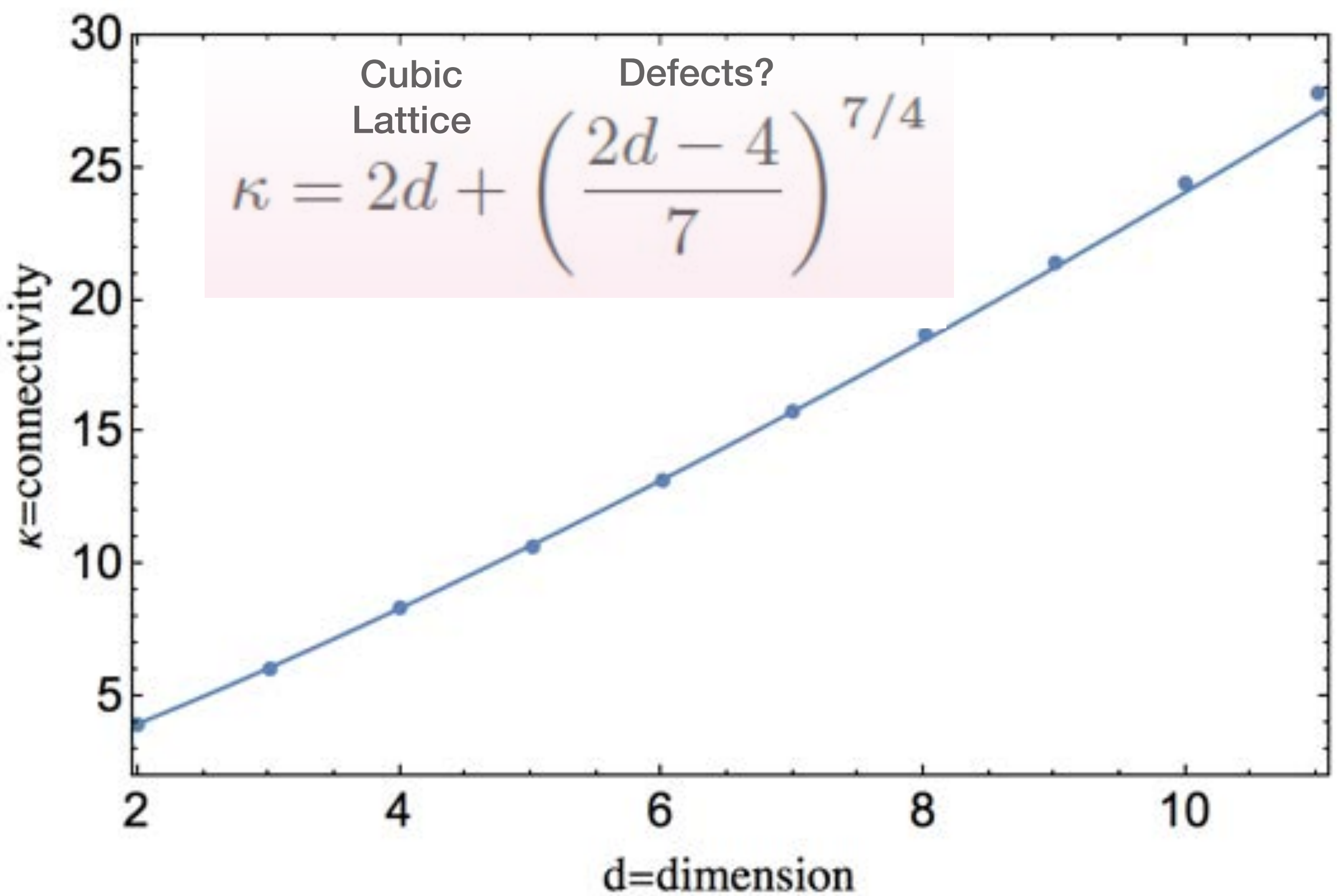
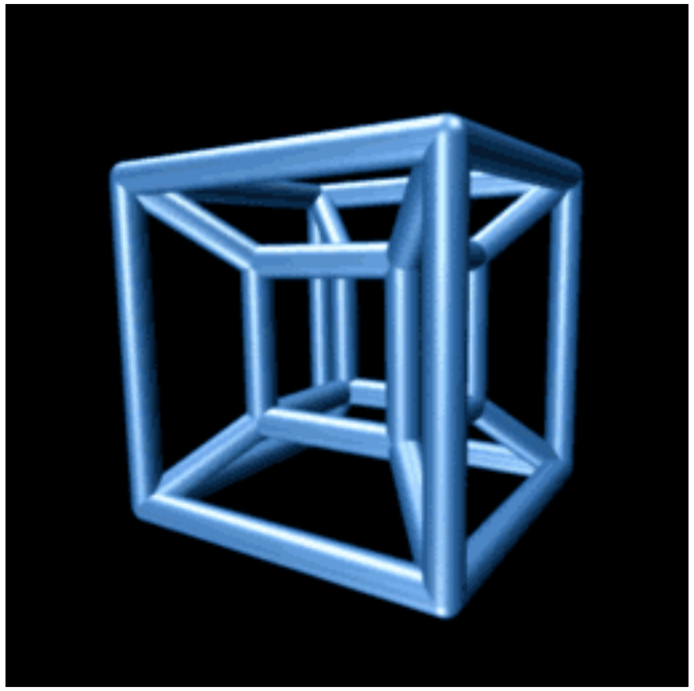


In d dimensions, (relying on numerical integrations):



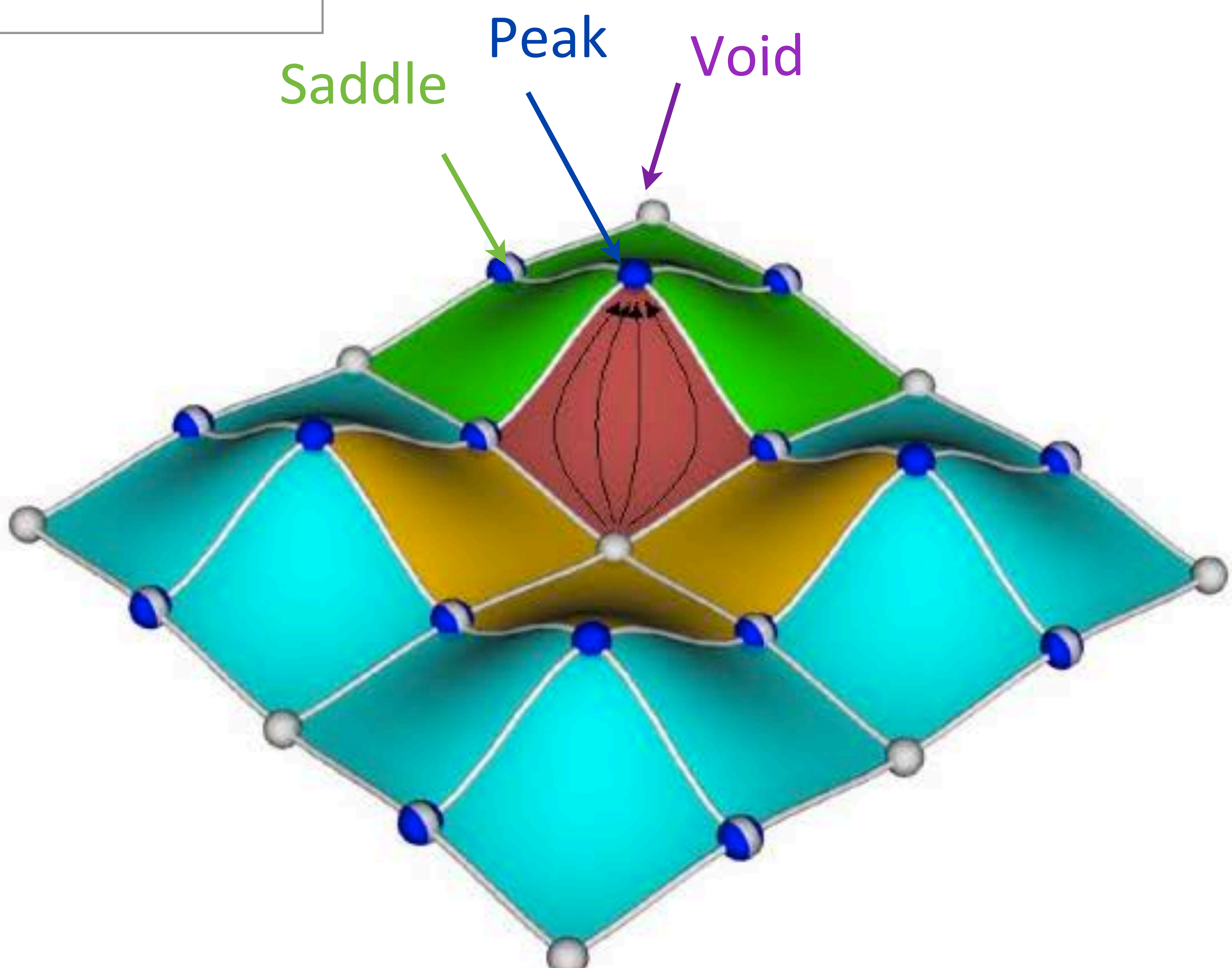
Asymptotic result?



2D connectivity: topology

2D "ideal" cosmic environment :

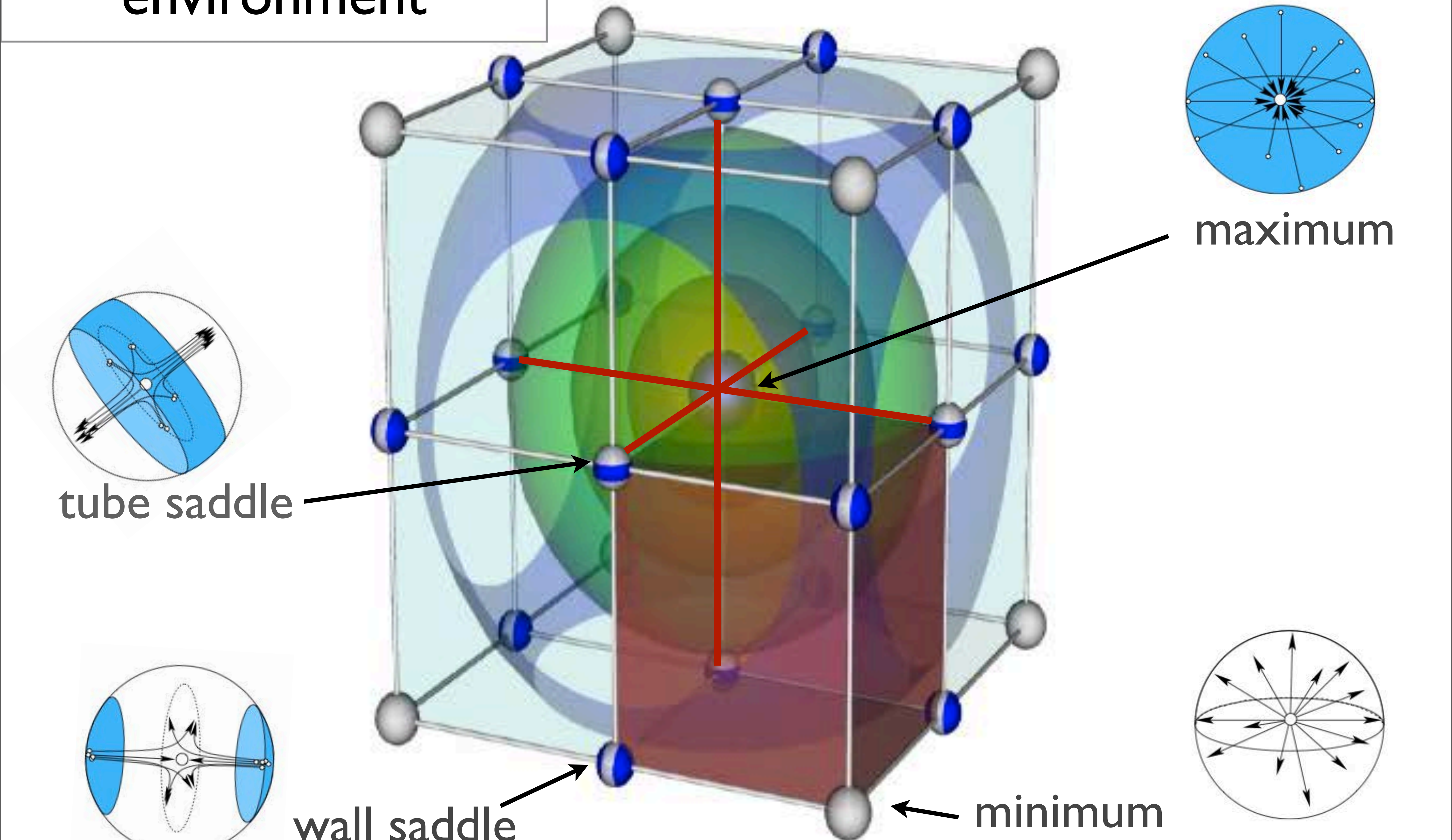
Mean local cosmic initial condition **homeomorphic** to such crystal



3D connectivity: topology

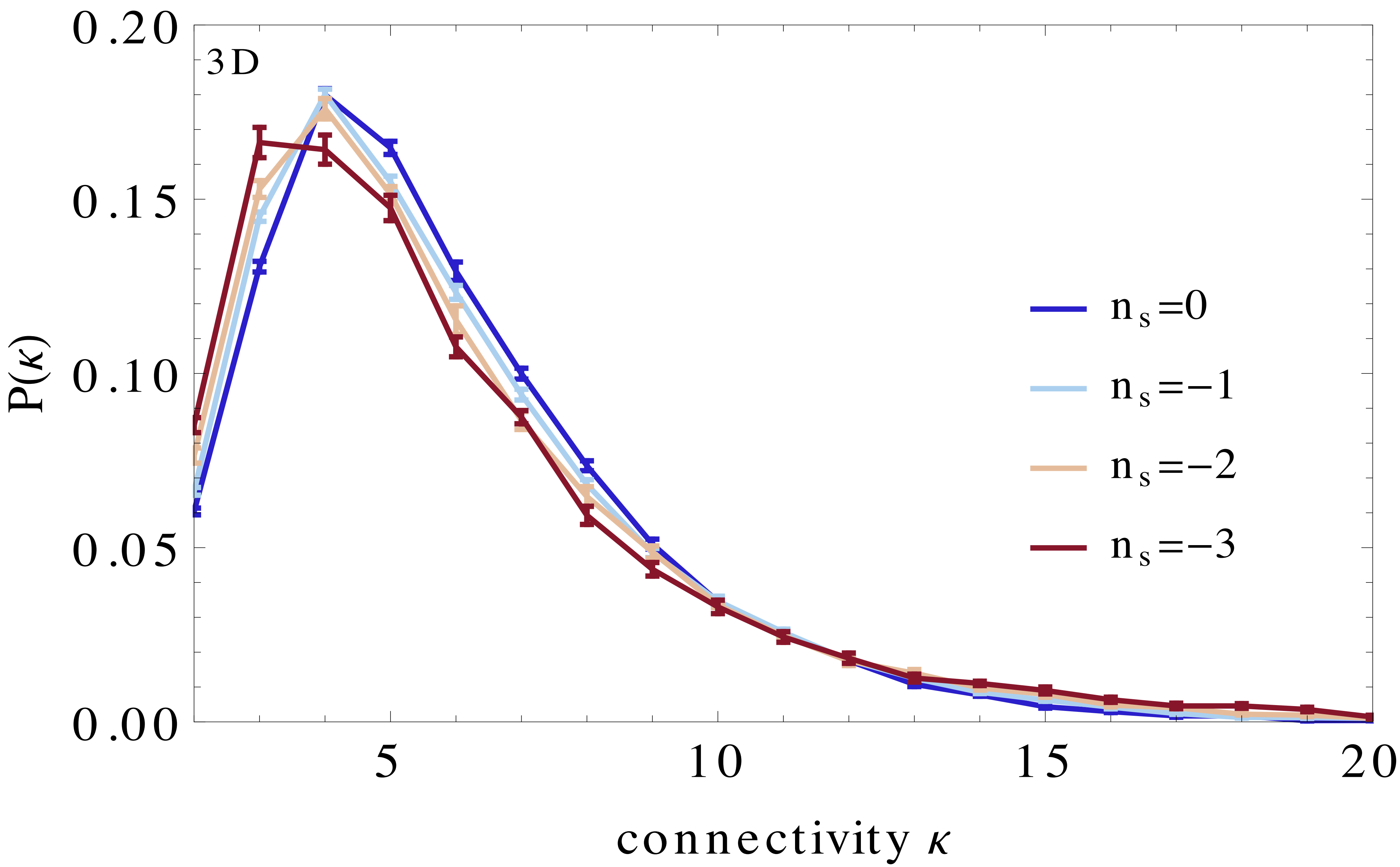
3D "ideal" cosmic environment

Mean local cosmic field quasi **homeomorphic** to such crystal



GRF connectivity PDF: dependence with scale/n_s

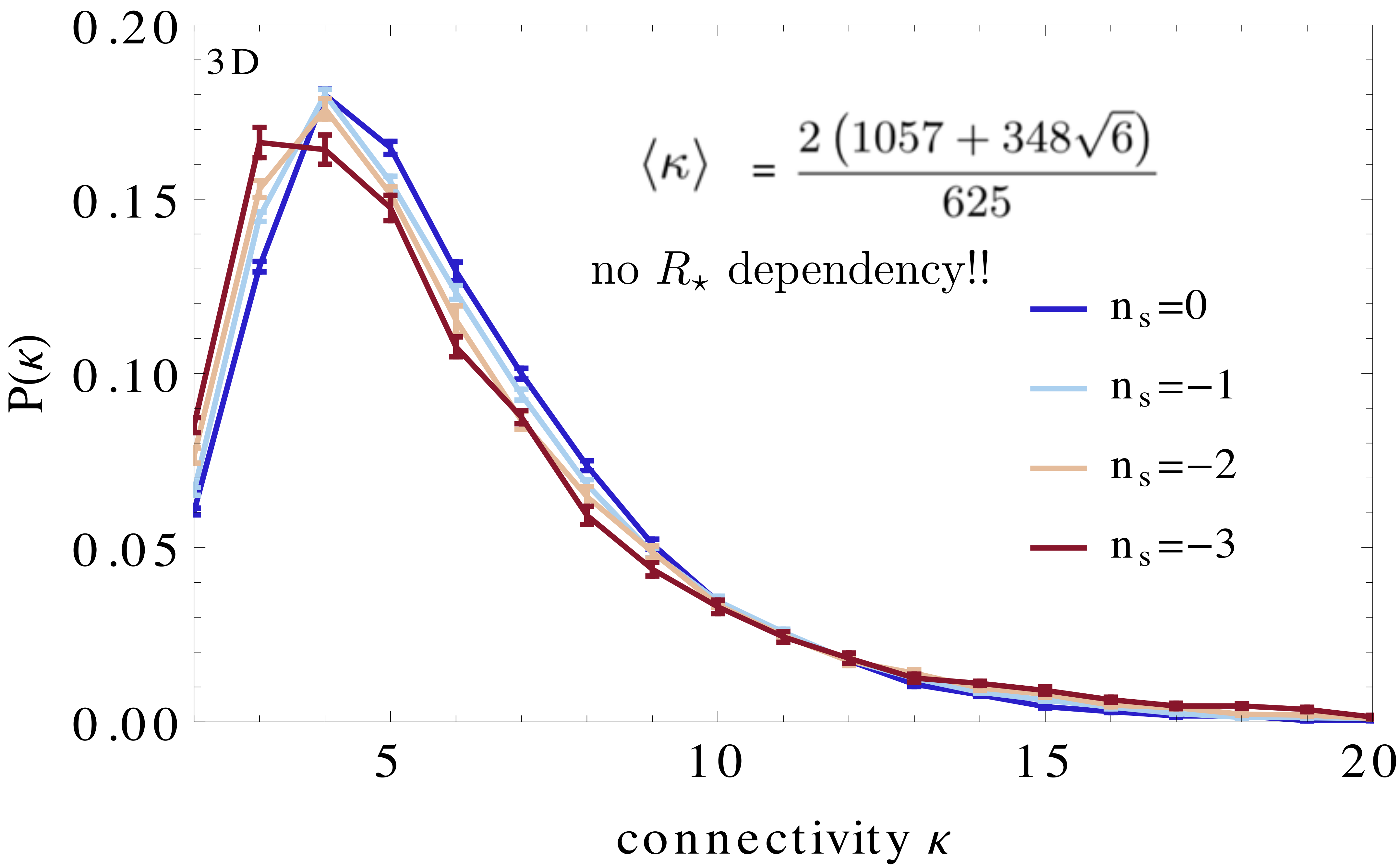
Full distribution of connectivity:



Weak dependency on n_s

GRF connectivity PDF: dependence with scale/n_s

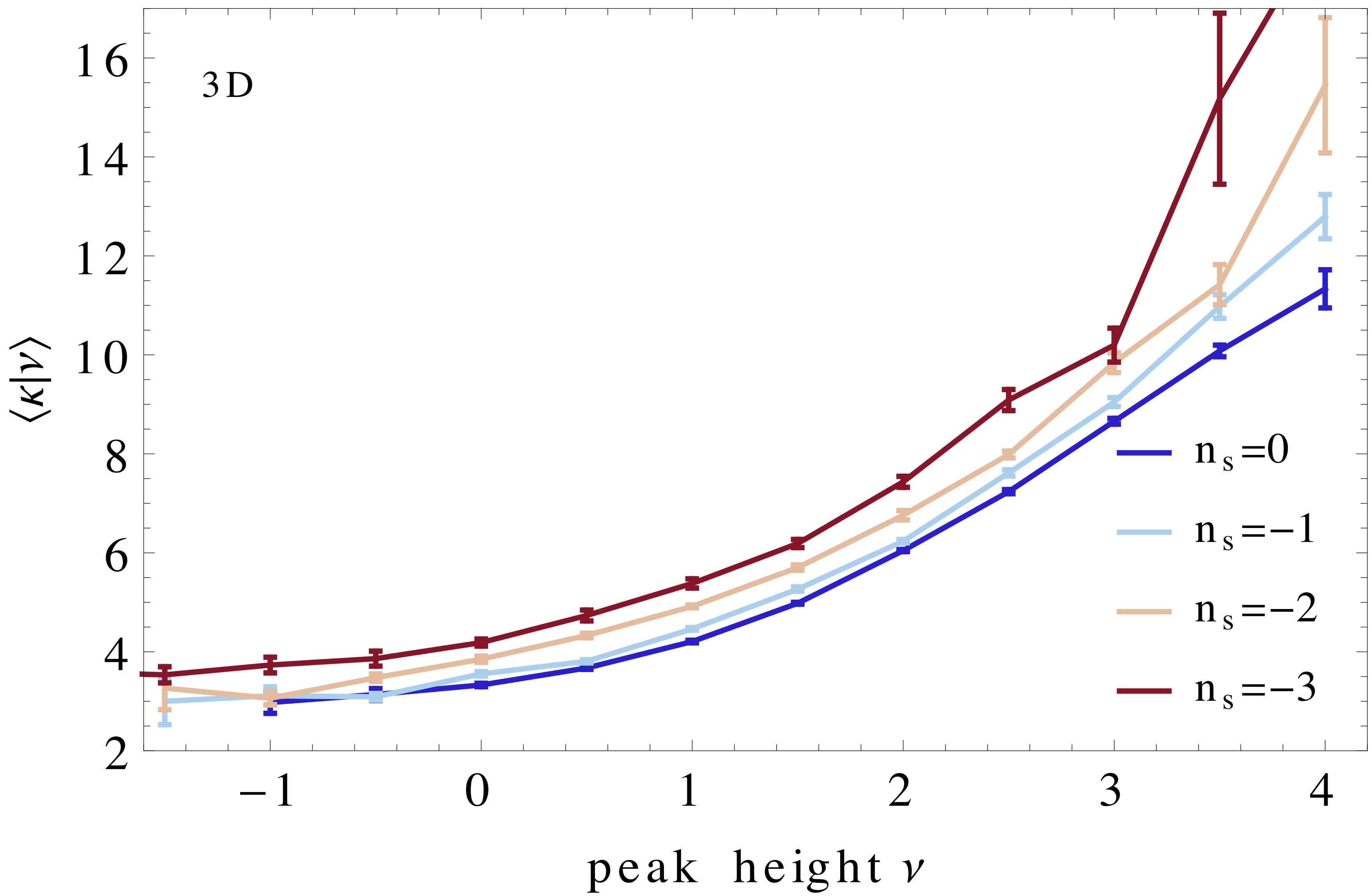
Full distribution of connectivity:

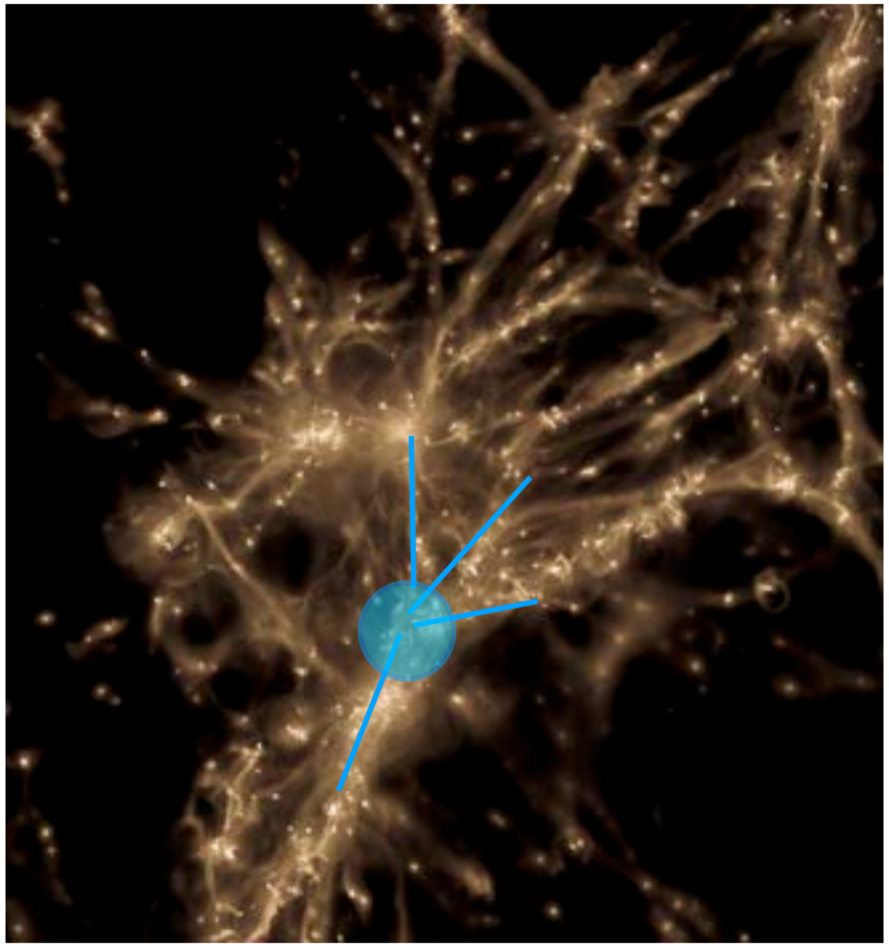


Weak dependency on n_s

GRF connectivity: dependence with peak height

Dependence with peak height:



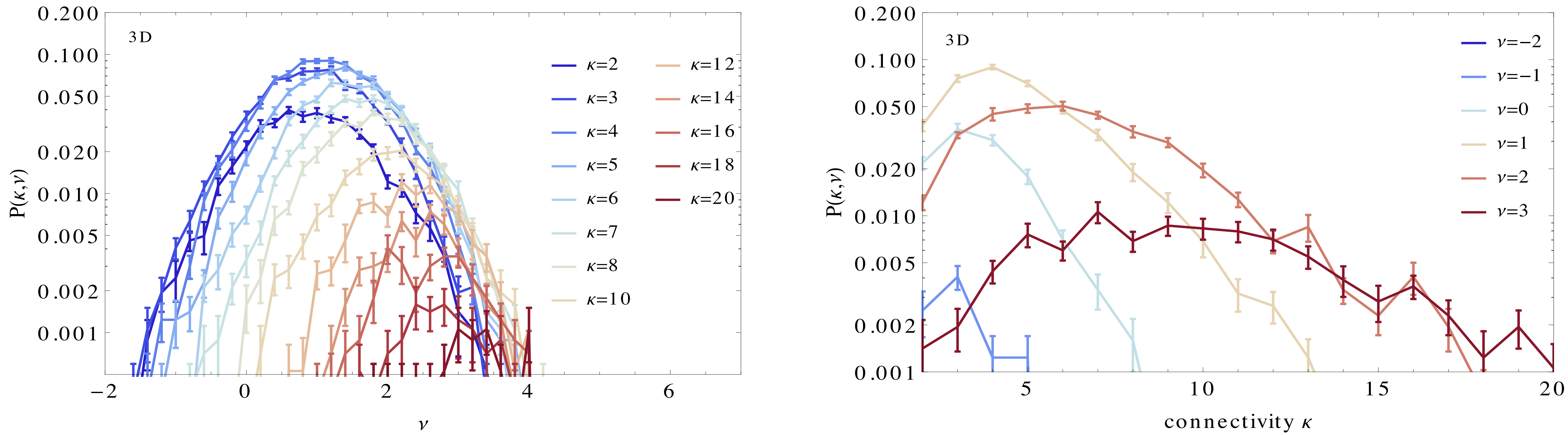


$$\nu = \delta/\sigma$$

The rarer the peak, the more connected

GRF connectivity: dependence with peak height

Joint PDF of κ and ν in 3D

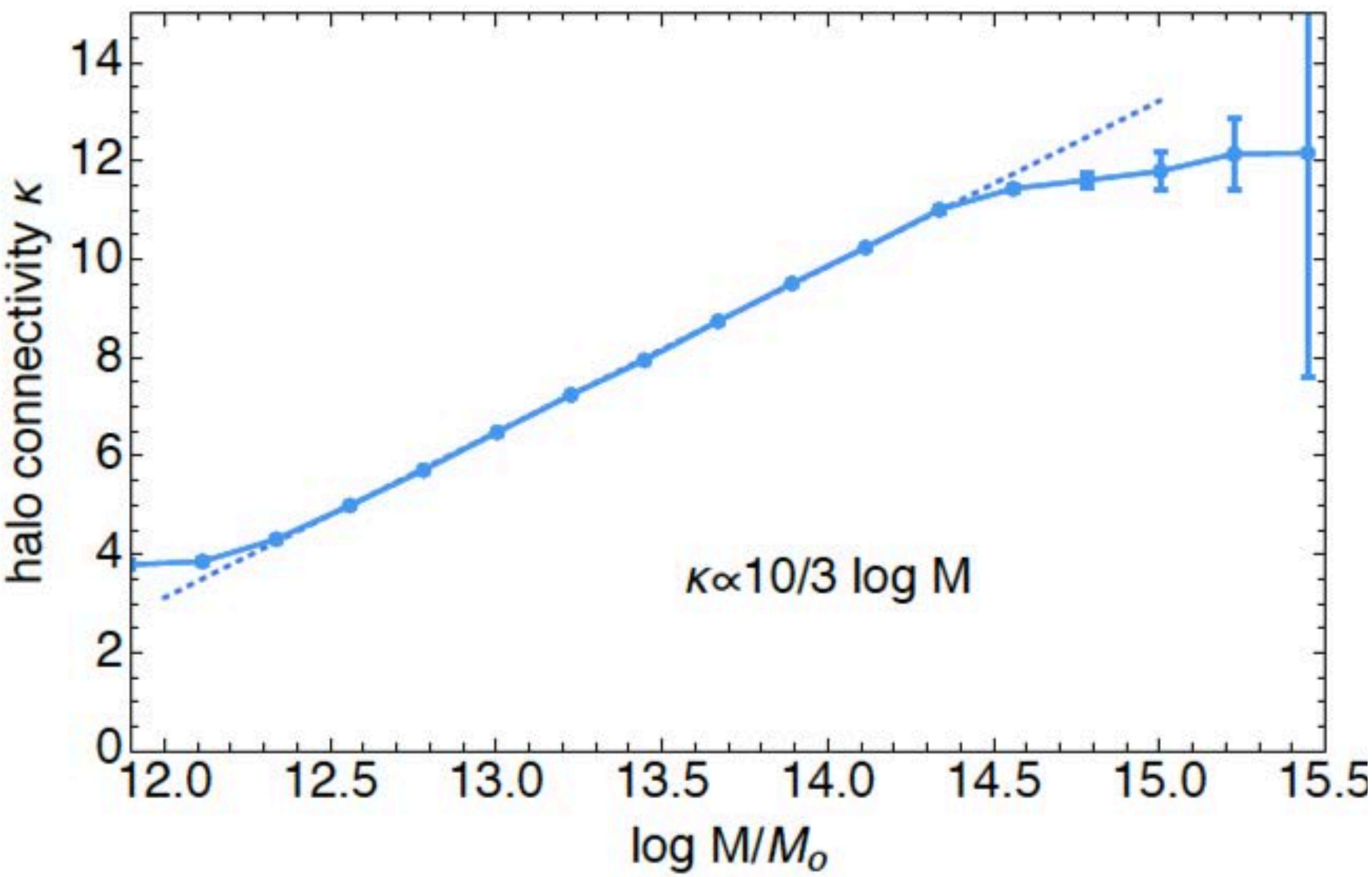


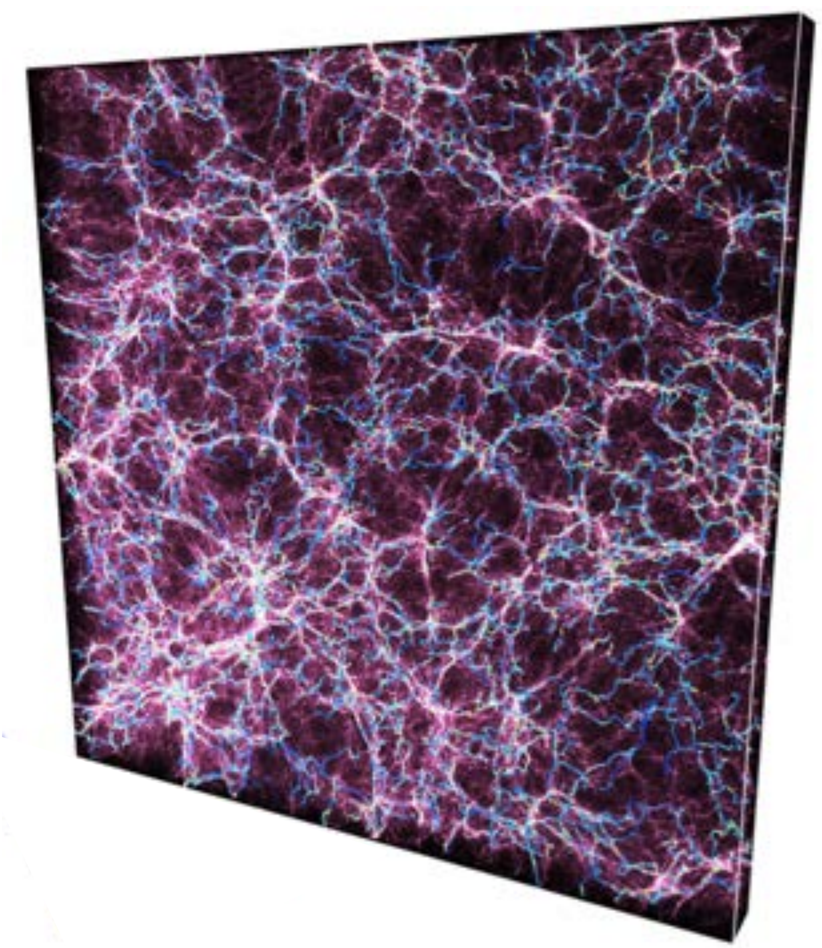
Notable Result:

- High peaks tend to have more connections
- Peaks with large number of connections are predominantly high
- mean $\langle \kappa | \nu \rangle$, $6.5\ (\nu = 2)$, $10\ (\nu = 3)$

Connectivity versus Mass in LCDM

Connectivity as a function of Mass in Horizon-4π: 1 000 000 halos.





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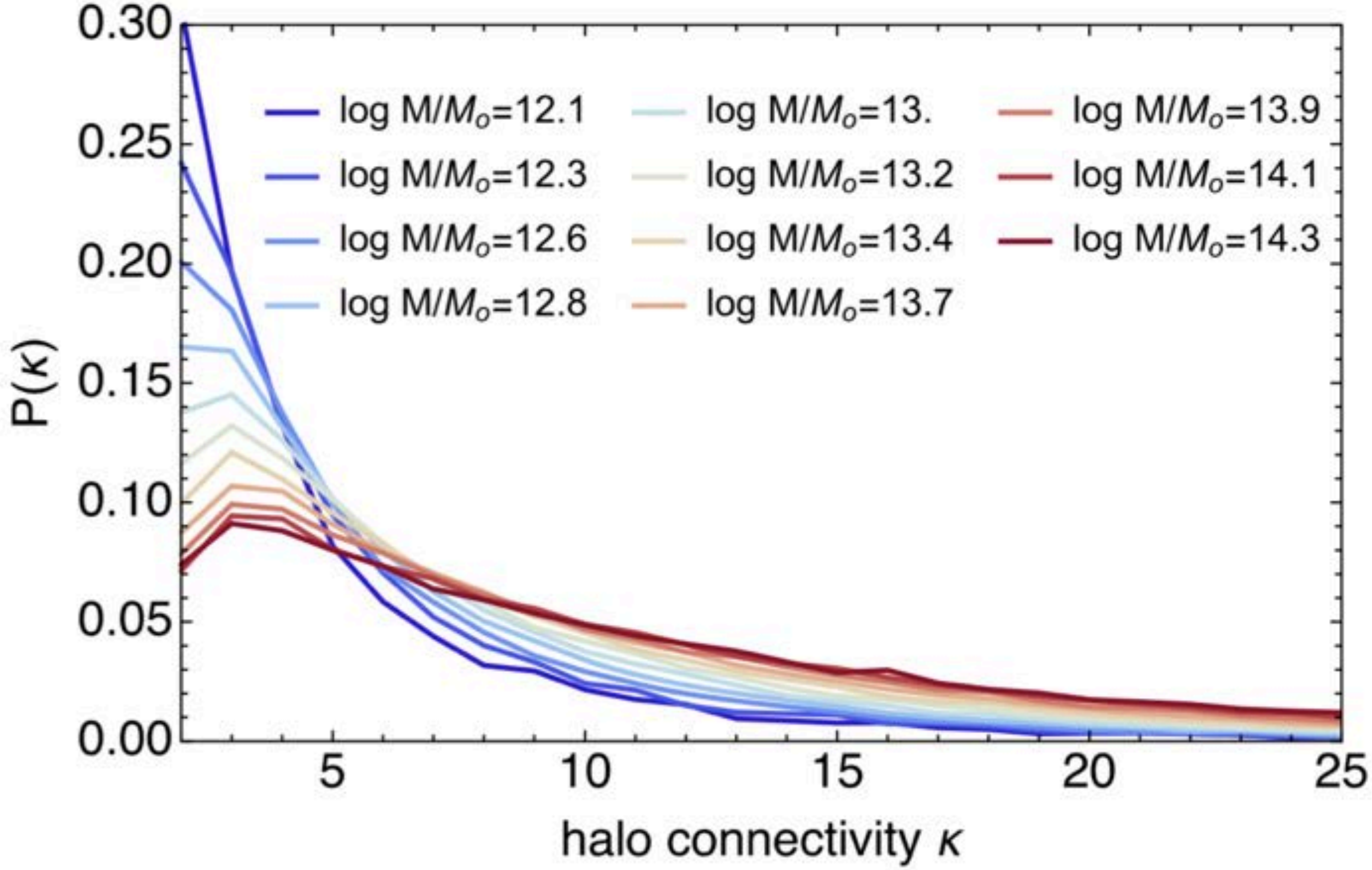
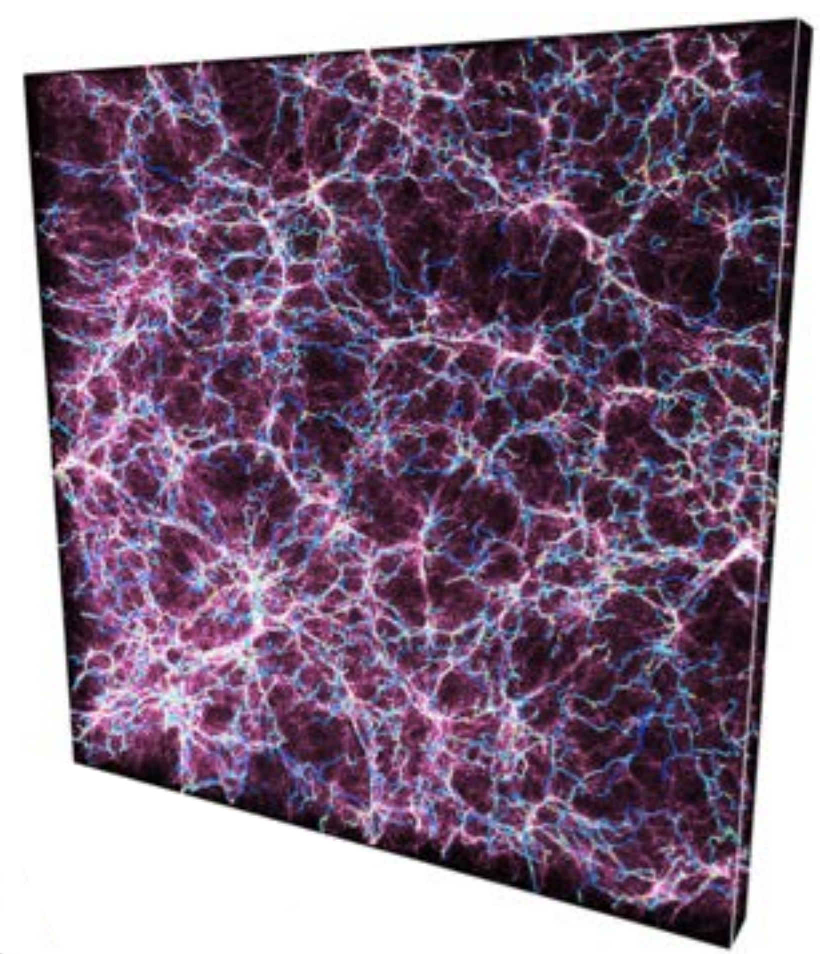
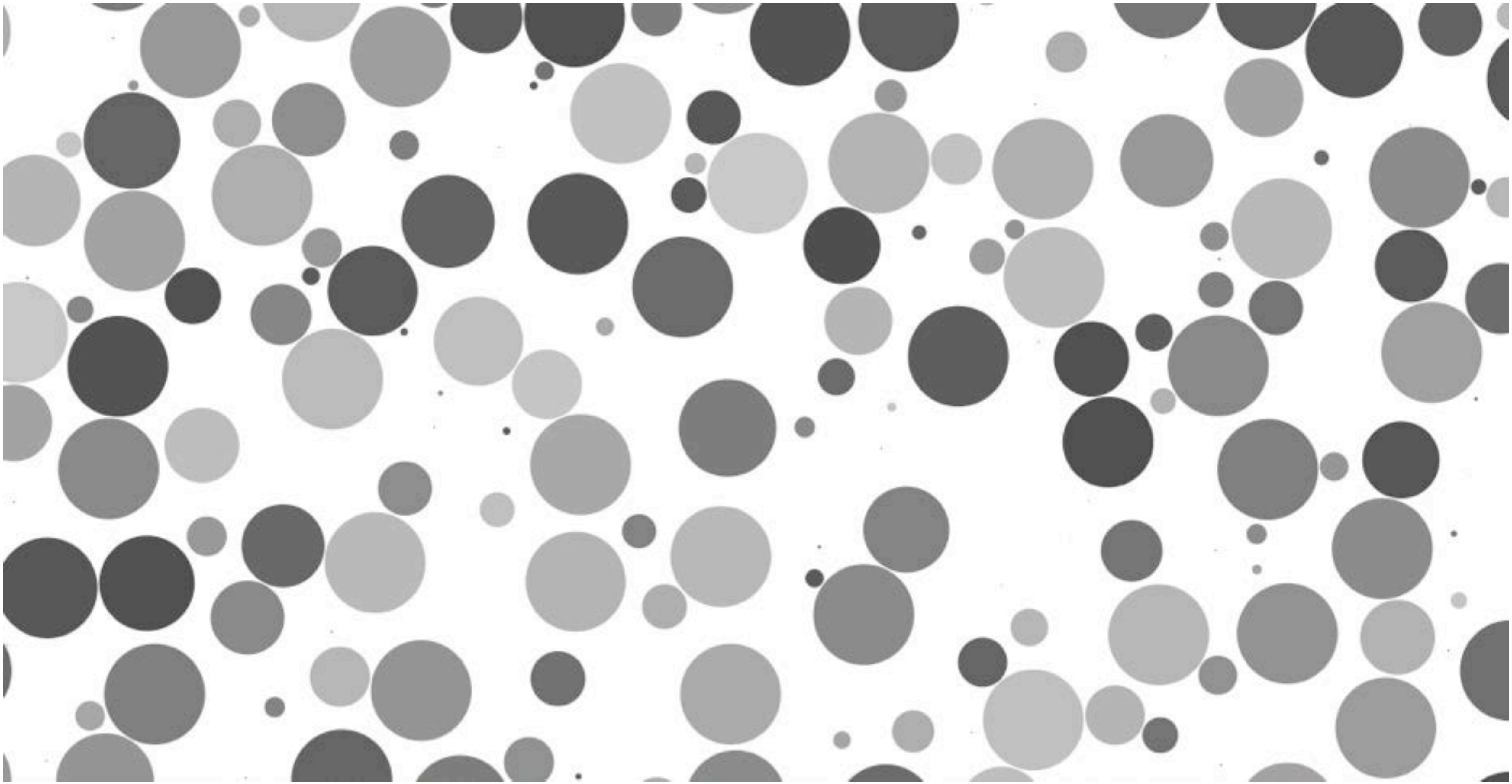




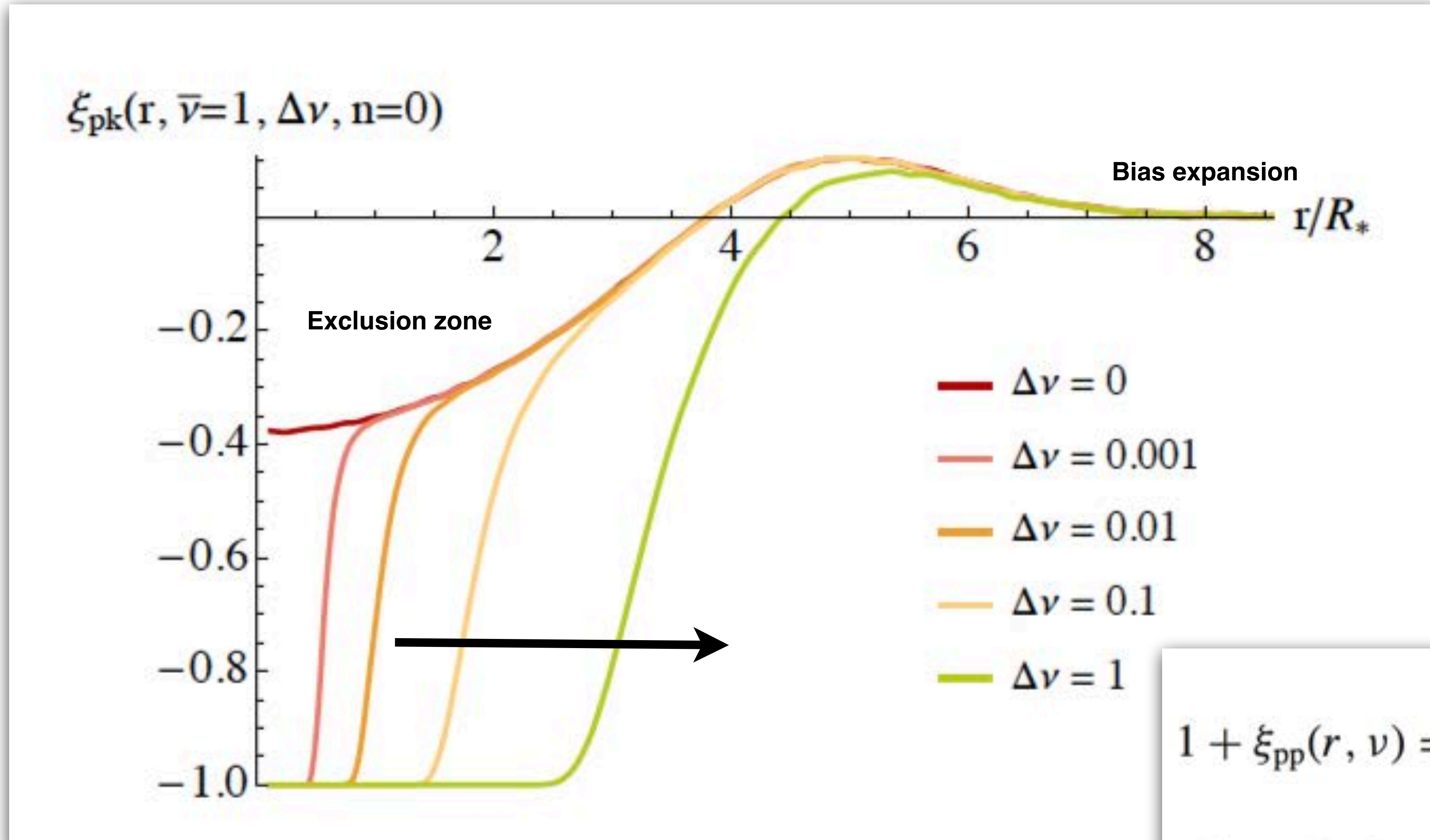
Global connectivity for GRF: IDEA?

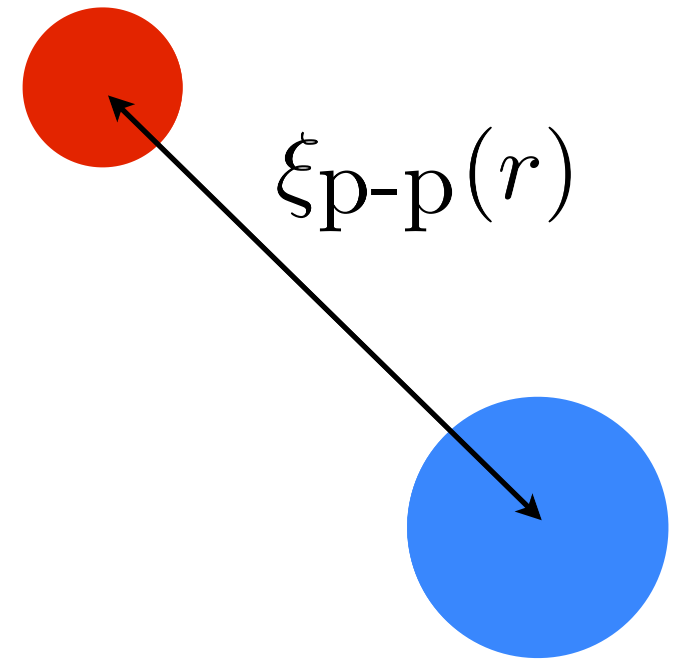
Analogy with sphere packing pb



Peak theory: Clustering (2 point statistics)

Same ideas can be used to also predict the **clustering of peaks** by means of their 2 point correlation function (also applies to peak saddle etc.):





$$1 + \xi_{\rm pp}(r, \nu) = \frac{\langle \rho_{\rm pk}(\mathbf{X}) \rho_{\rm pk}(\mathbf{Y}) \rangle}{\langle \rho_{\rm pk}(\mathbf{X}) \rangle^2} .$$

where the localized peak number density $\rho_{\rm pk}(\mathbf{X})$,

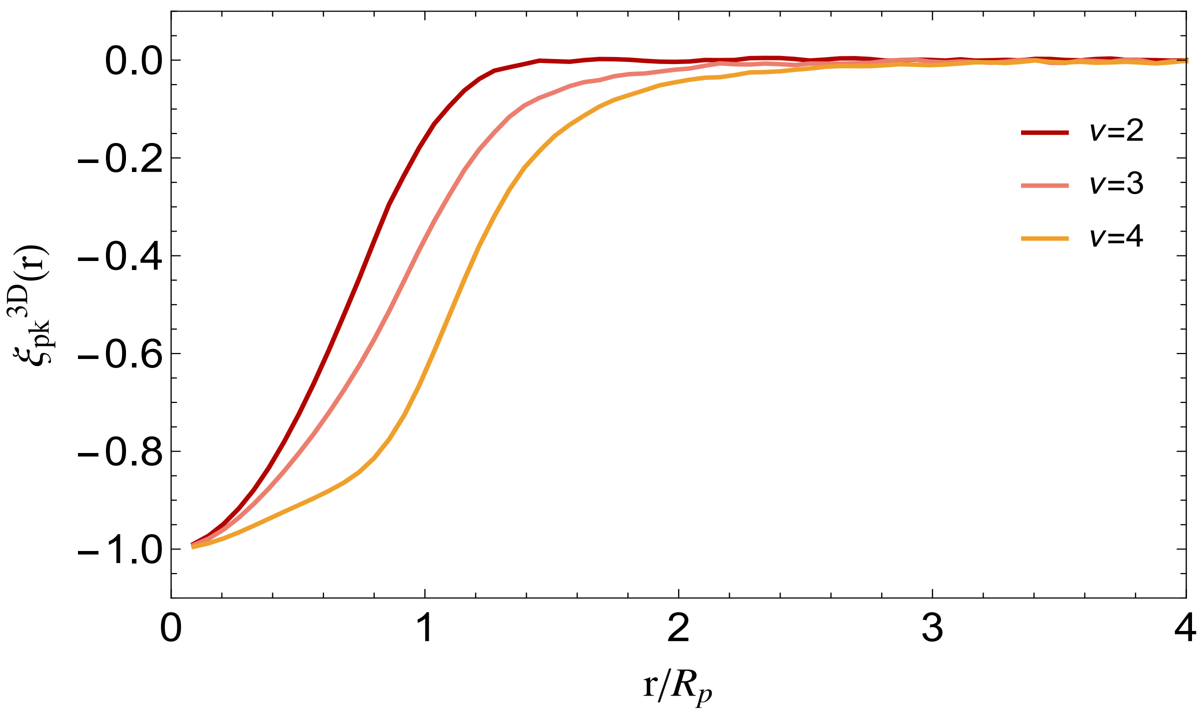
$$\rho_{\rm pk}(\mathbf{X}) = \frac{1}{R_\star^{\rm d}} |\det(x_{ij})| \delta_{\rm D}(x_i) \Theta_{\rm H}(-\lambda_i) \delta_{\rm D}(x - \nu) ,$$

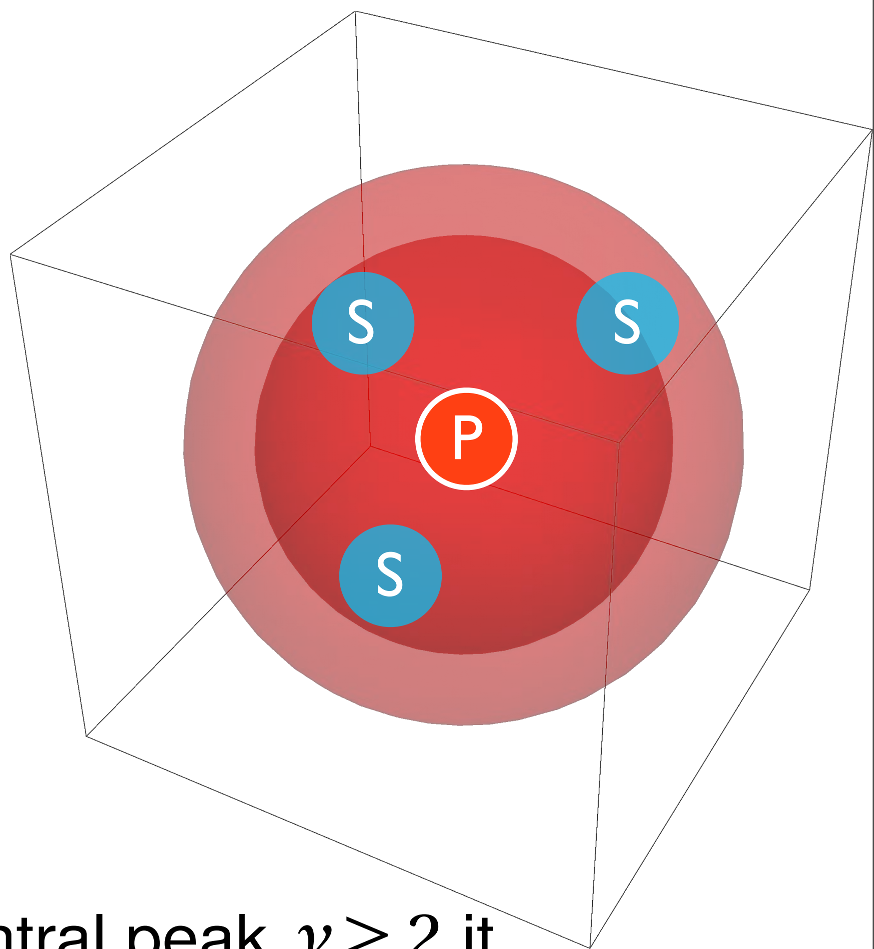
exclusion is essential to understand connectivity

Global connectivity for GRF: theory

Towards connectivity theory

Idea: Count the number of saddles up to $R_{\rm max}$ . . . , conditional on the properties of the peak. But what is $R_{\rm max}$? Some characteristic size of a peak-patch around the peak. Let us look (in 3D) where the neighbouring peaks are using peak-peak correlation function





They are at the end of the exclusion zone, which for high central peak $\nu \geq 2$ it increases with ν roughly linearly

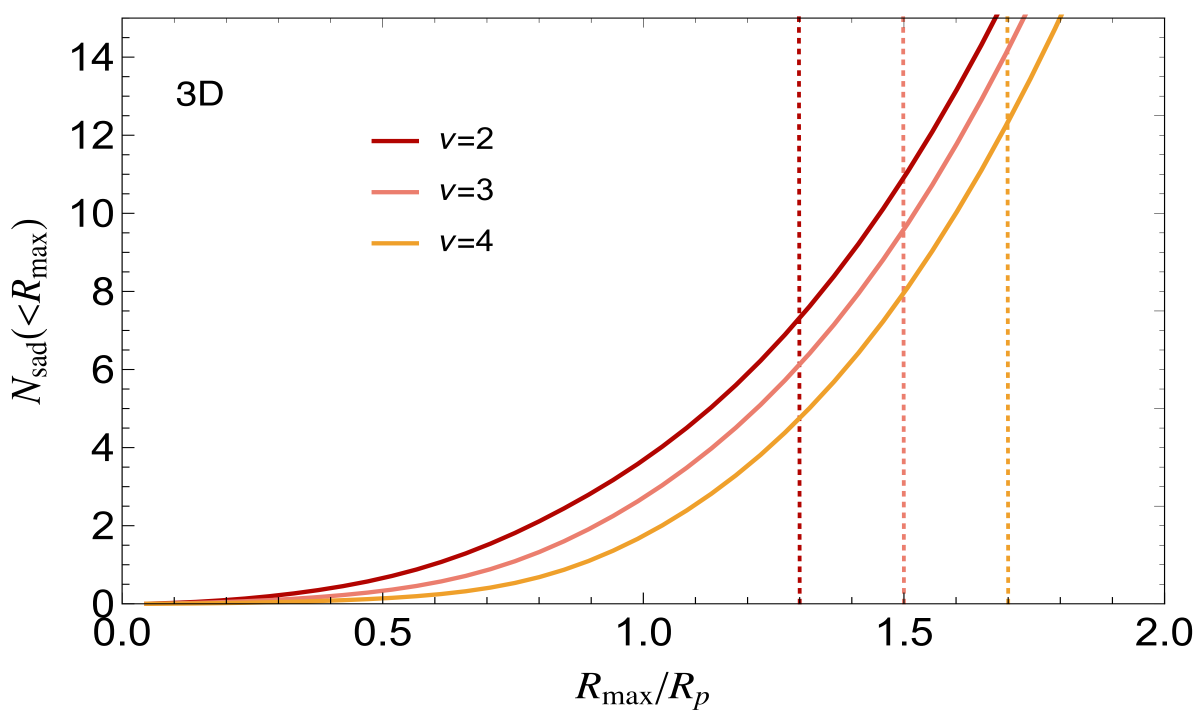
$$R_{max} \approx (0.9 + \nu/5) R_p \qquad R_{\rm max} = 1.2, 1.5, 1.8\ R_p, \quad \nu = 2, 3, 4$$

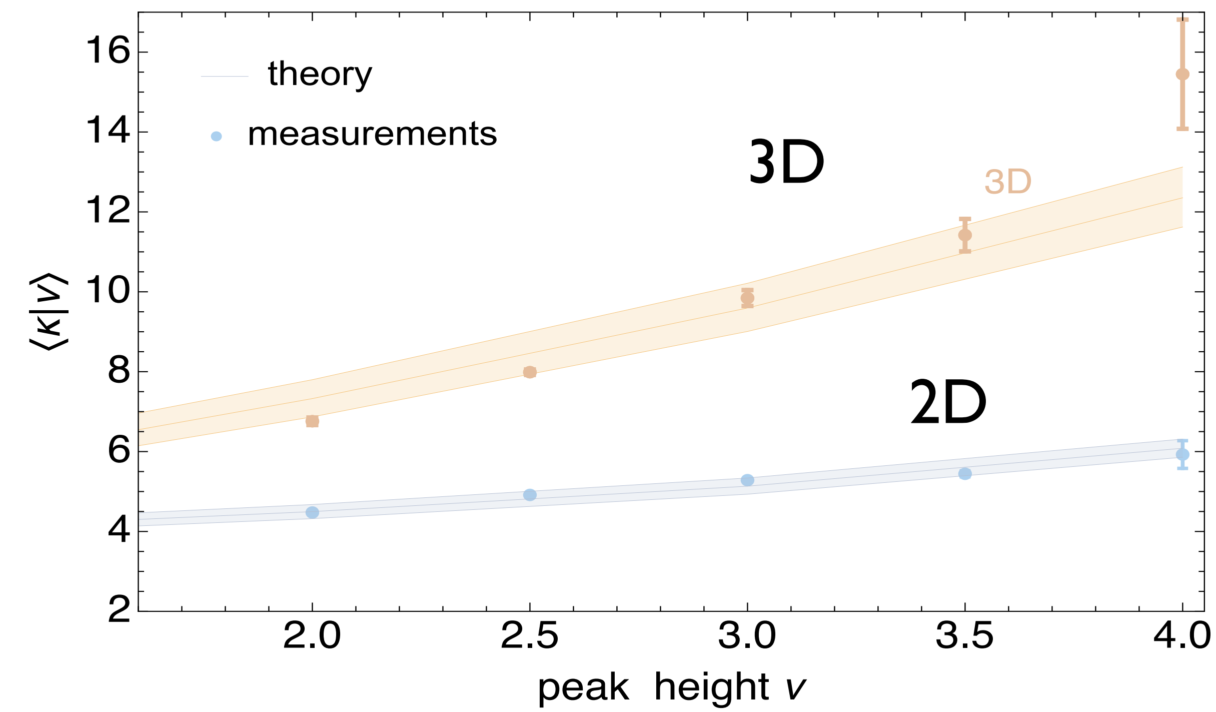
Global connectivity for GRF: theory

Estimating κ by counting saddles to the next peak

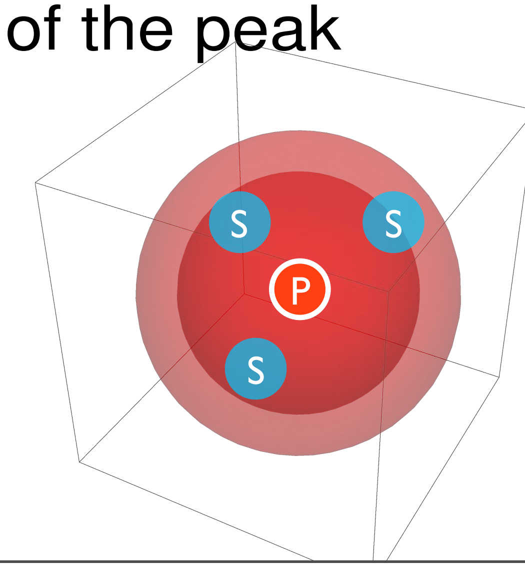
From the Peak-Saddle correlation function

$$\kappa(\nu) = \bar{n}_{\rm sad} \int_0^{R_{\rm pp}} {\rm d}^D r \, (1 + \xi_{\text{pk-sad}}(r, \nu))$$





Number of saddles to distance R_{max} conditional on the height ν of the peak translates to peak connectivity $\langle \kappa | \nu \rangle$



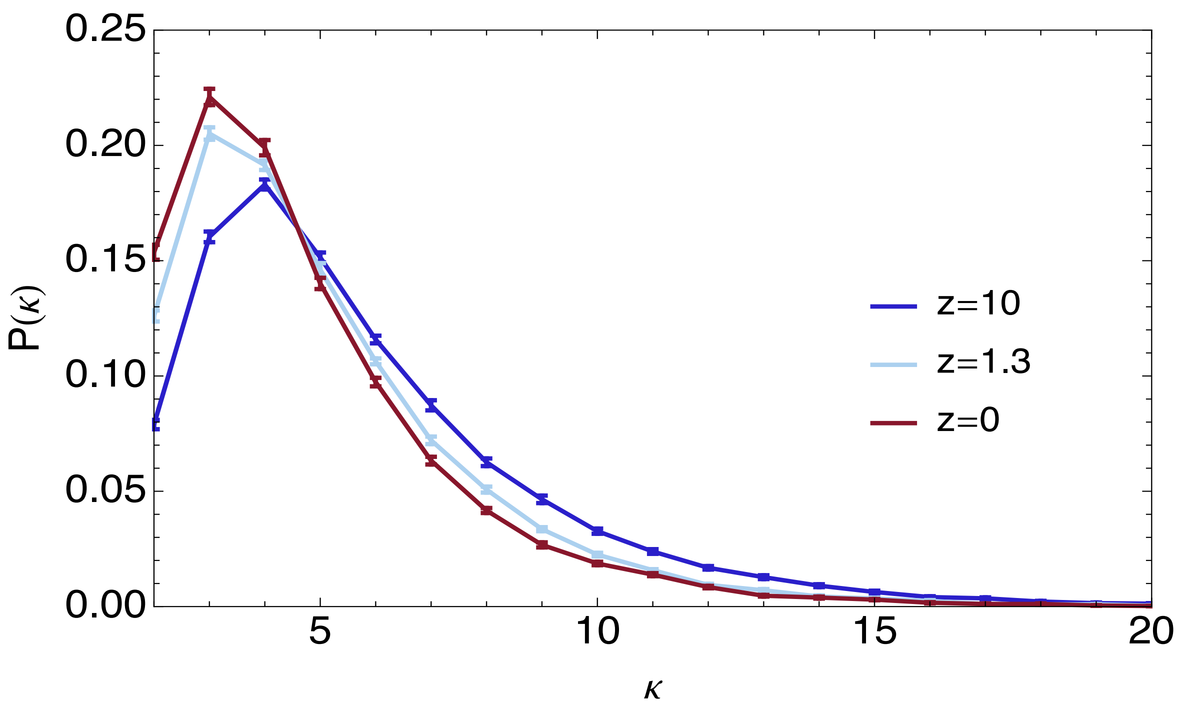
Subtle interplay between clustering of saddles and zone of influence of peak.

Connectivity: evolution with cosmic time

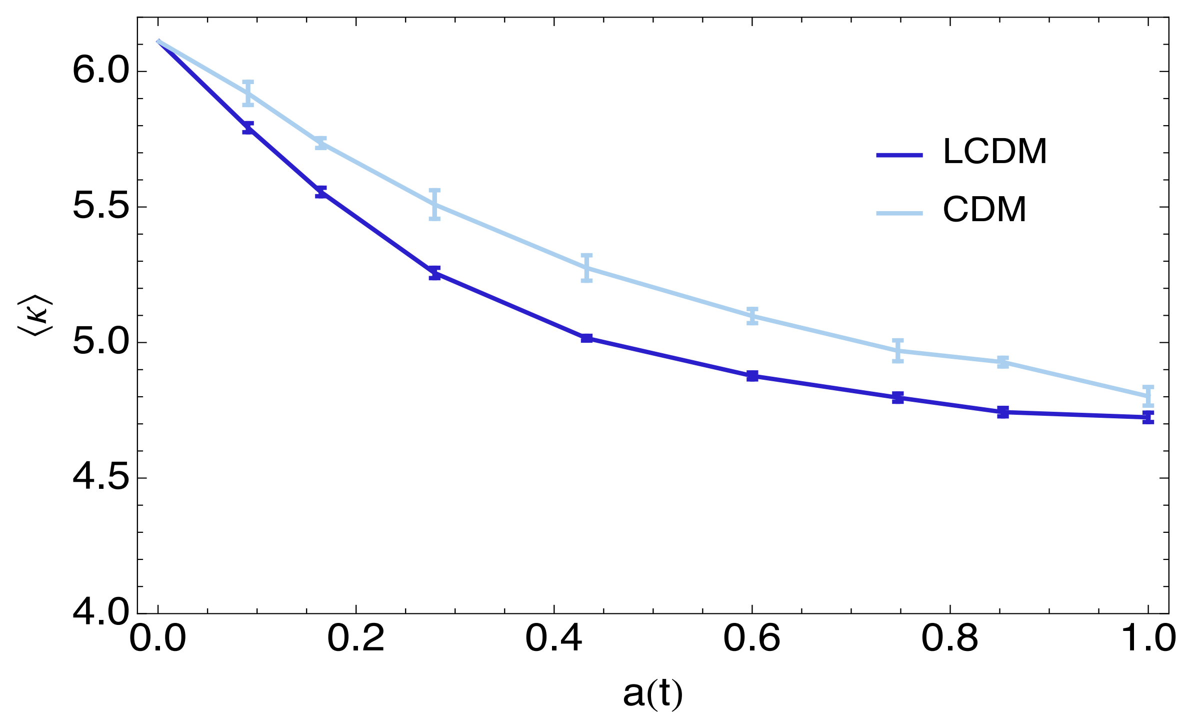
Connectivity of a non-Gaussian field differ from the Gaussian

- In cosmological simulations, as density becomes more non-Gaussian, connectivity of the Cosmic Web decreases
- This leads to model dependent history of the connectivity at different redshifts.

$P(\kappa)$



$\langle\kappa\rangle$

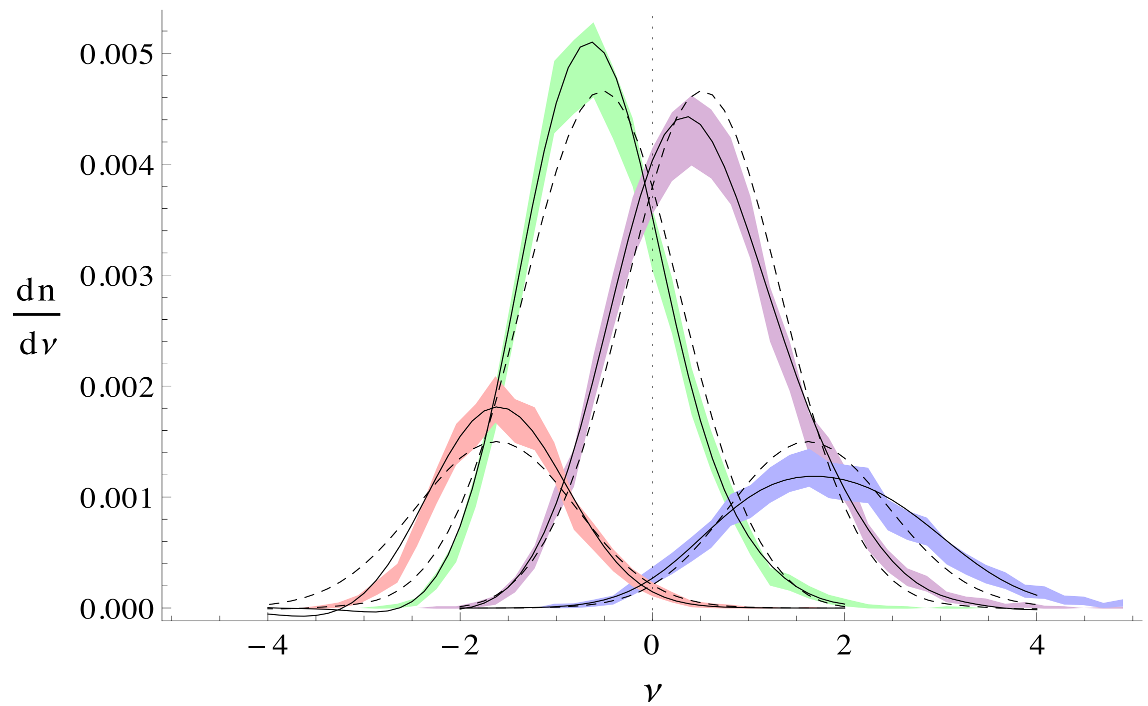


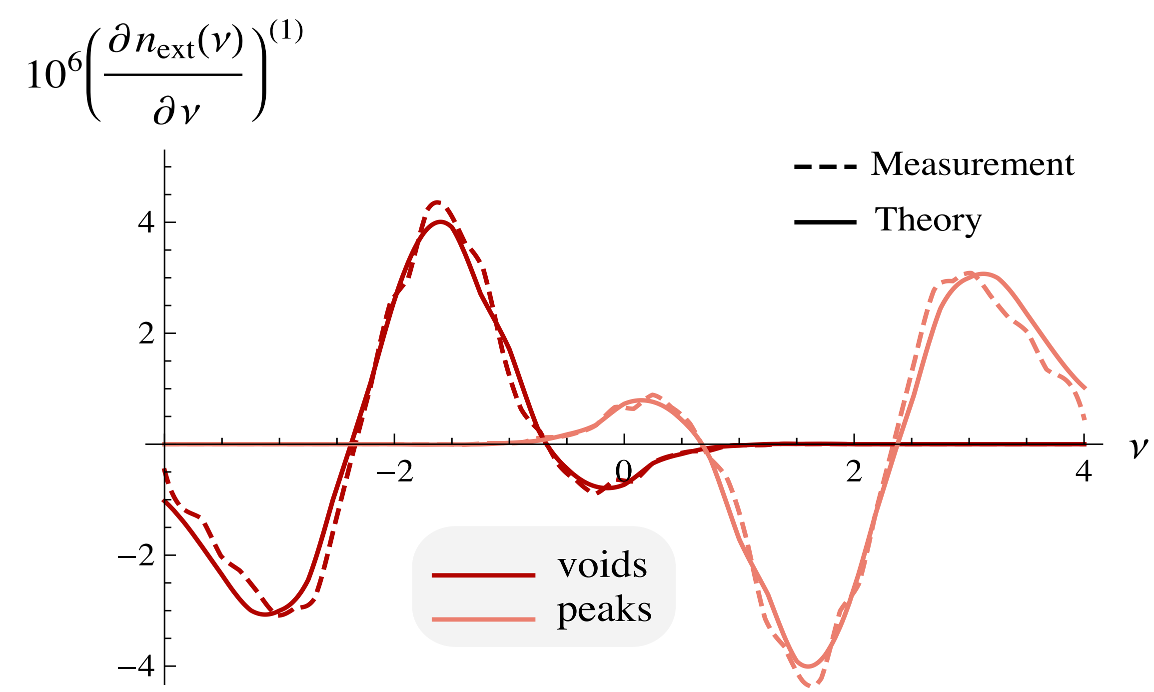
Filaments merge in a cosmology-dependent way!

Theory: evolution with cosmic time

Non-Gaussian 3D Extrema Counts (Gay et al, 2011)

$$\langle n_{\mp--}\rangle = \frac{29\sqrt{15}\mp 18\sqrt{10}}{1800\pi^2 R_*^3} + \frac{5\sqrt{5}}{24\pi^2\sqrt{6\pi}R_*^3}\left(\overbrace{\langle q^2 J_1\rangle - \frac{8}{21}\langle J_1{}^3\rangle + \frac{10}{21}\langle J_1 J_2\rangle}^{\text{scales like D(z)} \times \text{a number}}\right)$$





$$\boxed{\langle\kappa\rangle = \kappa^{\rm G}\left(1 + \sum_{i\geqslant 1}\kappa^{(i)}\sigma_0^i\right)}$$

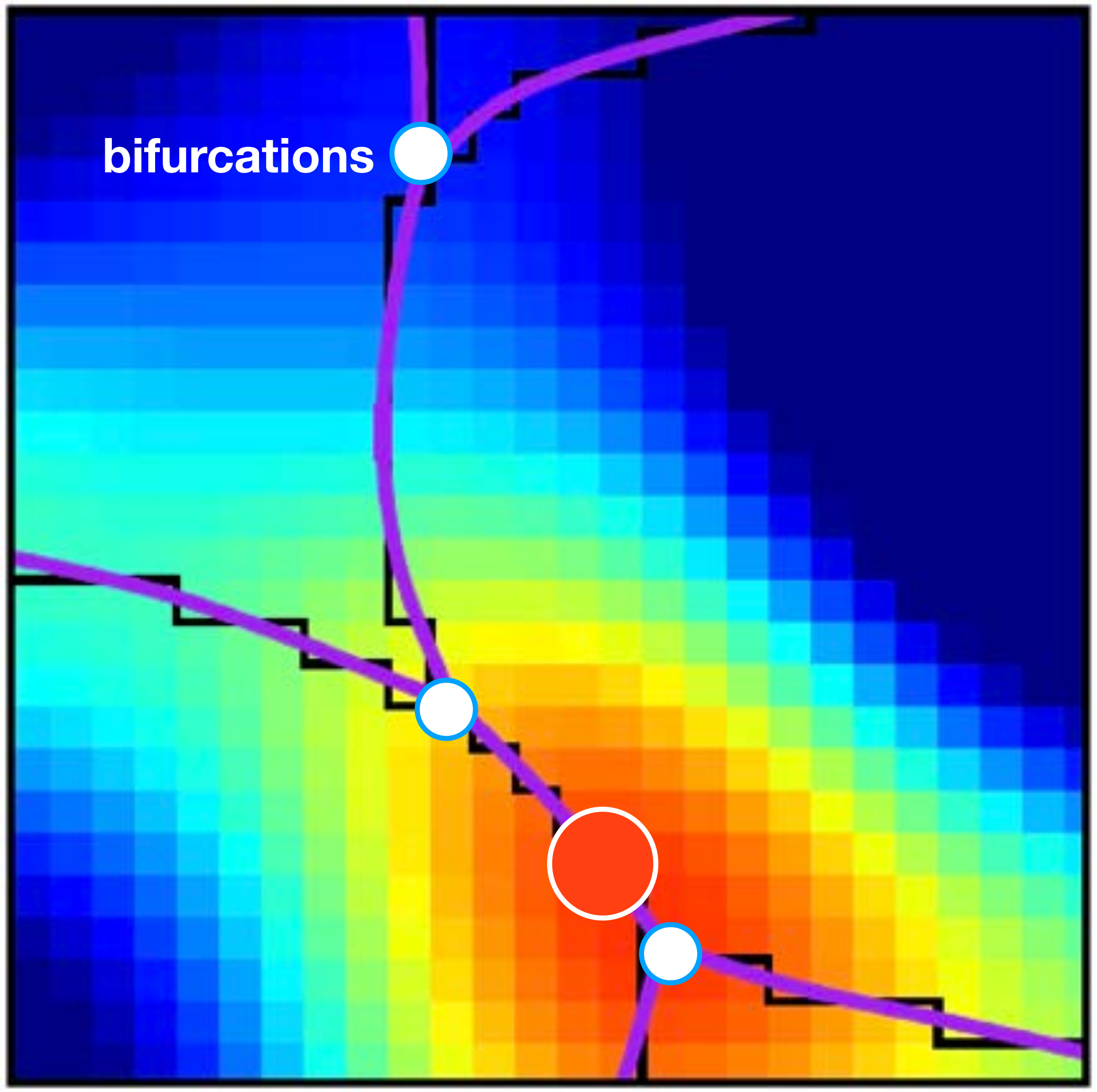
With Gram Charlier expansion, prediction at arbitrary order

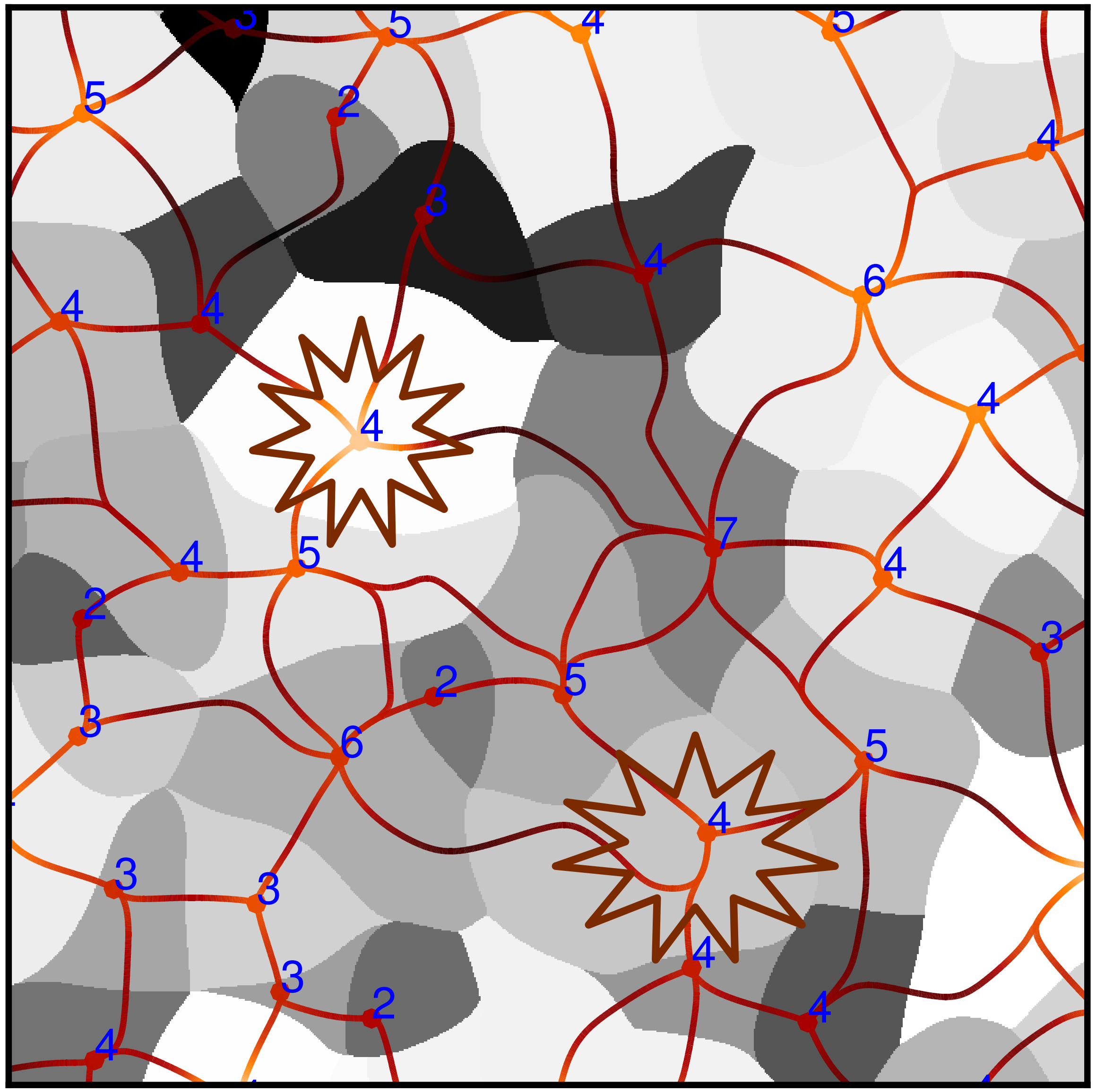
At $\sigma \approx 0.2$

$$\frac{\langle n_{saddle}\rangle}{\langle n_{peak}\rangle} \approx 2.8 \quad \Rightarrow \langle\kappa\rangle \approx 5.6$$

Local multiplicity and bifurcation points

For galaxy formation, what matters most is how many filament connect **locally** onto a galaxy. At small enough scale, a peak is always **ellipsoidal** so that only two branches of filament stick out. Then those branches **bifurcate**. Some bifurcations appear so close to the peak that they are physically irrelevant. Hence we will define the **multiplicity** as the local number of filaments.





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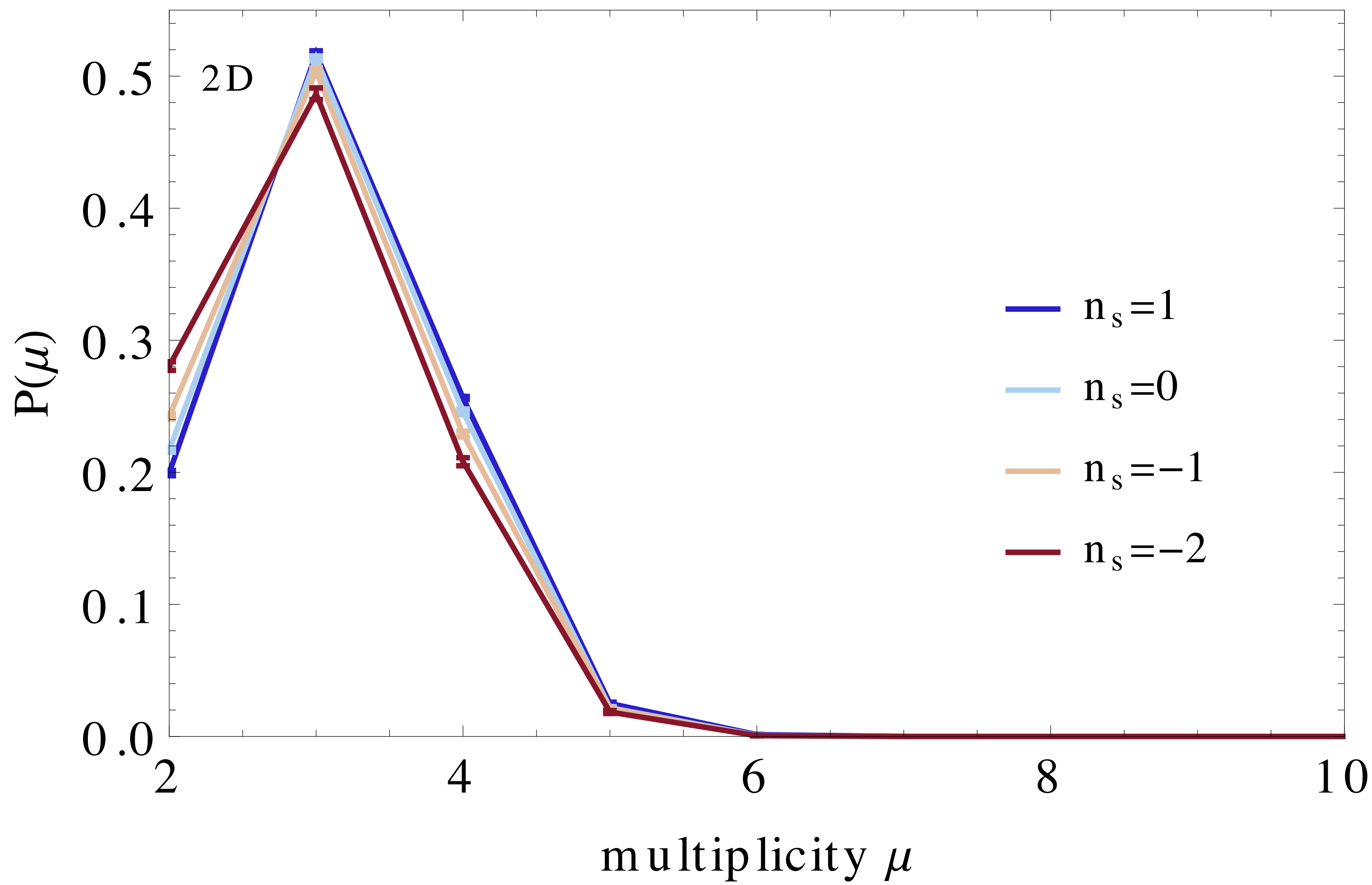
$$\boxed{\mu = \kappa - n_{\rm bifurcations}}$$

$\mu \approx 3$ in 2D

$\mu \approx 4$ in 3D



2D: $\langle\mu\rangle = 3$

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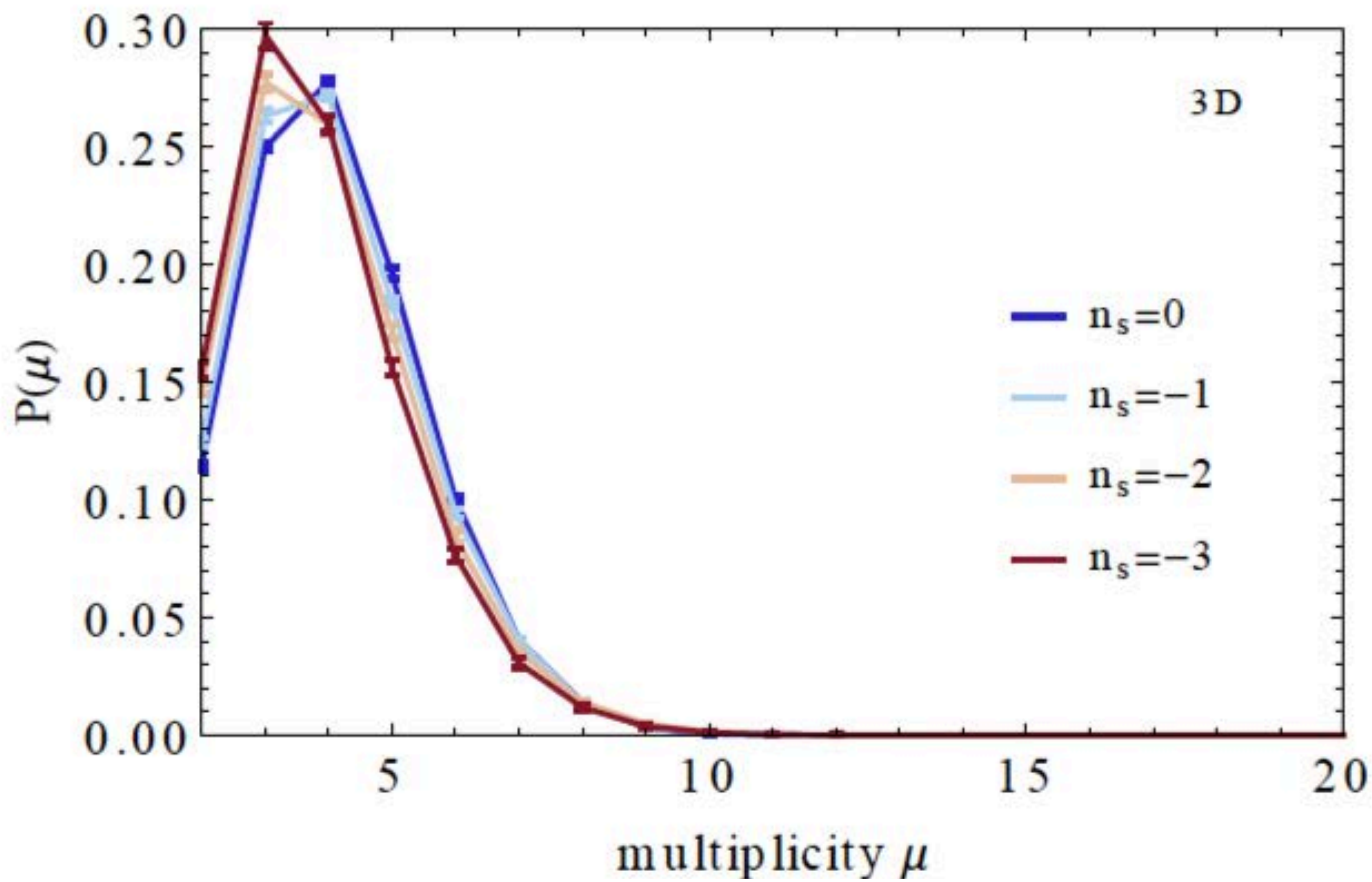
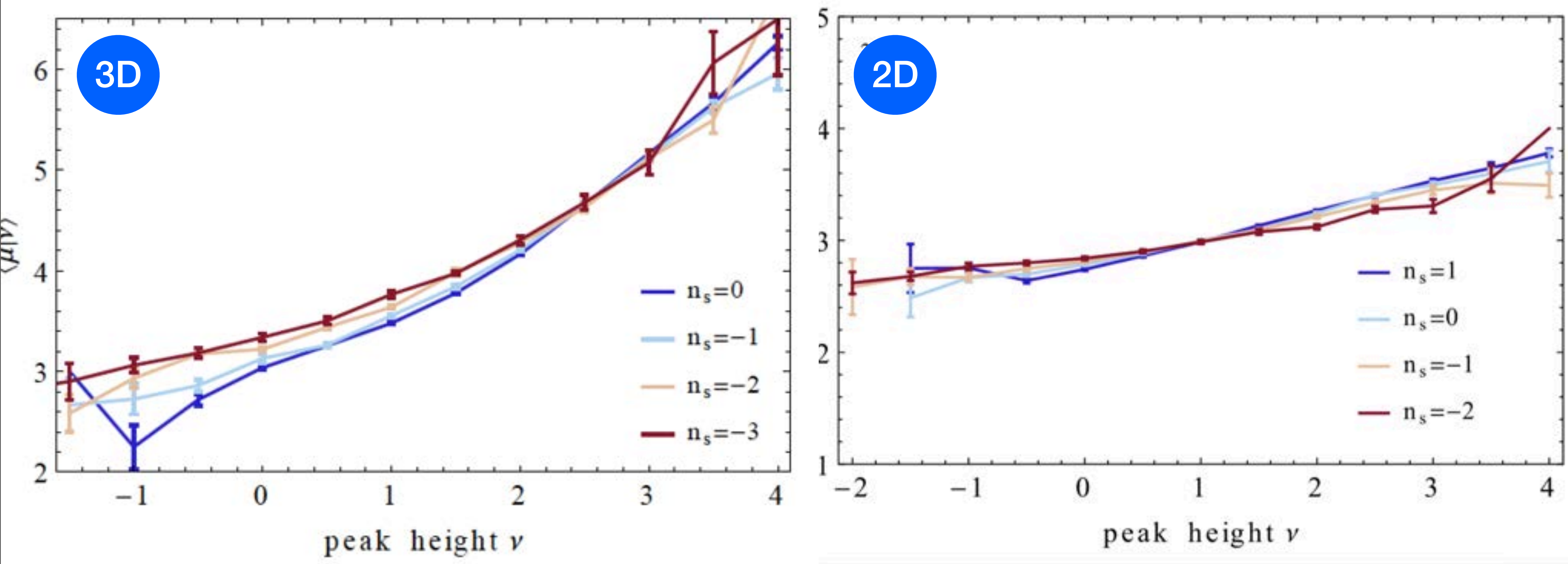


3D: $\langle\mu\rangle = 4$

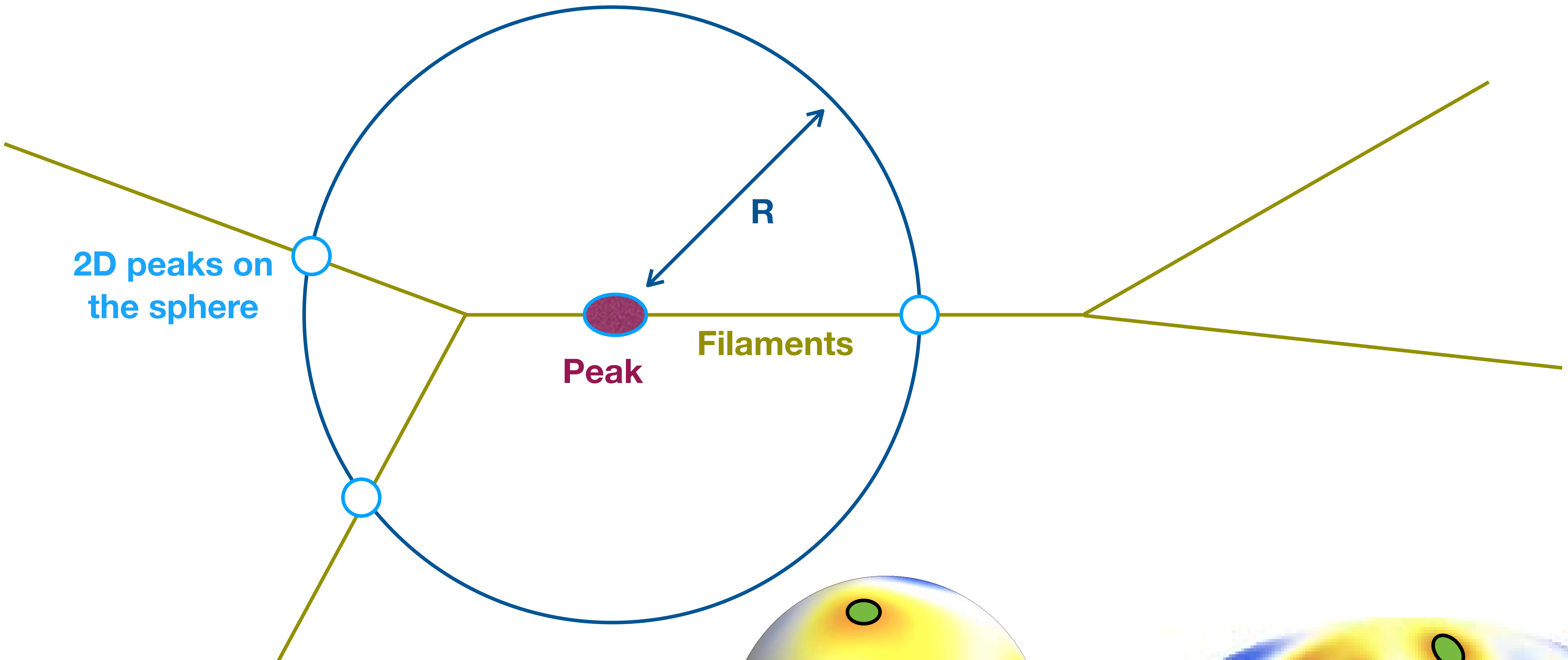
Local multiplicity

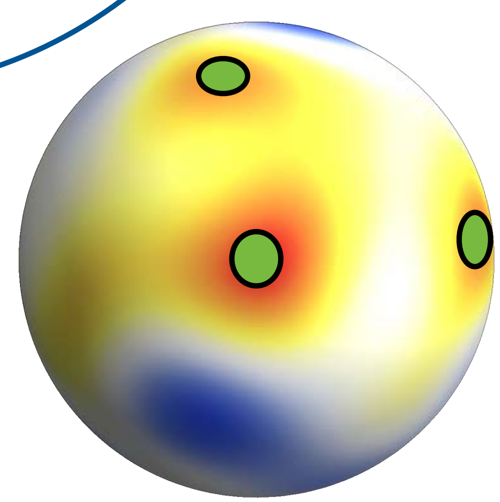
The denser the environment, the higher the multiplicity
(e.g. bringing less coherent angular momentum and generating more ellipsoidal galaxies)

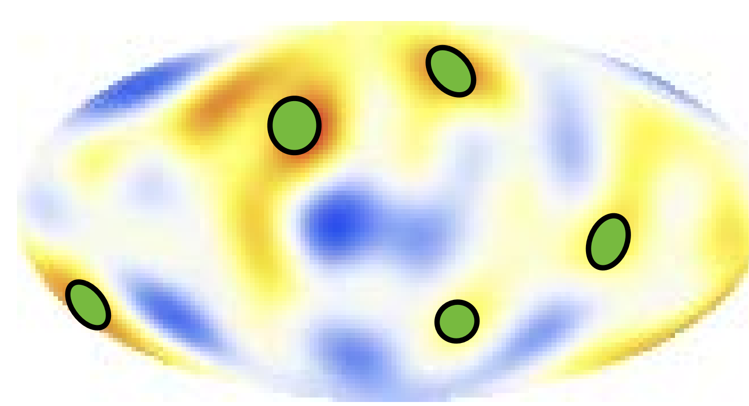


Local multiplicity: towards a theoretical prediction

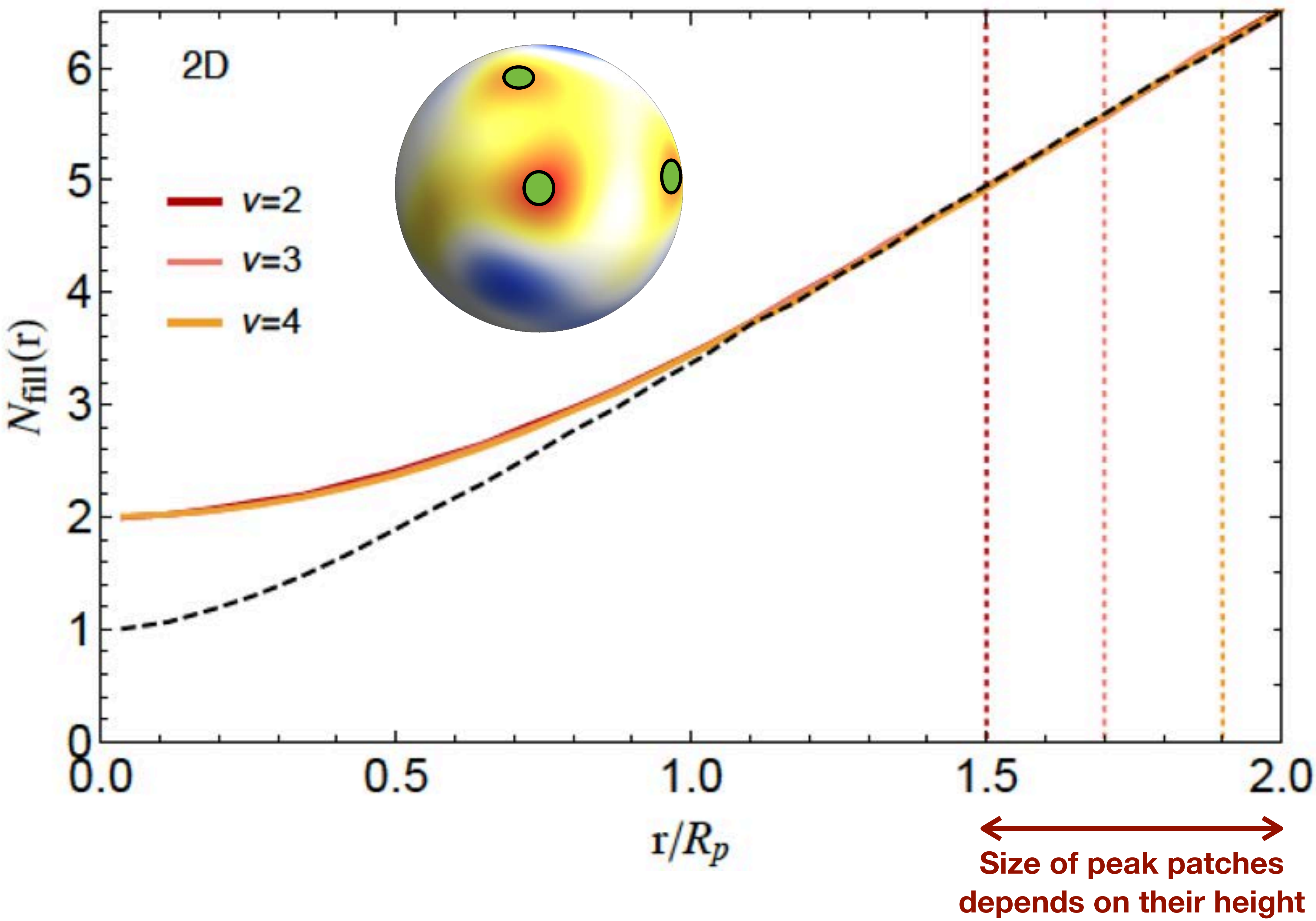
Let us count filament crossings at a sphere of radius R around the central peak…







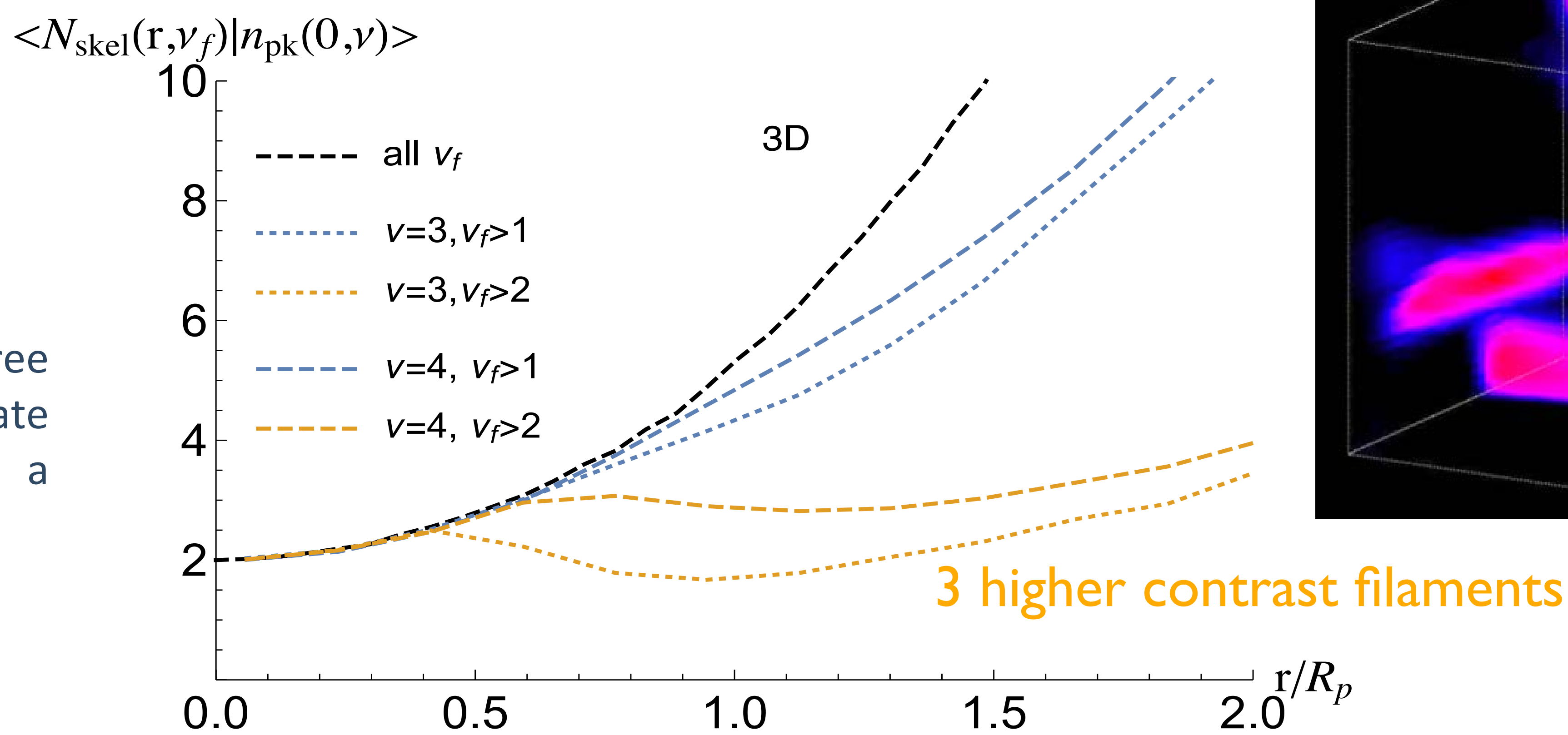
Local multiplicity: towards a theoretical prediction

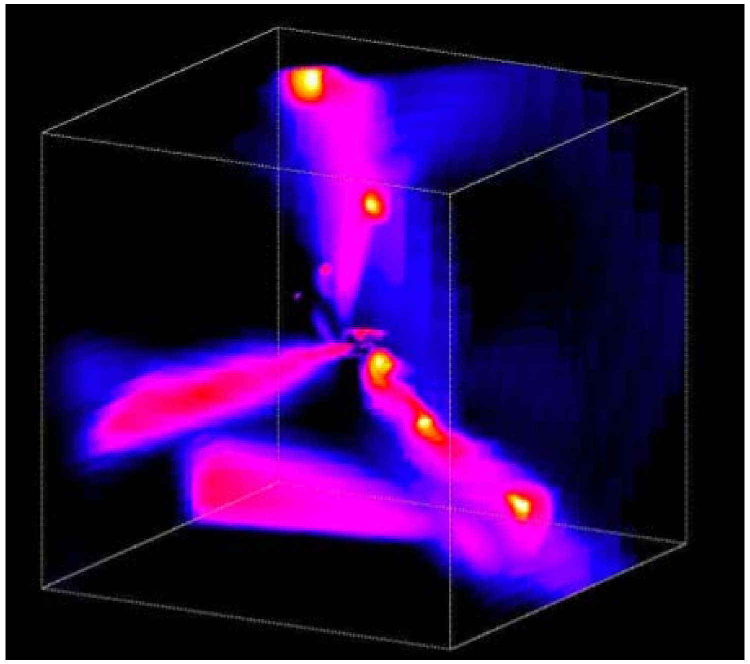


Local multiplicity: towards a theoretical prediction

Not all filaments are equally prominent. Counting important ones

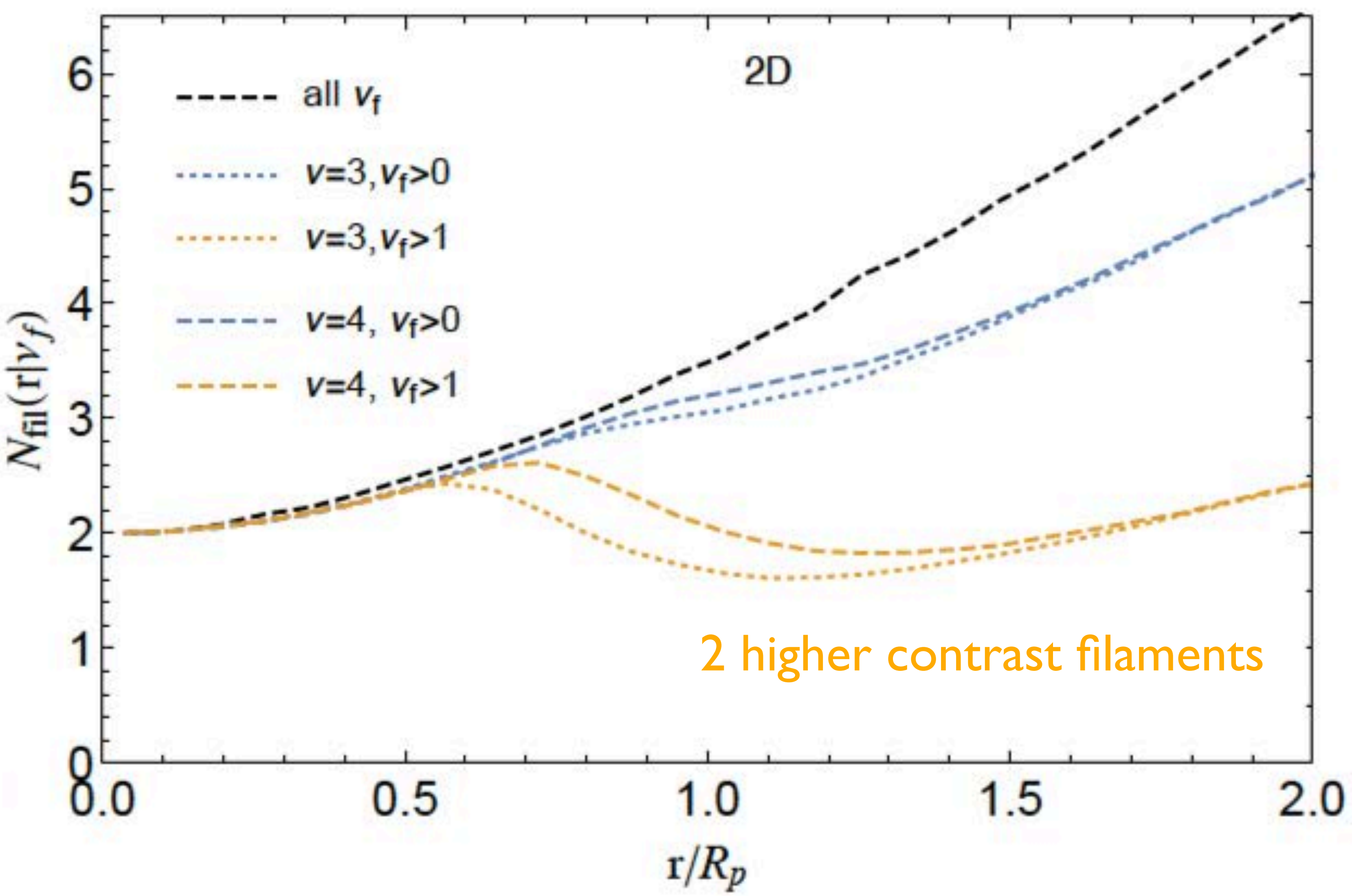
Typically, two to three dense filaments dominate and therefore define a plane of accretion....

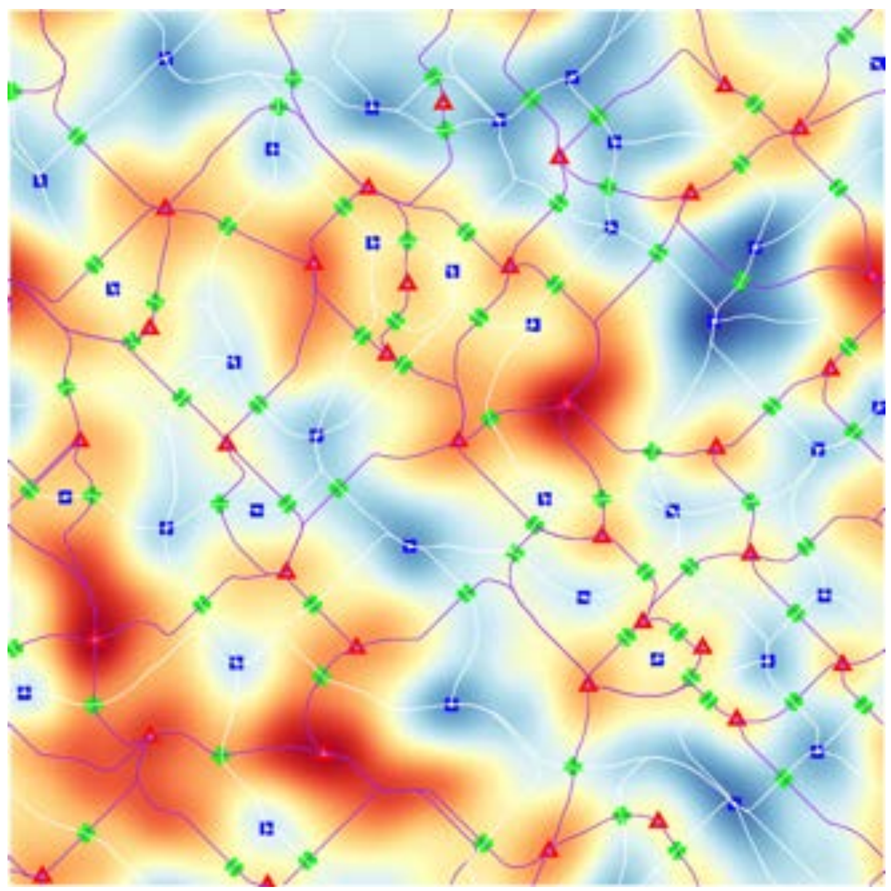




- Number of dense $\nu_f > 2$ filamentary bridges is increasing with the height of the central peak
- Not very rare $\nu = 3$ central peak has two (branches of) dense filaments, i.e it sits in one dominant filament on average
- Rare $\nu = 4$ peak is at intersection of three prominent branches.

Local multiplicity: towards a theoretical prediction



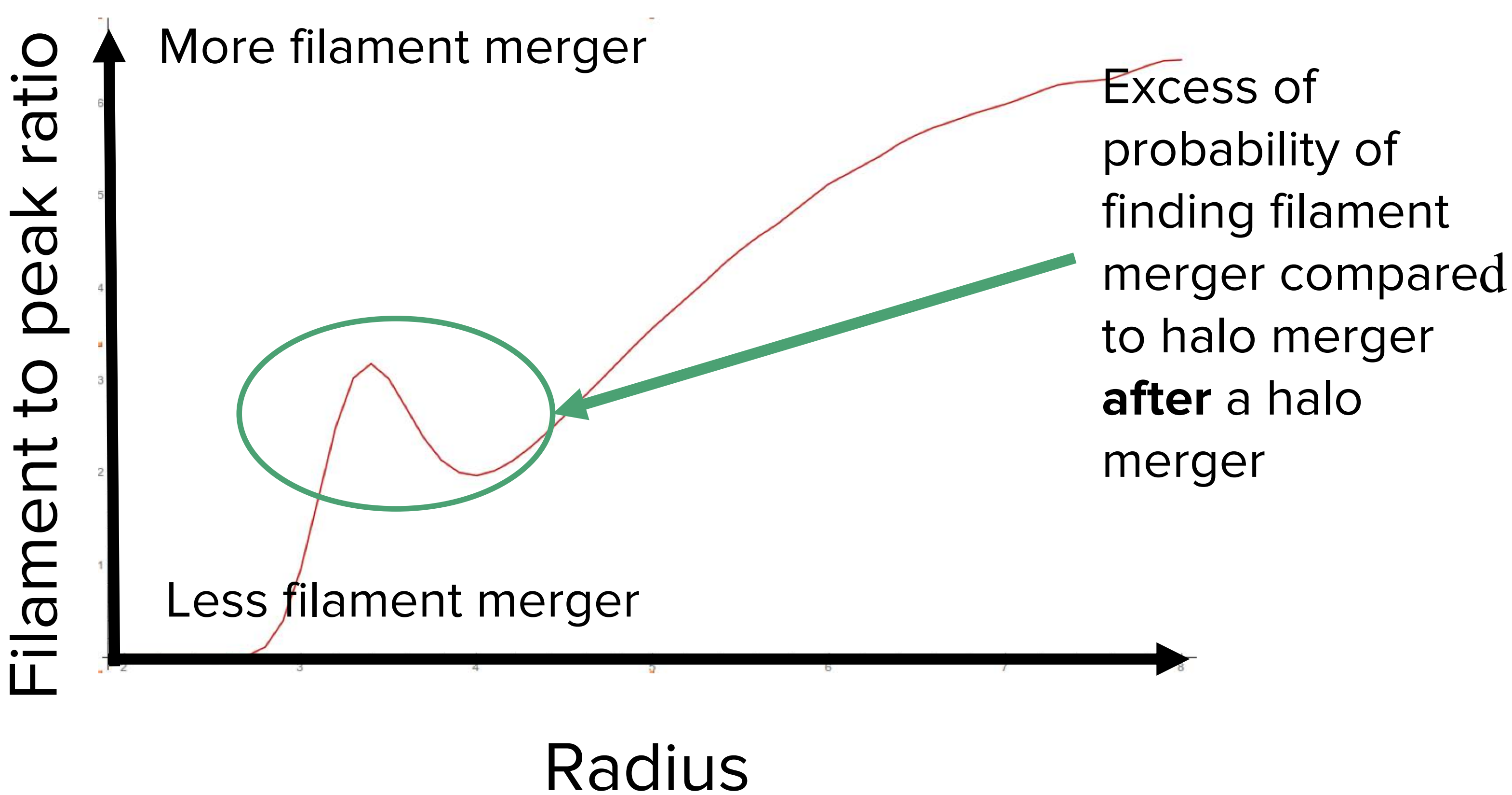


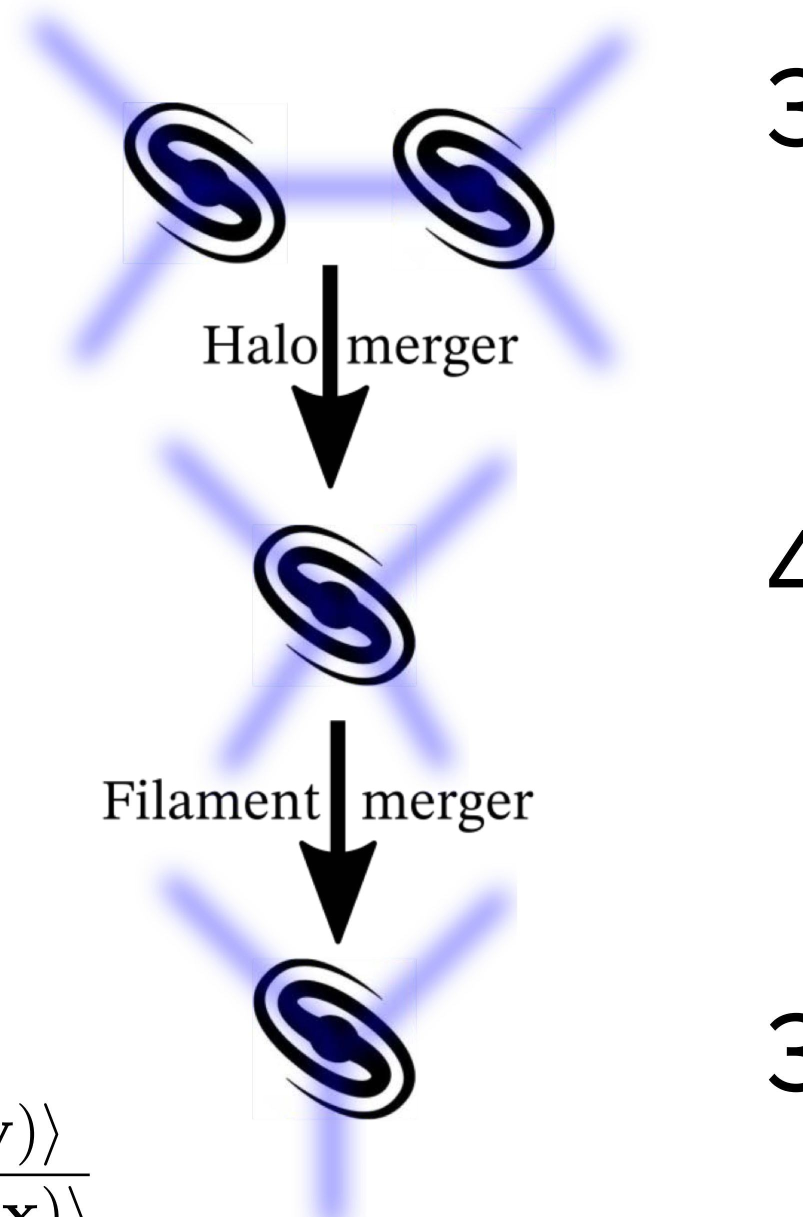
Typically, two to three dense filaments dominate and therefore define a plane of accretion… in agreement with numerical simulation (Danovich+12) and observations of plane of satellites around galaxies.

Application: preserving cosmic connectivity

On the connectivity of halos

Compute frequency of filament merger compared to halo merger in the vicinity of a halo merger event $\xi_{\rm hf}(r)/\xi_{\rm hh}(r)$.





$$1+\xi_p=\frac{\langle {\rm cond}_p(\mathbf{x}){\rm cond}_p(\mathbf{y})\rangle}{\langle {\rm cond}_p(\mathbf{x})\rangle^2},\; 1+\xi_f=\frac{\langle {\rm cond}_f(\mathbf{x}){\rm cond}_p(\mathbf{y})\rangle}{\langle {\rm cond}_f(\mathbf{x})\rangle\langle {\rm cond}_p(\mathbf{x})\rangle}$$

Upshot

Connectivity is a packaging pb because of exclusion

- Connectivity = number of filament connected
  - κ= 4 in 2D κ = 6.11 in 3D (for GRF)
- Mutiplicity = number of *local* filament connected
  - μ~3 in 2D μ ~ 4 in 3D
- Both can be predicted *from first principle*
- Hence useful for cosmology & galaxy formation

horizon-AGN

Upshot

- Set of critical events = useful topological compression of ICs
  - impacts 'dressed' mergers: ML on morphology?

  (i.e. cosmic evolution of peaks and their filaments and walls).

- Clustering of filament disappearance is consistent

  with *preserving connectivity* of peaks as they merge:
  - the rarer the peak the higher the rate of filaments merging.

- Rate of wall disappearance = dark energy probe,

  depend on the growth rate of structure and $\sigma_2/\sigma_1\sigma_3$.

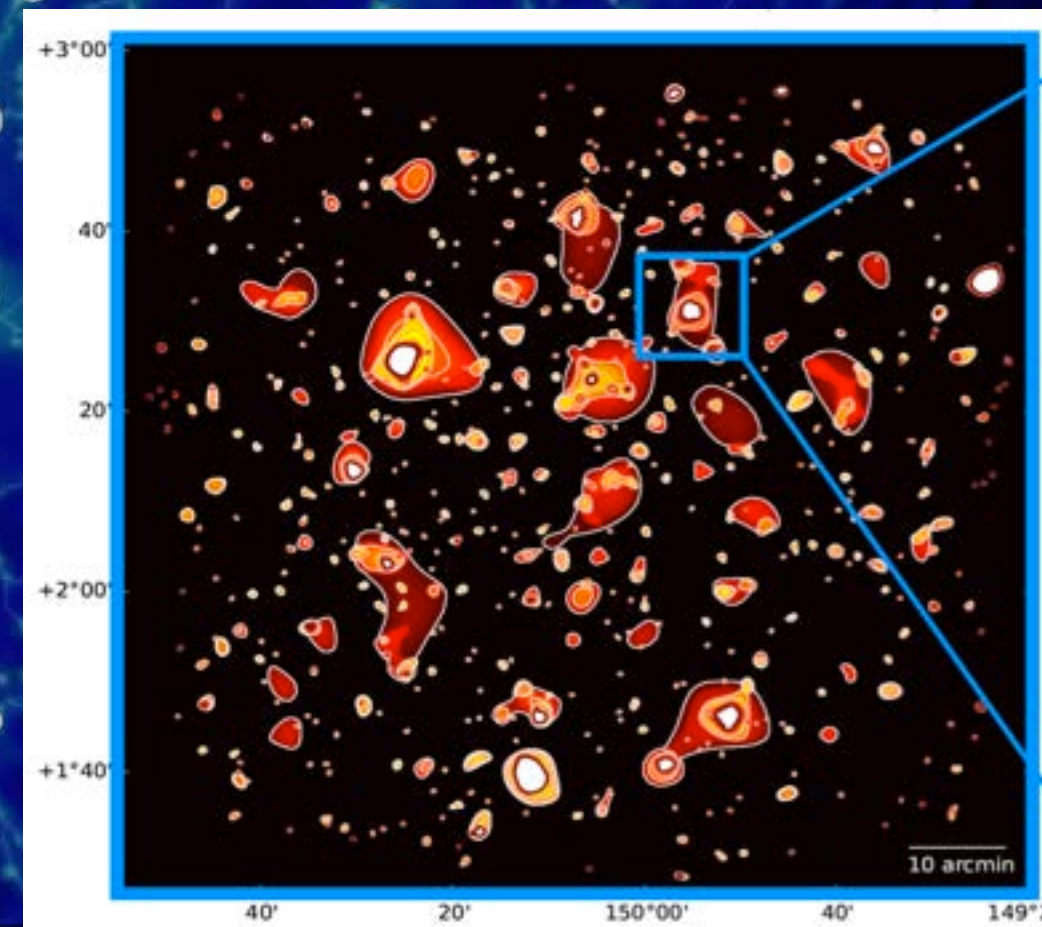
Conclusion

- Peak and constrained random field theories are paramount to understand the birth and growth of the cosmic web
- Many analytical results can be obtained in the weakly non-linear regime
- The topology and geometry of the cosmic web carries important cosmological information and is key for galaxy evolution.
- In particular, we now have a precise understanding of the connectivity of the cosmic web (the cosmic crystal) and its evolution through statistics of critical events.

IMHO of interest beyond cosmology

Application: impact of AGN feedback?

X-Ray detected groups



filaments from galaxy distribution

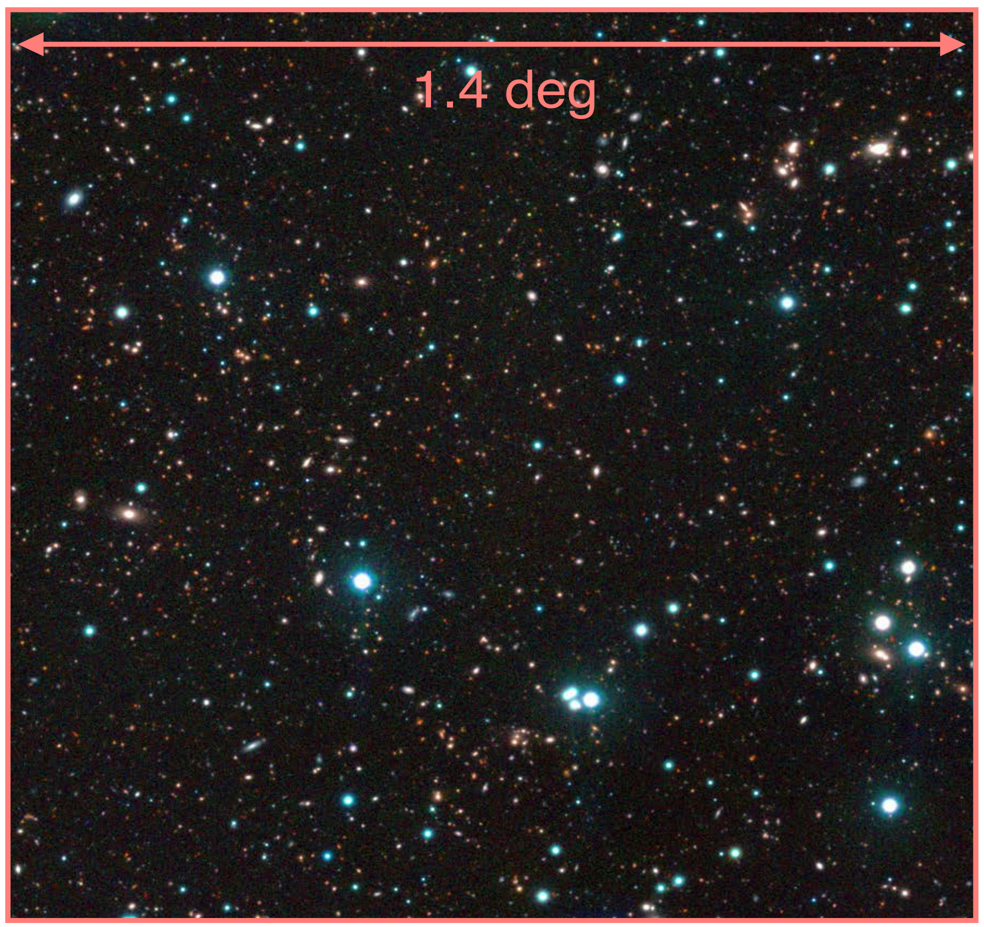
Elise Darragh-Ford
Laigle, Gozaliasl, Pichon, Devriendt, Slyz et al.

Galaxy distribution, gas density — Horizon-AGN simulation (Dubois+14)

Filament extraction in 2D around groups

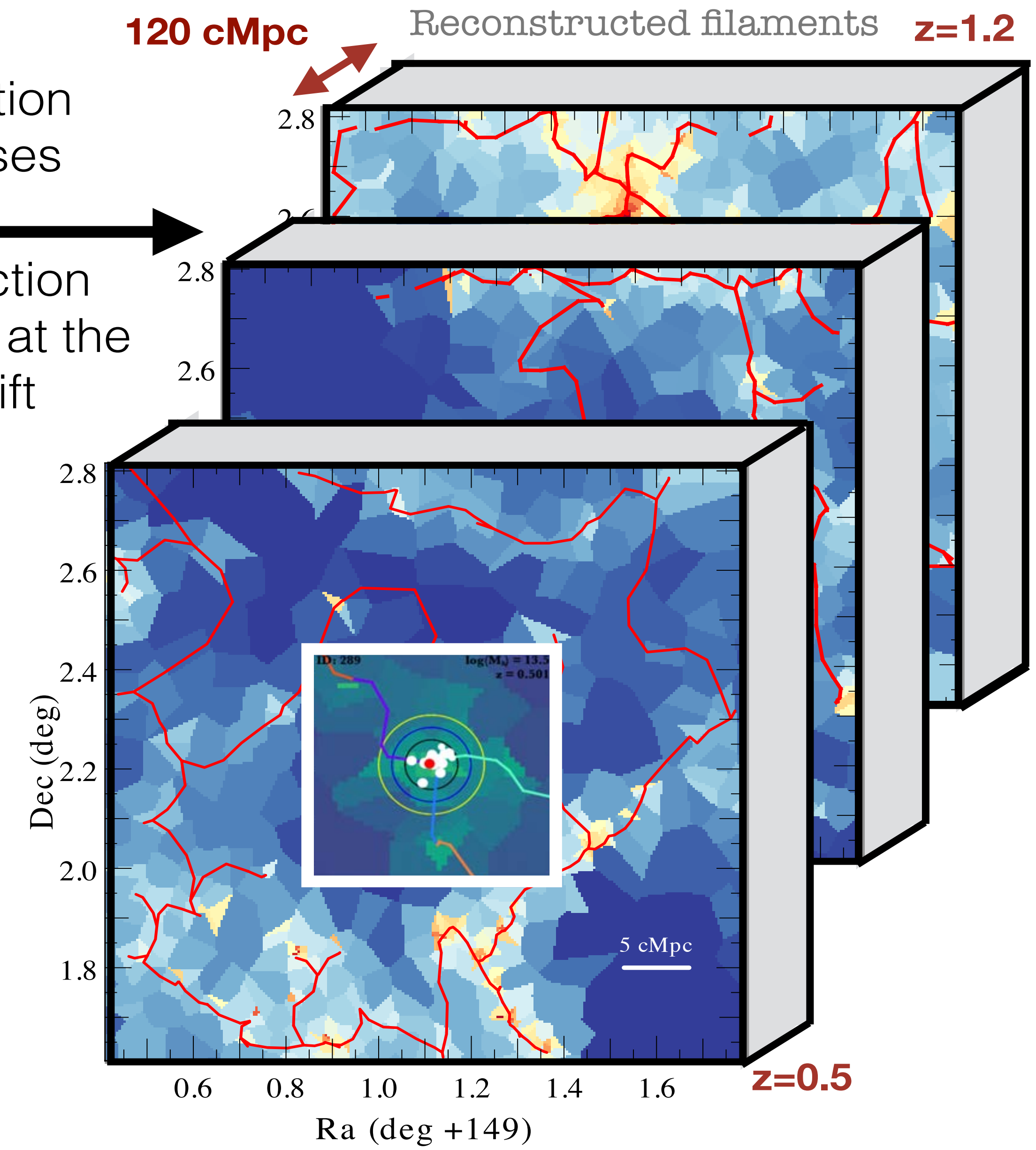
Darragh-Ford, Laigle et al. in prep

COSMOS
reference photometric field
1 million of galaxies



Galaxy extraction
redshift, masses

Filament extraction
in slices centred at the
group redshift



Redshift and mass range constrained by galaxy photometric properties:
We work in $0.5<z<1.2$ with all galaxies more massive than 10^{10} solar mass

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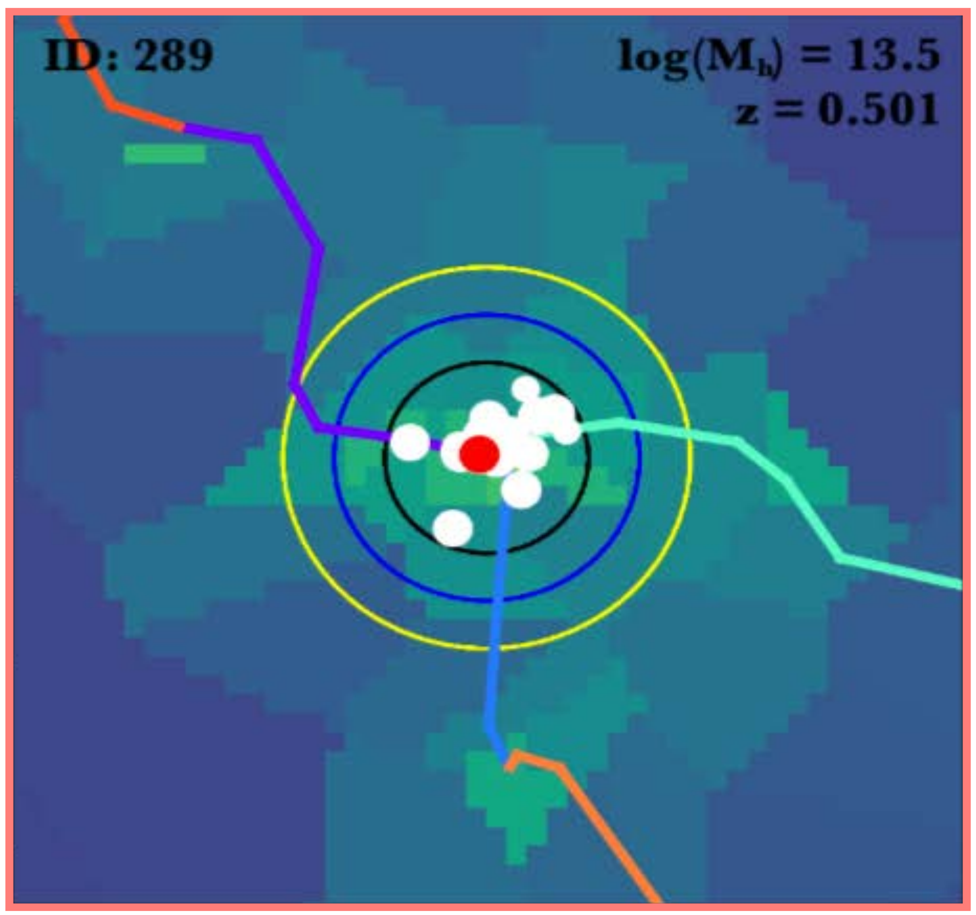

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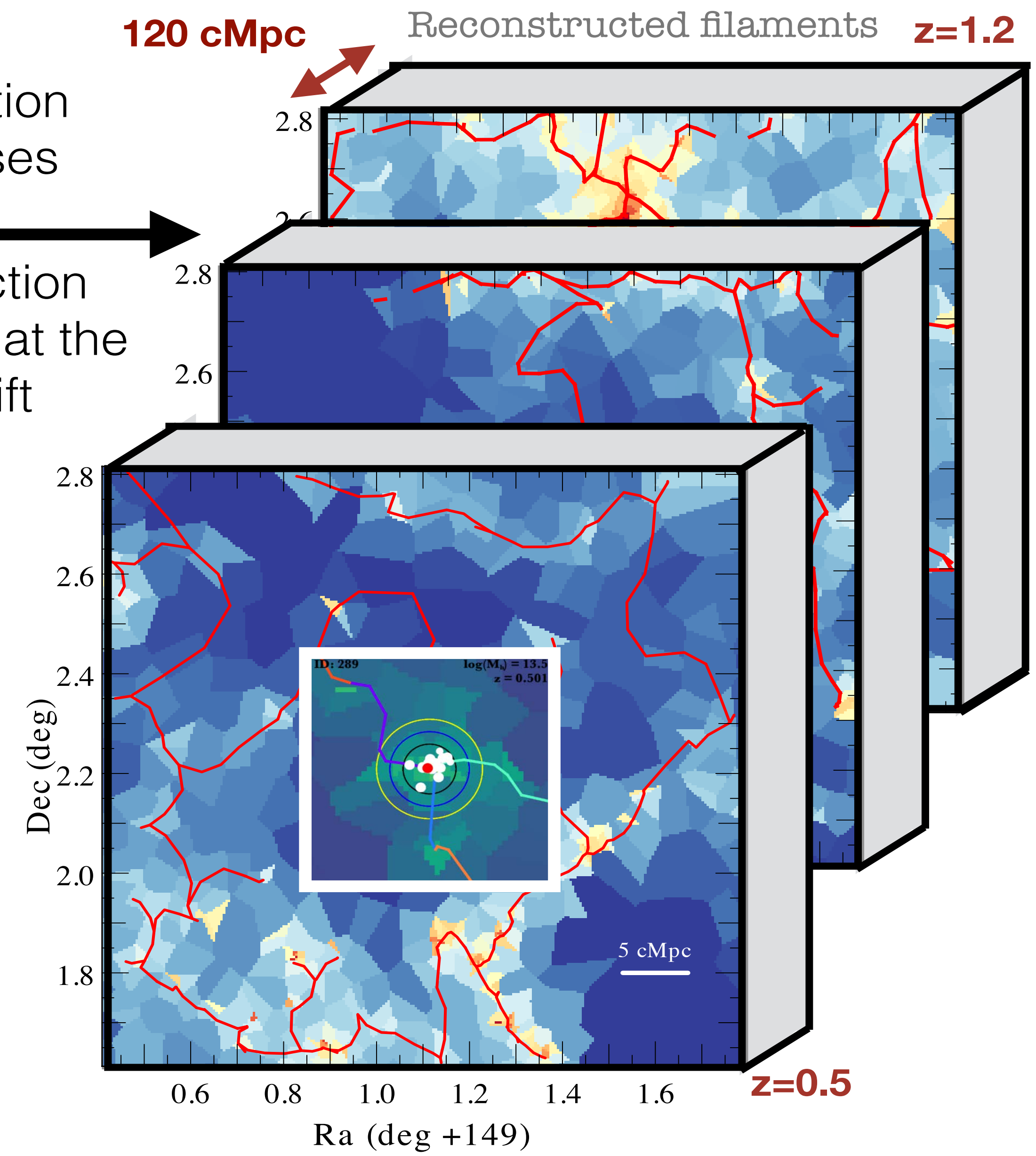

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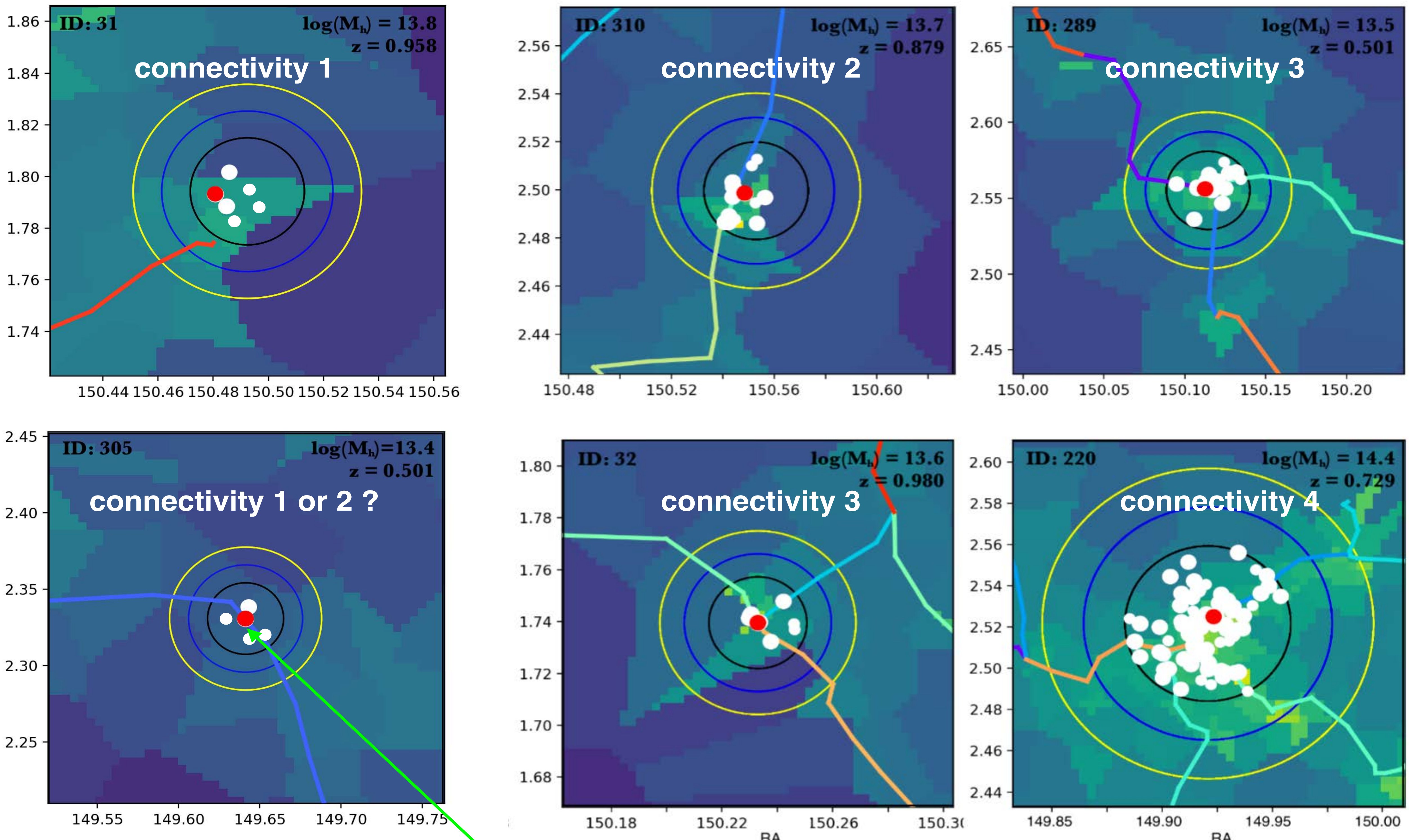
Group Multiplicity

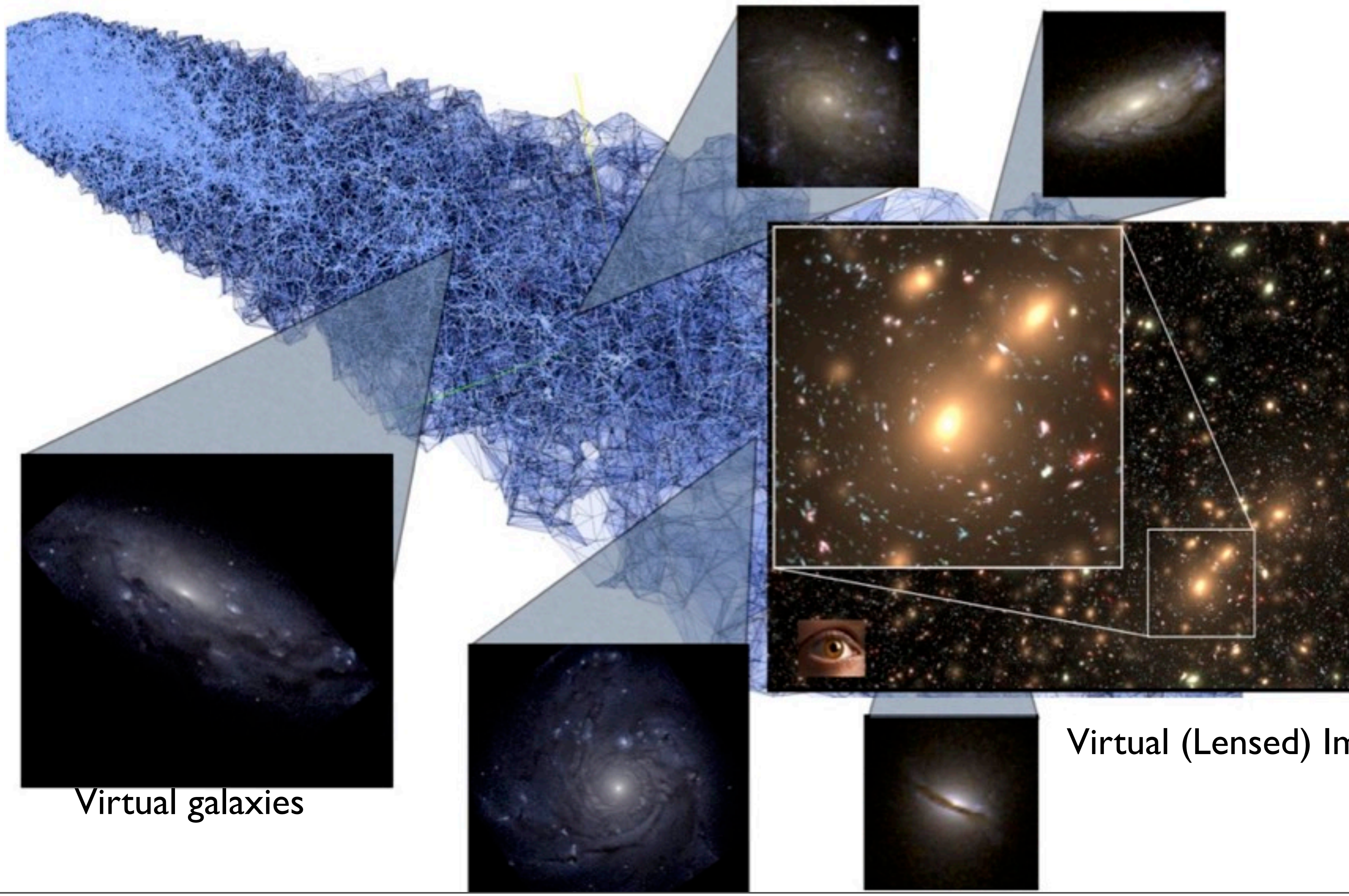
Measuring connectivity with photometric filaments



BGG: brightest group galaxy

Group Multiplicity in Horizon-AGN

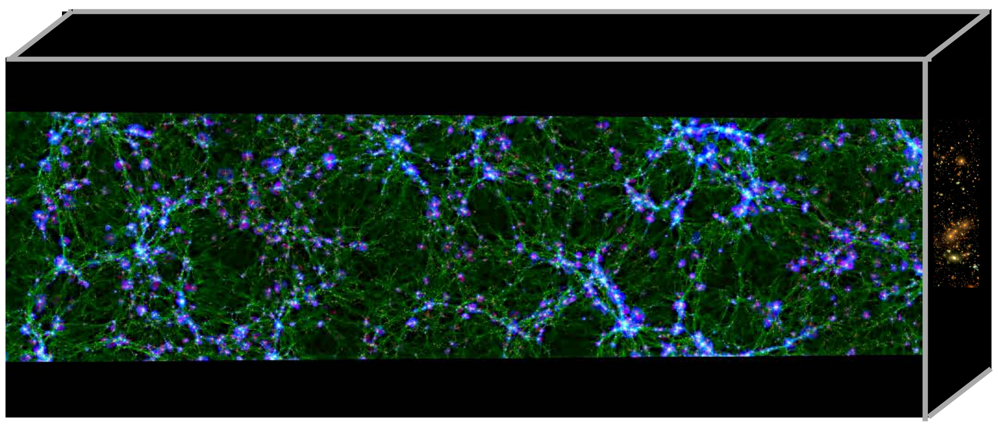
Testing the impact of photometric uncertainties



Group Multiplicity in Horizon-AGN

Testing the impact of photometric uncertainties

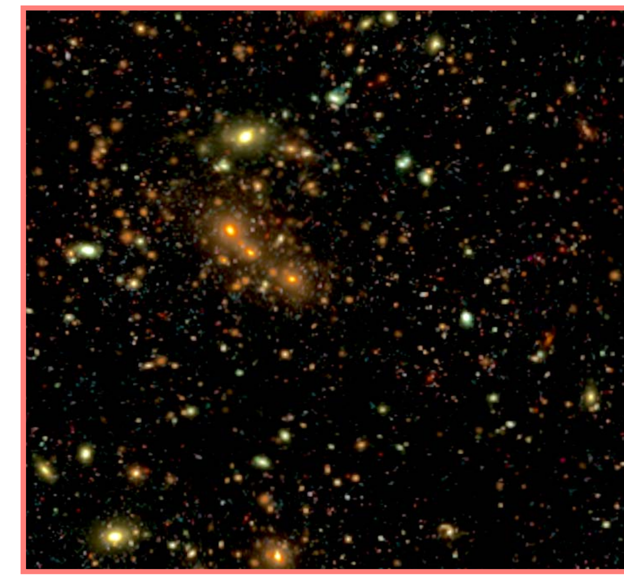
Hydrodynamical simulation Horizon-AGN
Dubois+14



Mock observation generation
(photometry, photo-z, photometric masses)

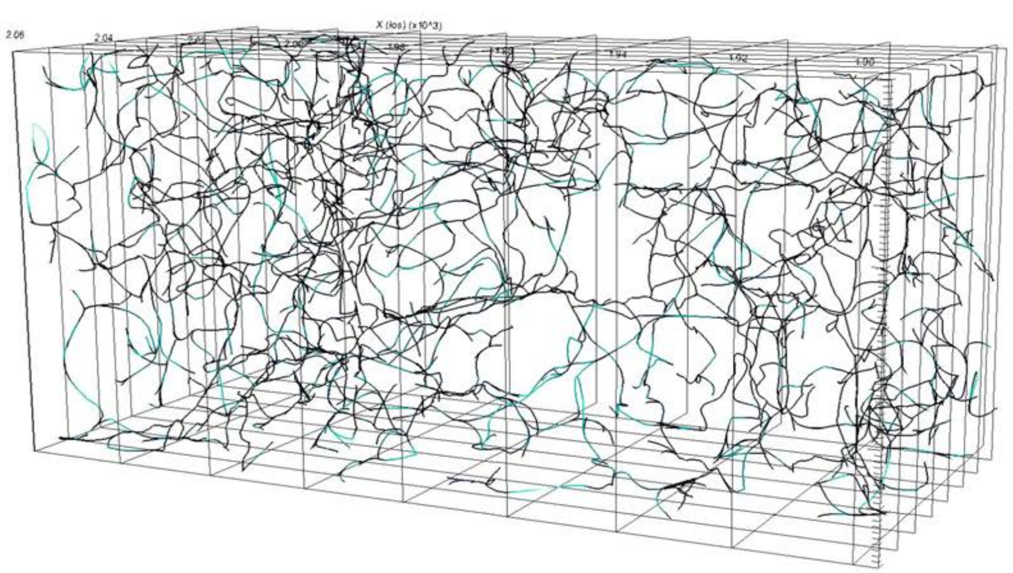
Adding errors
(including systematics)

Mock image

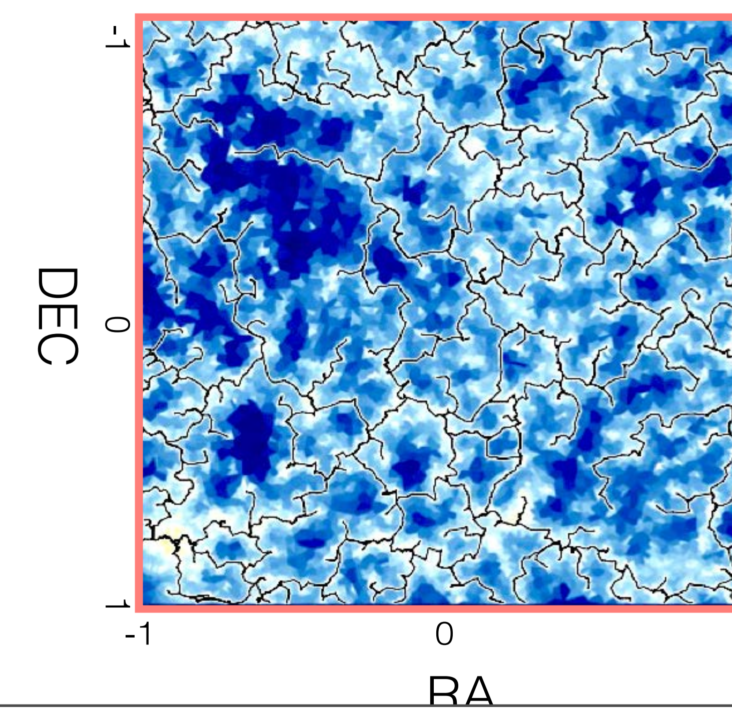


3D reference skeleton

Virtually observed
skeleton



Confrontation



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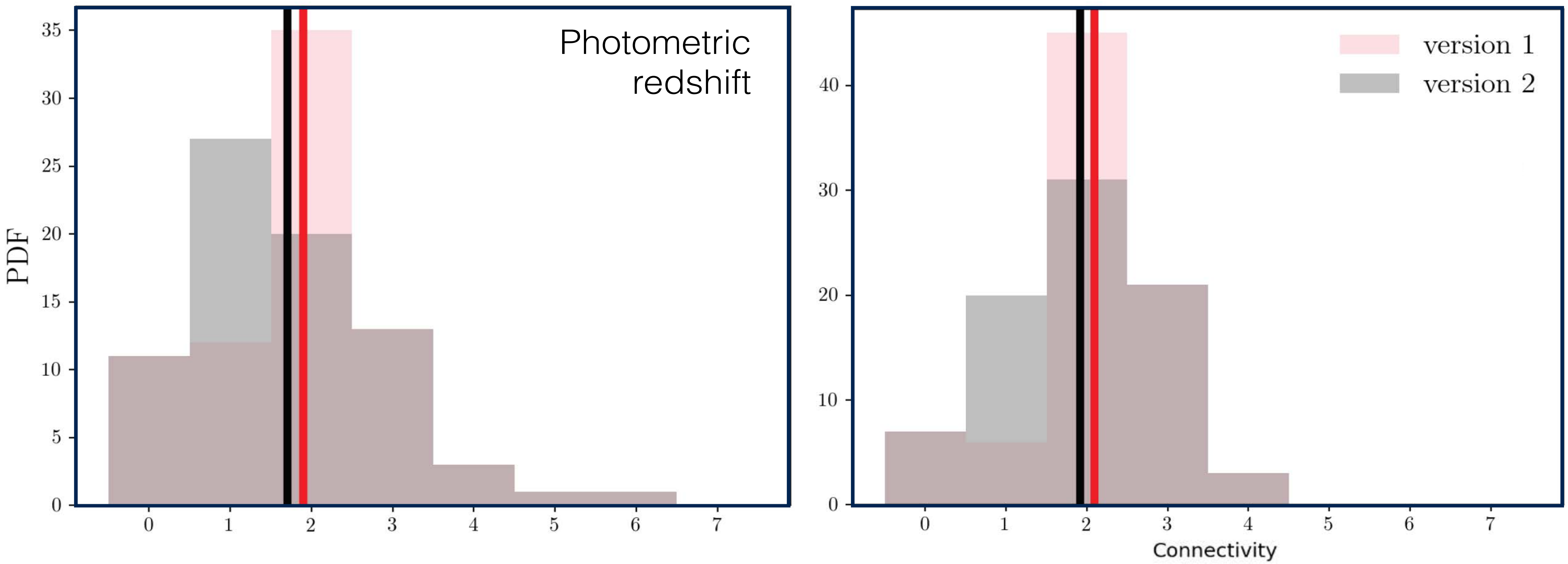
Measuring connectivity with photometric filaments

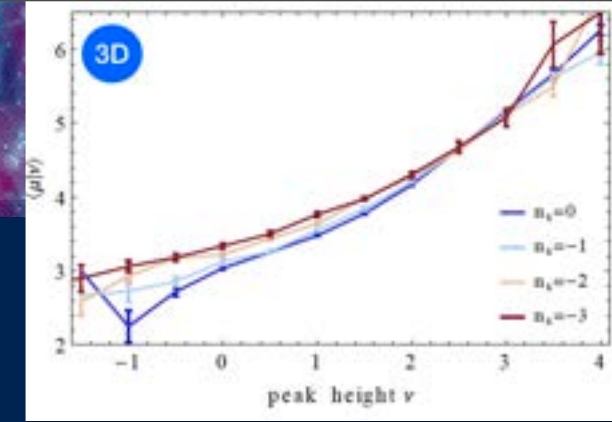
Darragh-Ford, Laigle et al. in prep

photo-z uncertainties decrease connectivity

Hz-AGN connectivity

COSMOS connectivity





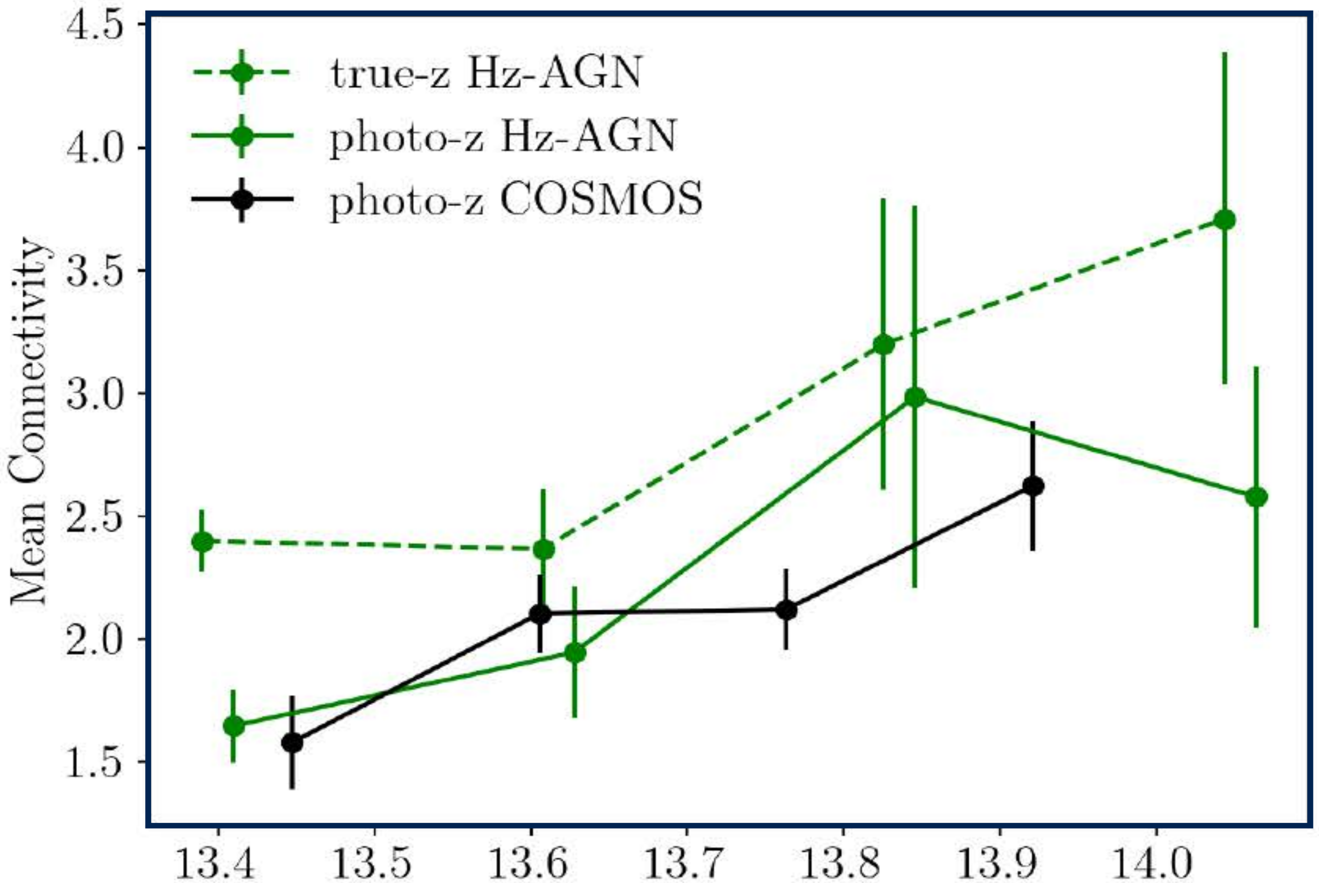
Group Multiplicity

Impact of connectivity on group properties

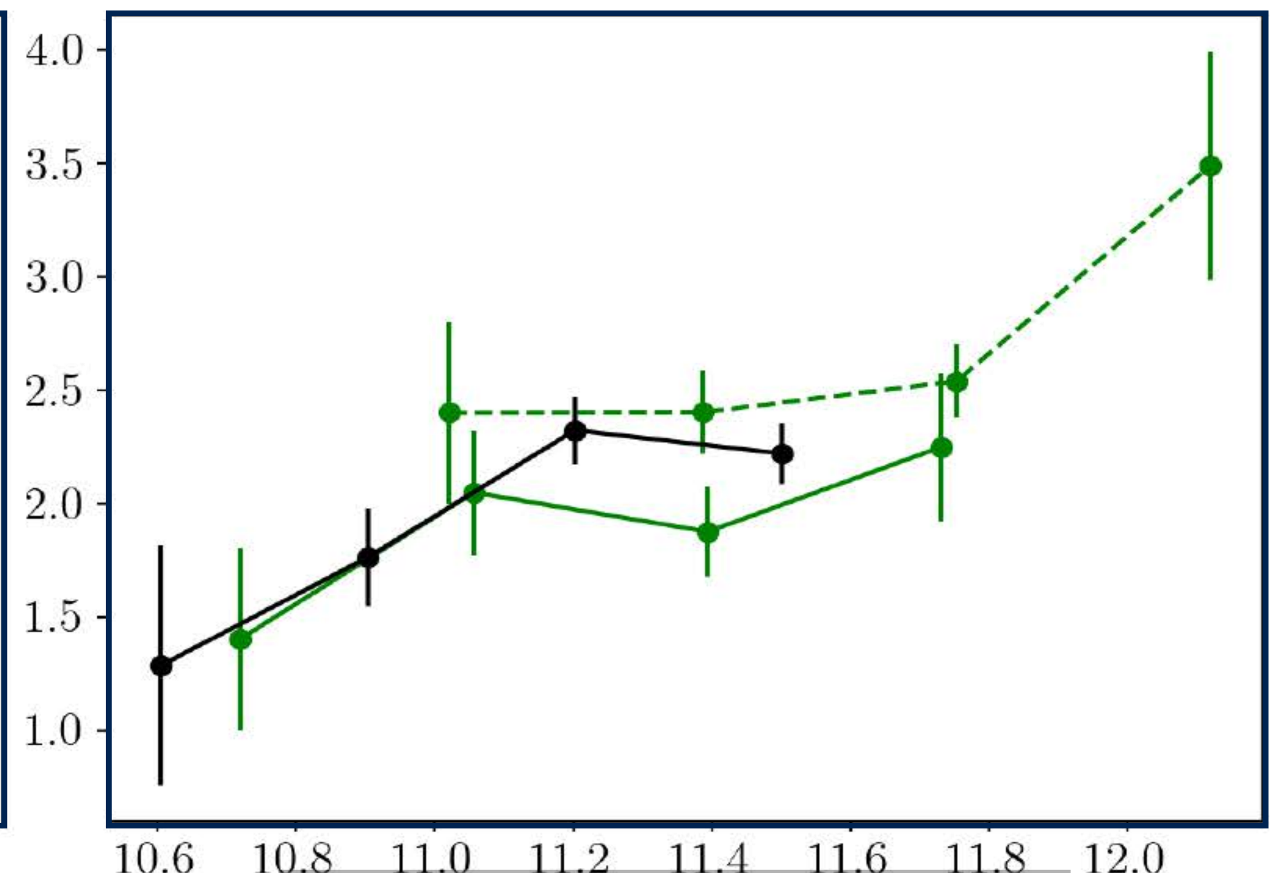
Darragh-Ford, Laigle et al. in prep

see also
Theoretical predictions from
Codis et al. 2018

Mean connectivity increases with halo/BGG mass



Halo Mass

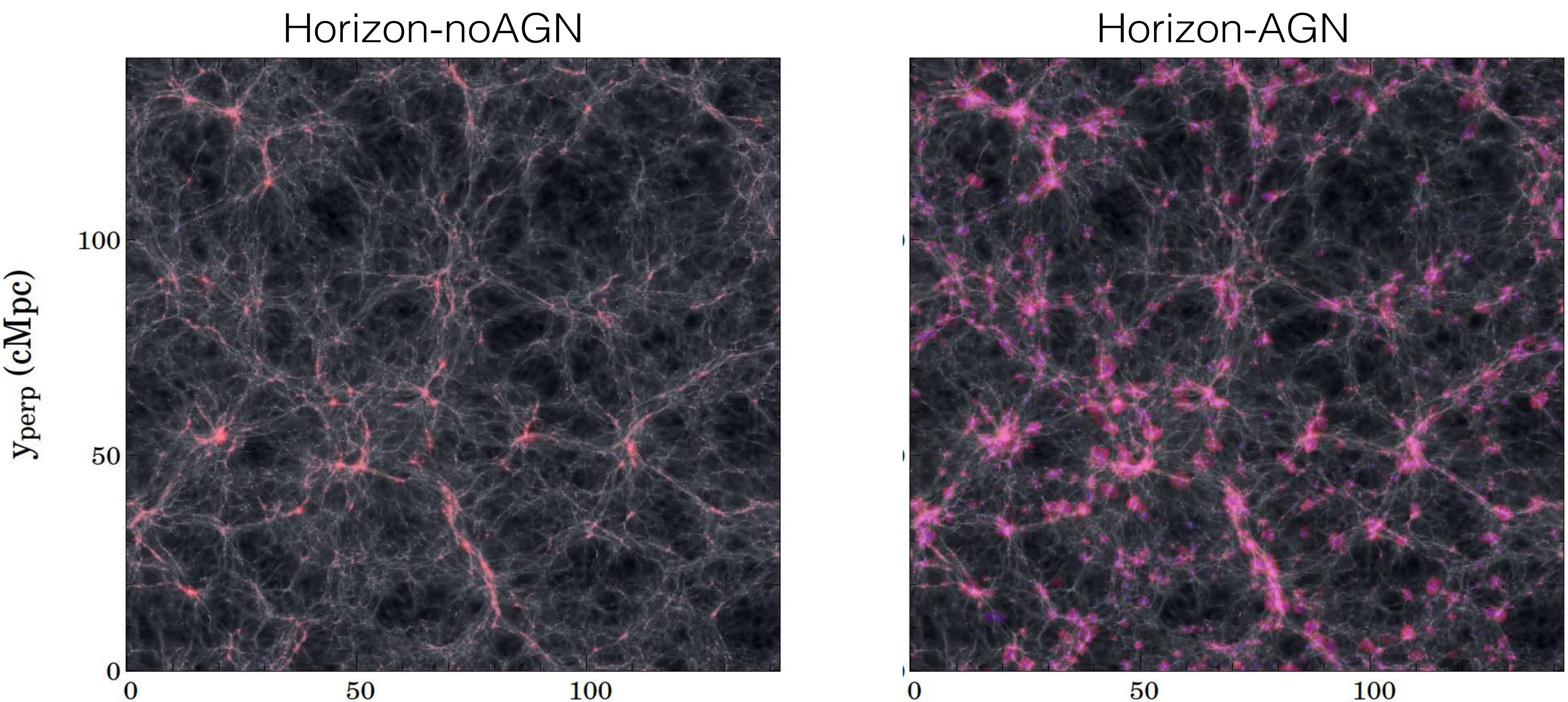


Brightest group galaxy Mass

The impact of Multiplicity on BGG properties

Interpretation from Horizon-AGN simulation

Darragh-Ford, Laigle et al. in prep



Group Multiplicity

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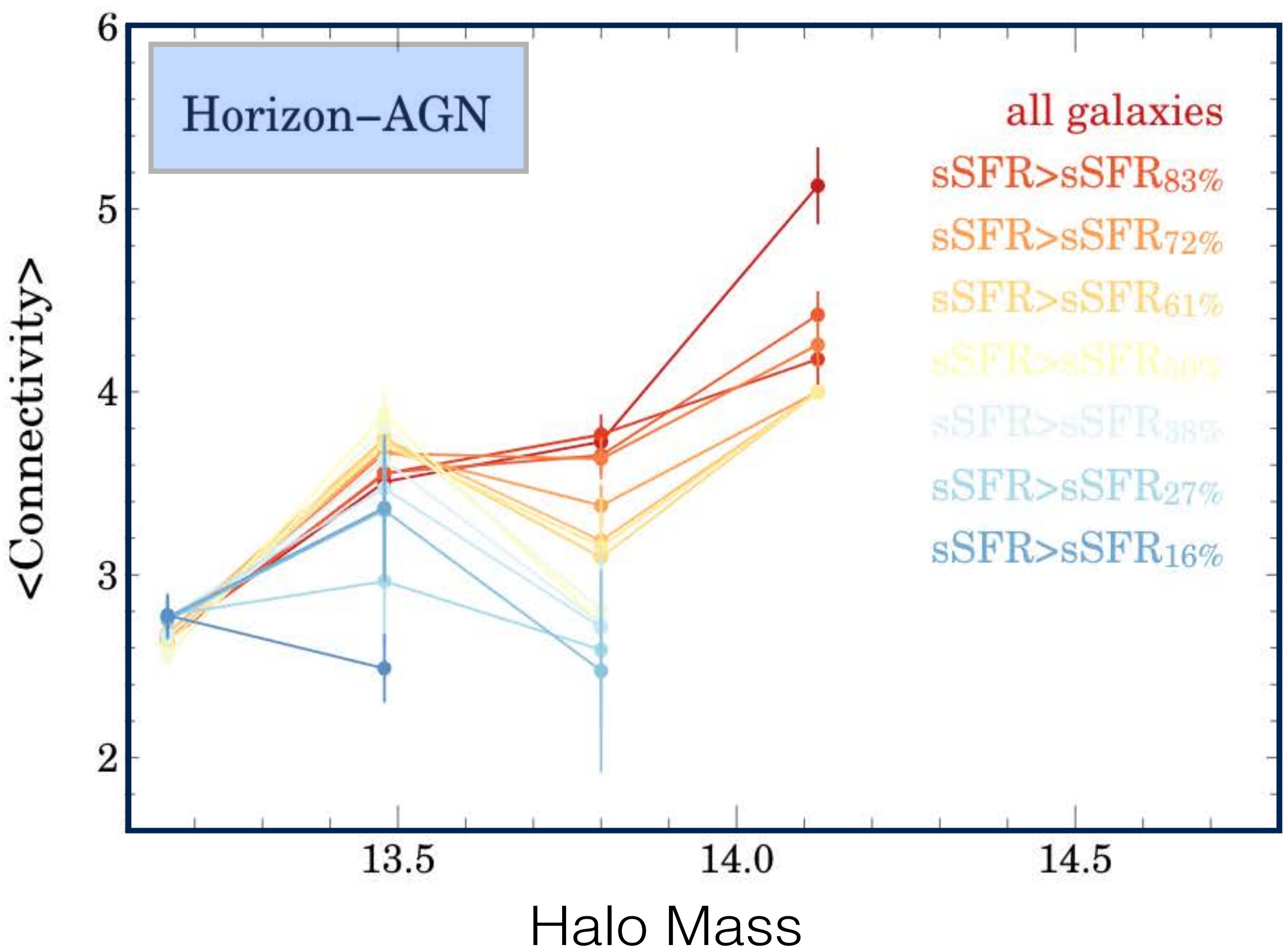
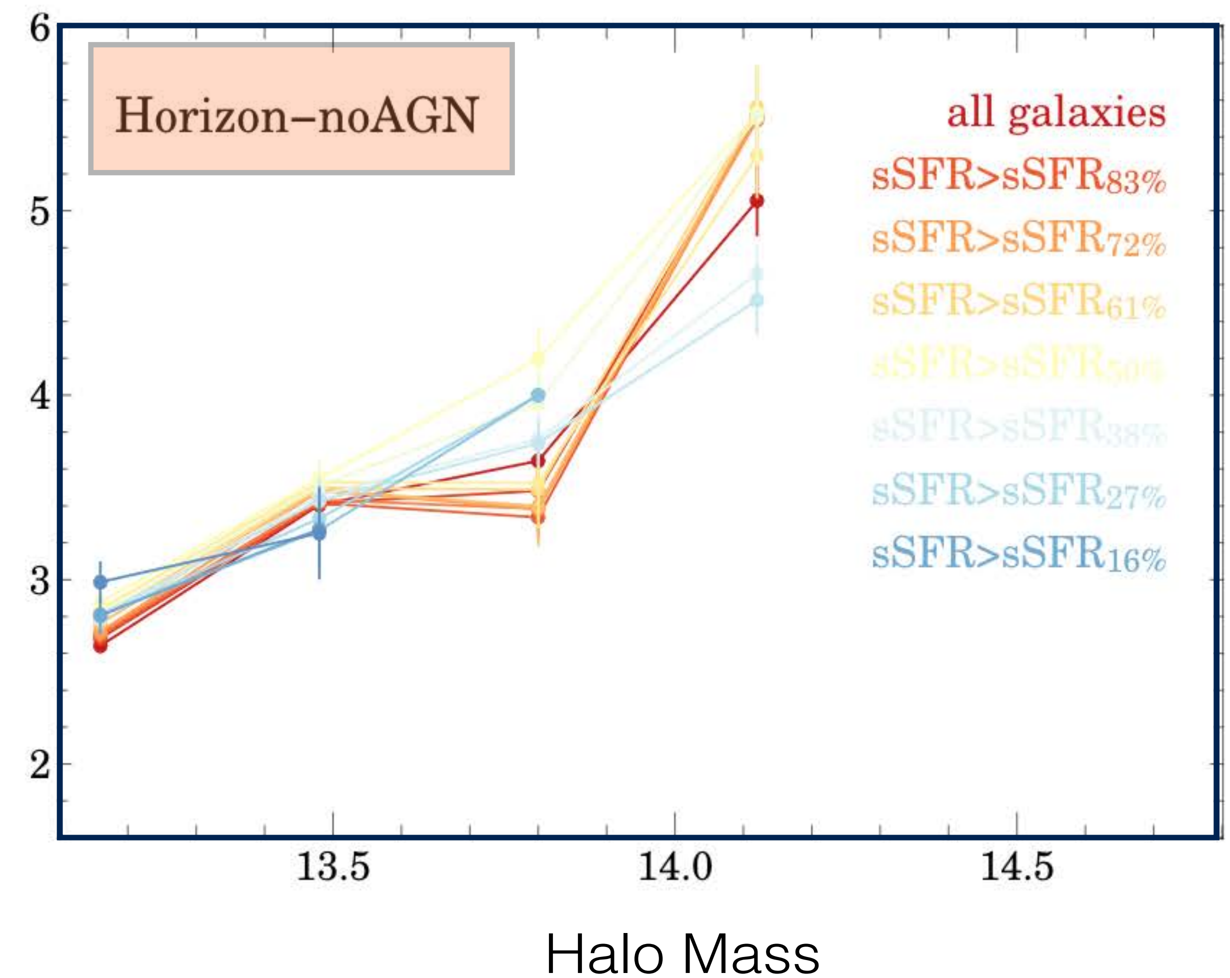
HORIZON-AGN simulation result:

At a given halo mass, “AGN quenching efficiency” is higher at higher connectivity





- Connectivity: proxy for mass of accreted matter; more accretion ➡ higher feedback?
- higher connectivity ➡ accretion more isotropic

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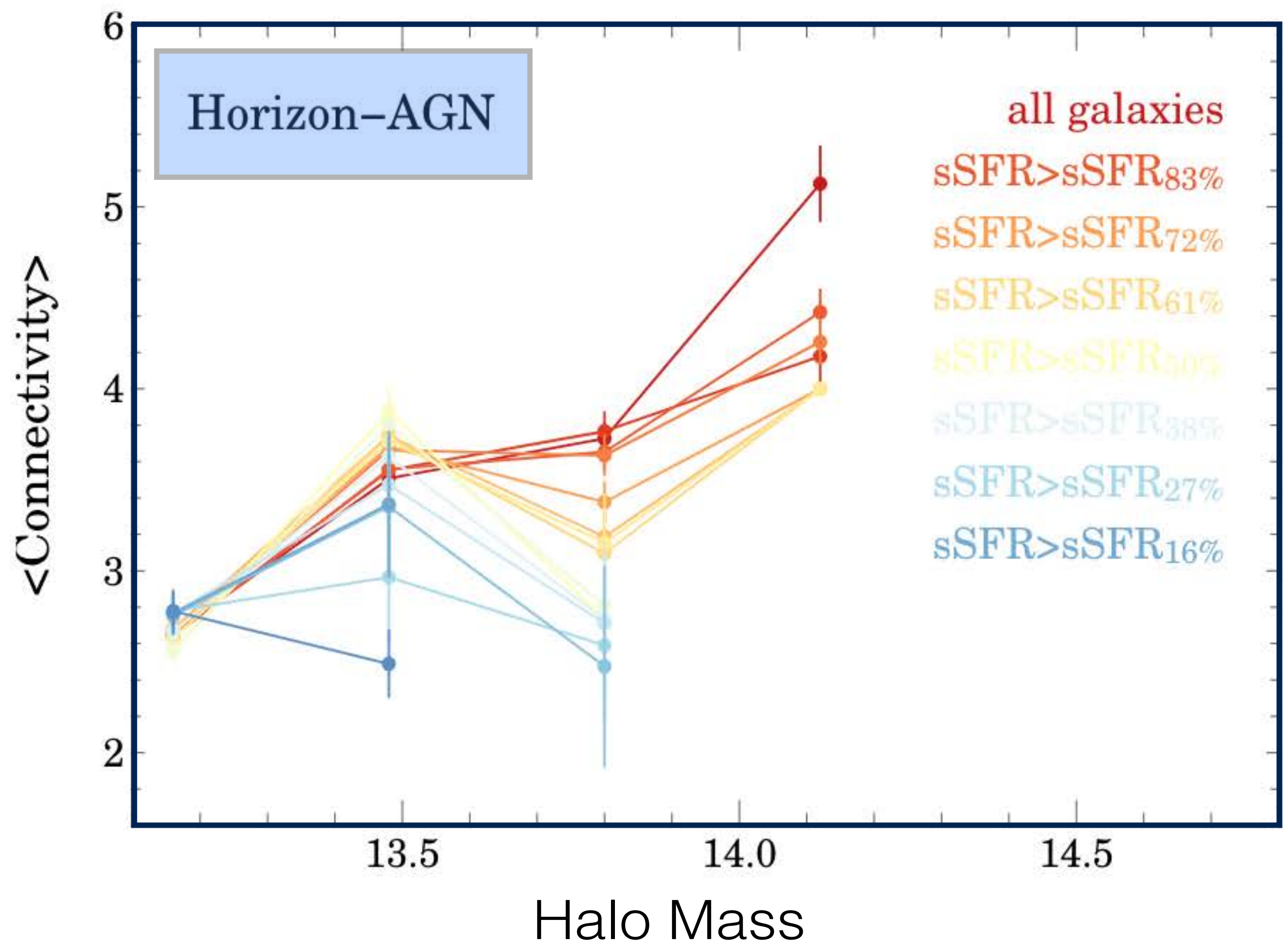
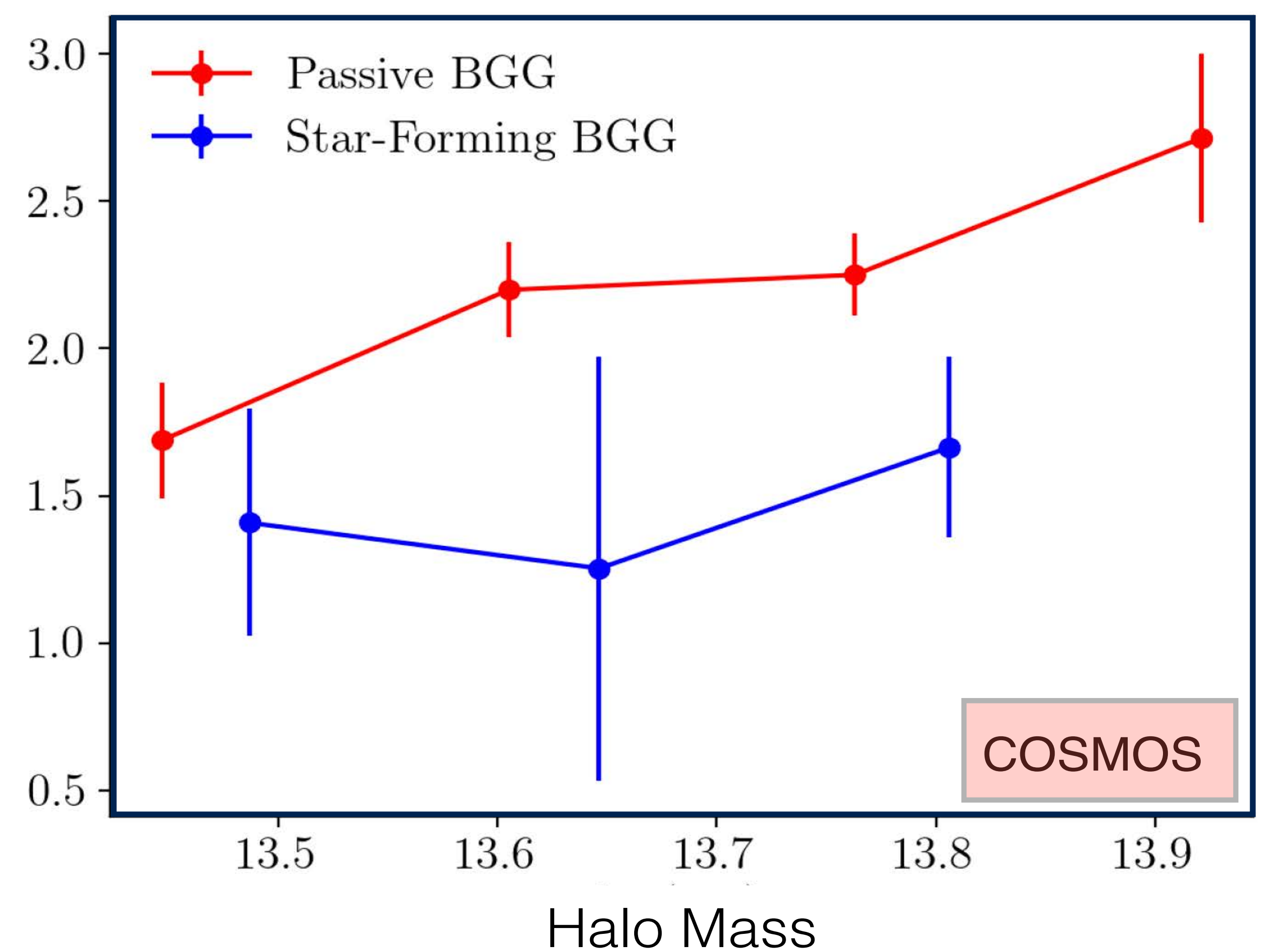




- Connectivity: proxy for mass of accreted matter; more accretion ➡ higher feedback?
- higher connectivity ➡ accretion more isotropic

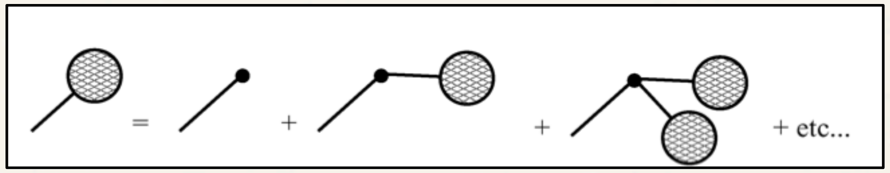
Generalized geometrical S_n

Purpose: Express the invariant **cumulants** in terms of σ (hence D(z)) through Perturbation theory $e.g. \quad \langle J_1 x \rangle = \text{function}(\sigma)$

$$F_2(\mathbf{k_1},\mathbf{k_2}) = \frac{5}{7} + \frac{\mathbf{k_1}\cdot\mathbf{k_2}}{{k_1}^2} + \frac{2}{7}\frac{(\mathbf{k_1}\cdot\mathbf{k_2})^2}{{k_1}^2{k_2}^2} \implies \mathcal{F}_{\alpha,\beta,\gamma}(\mathbf{k_1},\mathbf{k_2}) = F_2(\mathbf{k_1},\mathbf{k_2})\mathcal{G}_{\alpha,\beta,\gamma}(\mathbf{k_1},\mathbf{k_2})$$

GRAVITY

Geometric shape factor= powers of k



$$\delta^{(n)}(\vec{k}) = \int d^3\vec{k_1}\dots d^3\vec{k_n}\delta^D(\vec{k}-(\vec{k_1}+\dots+\vec{k_n}))F_n(\vec{k_1},\dots,\vec{k_n})\delta^{(1)}(\vec{k_1})\dots\delta^{(1)}(\vec{k_n}),$$

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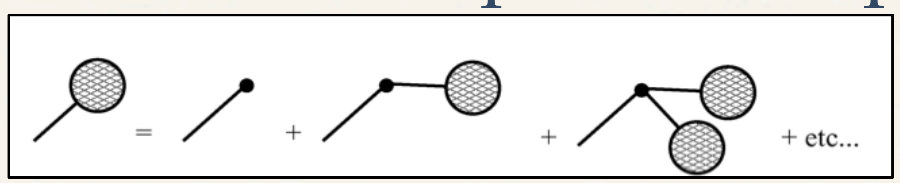

power spectrum index

$$\frac{1}{\sigma}\langle x^3\rangle = 3\,{}_2F_1\left(\frac{3+n}{2},\frac{3+n}{2},\frac{3}{2},\frac{1}{4}\right) - \frac{1}{7}(8+7n)\,{}_2F_1\left(\frac{3+n}{2},\frac{3+n}{2},\frac{5}{2},\frac{1}{4}\right).$$

Lokas 94

skewness of field

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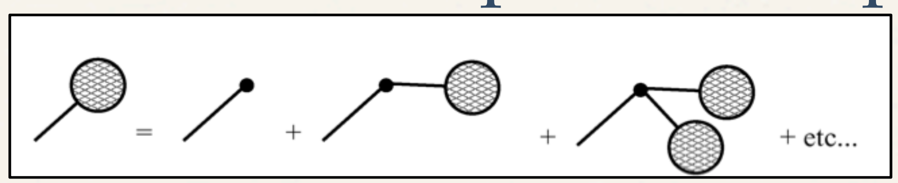

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3pt field- gradient cumulant

n= -3 : $\frac{1}{\sigma}\langle x^3\rangle = \frac{34}{7} \Longrightarrow \frac{1}{\sigma}\langle x x_1^2\rangle = \frac{34}{7}\frac{2}{3^2}$

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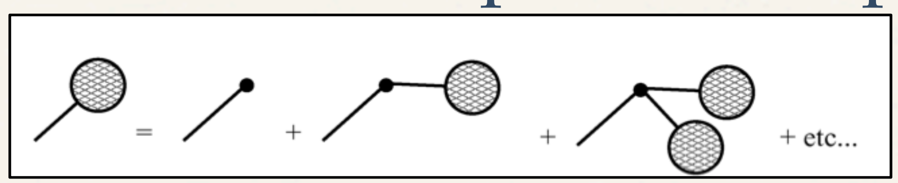
GRAVITY

Geometric shape factor= powers of k



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See also Bouchet for S4

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GRAVITY

G… powers of k

	$n_s = 0$	
	prediction	measurement
$\langle x^3 \rangle / \sigma$	3.144	3.08 ± 0.08
$\langle x q^2 \rangle / \sigma$	2.096	2.05 ± 0.03
$\langle x^2 J_1 \rangle / \sigma$	-3.248	-3.15 ± 0.06
$\langle x {J_1}^2 \rangle / \sigma$	3.871	3.75 ± 0.06
$\langle x J_2 \rangle / \sigma$	1.545	1.54 ± 0.02
$\langle q^2 J_1 \rangle / \sigma$	-1.335	-1.28 ± 0.02
$\langle {J_1}^3 \rangle / \sigma$	-4.644	-4.50 ± 0.08
$\langle J_1 J_2 \rangle / \sigma$	-0.679	-0.65 ± 0.01
$\langle J_3 \rangle / \sigma$	1.304	1.28 ± 0.03

power spectrum index

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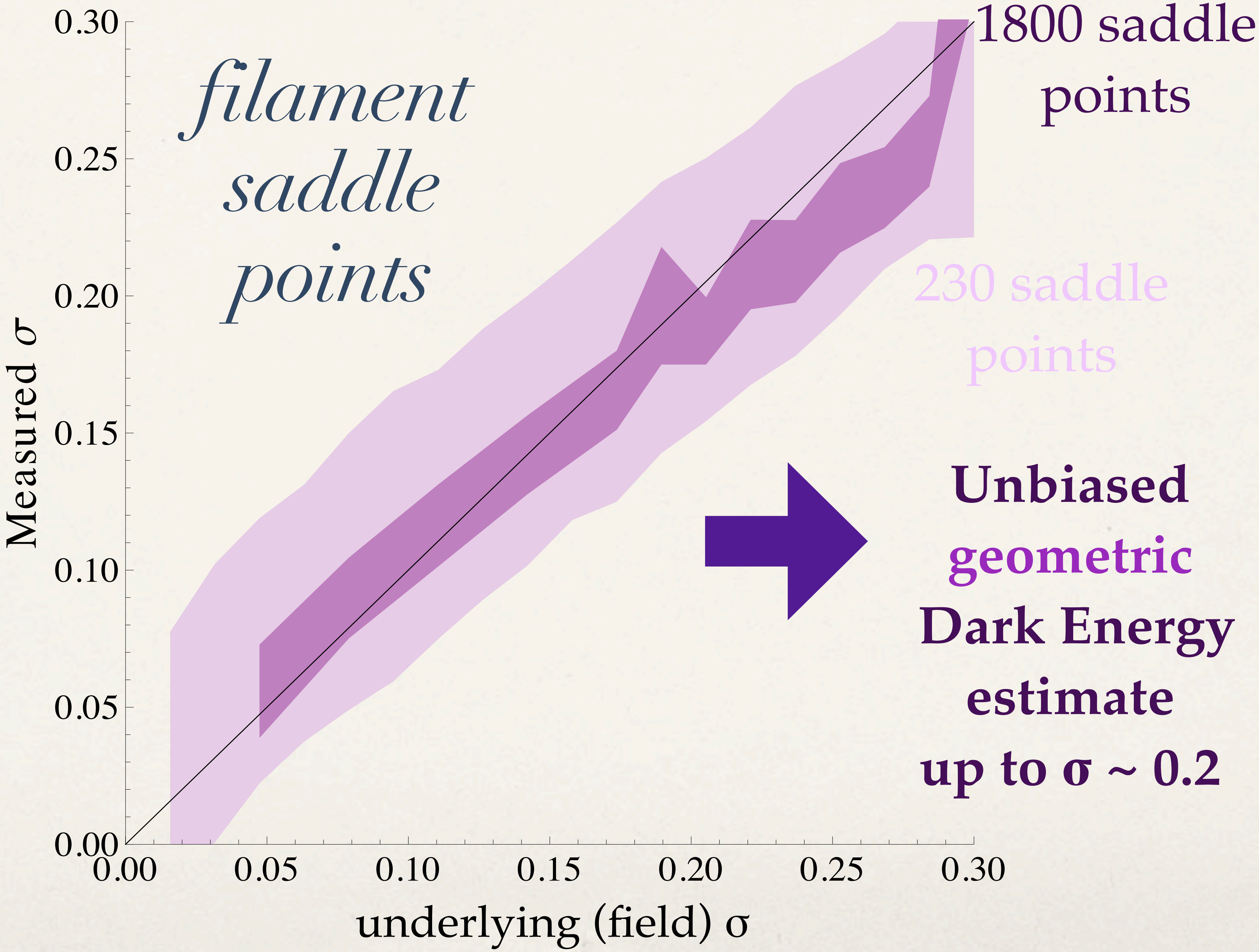
Fiducial DE experiment

- Generate scale invariant ICs
- Evolve them with gravity
- identify critical sets
- compute differential counts
- estimate amplitude of NG **distorsion** via PT
- deduce **geometric** critical set σ

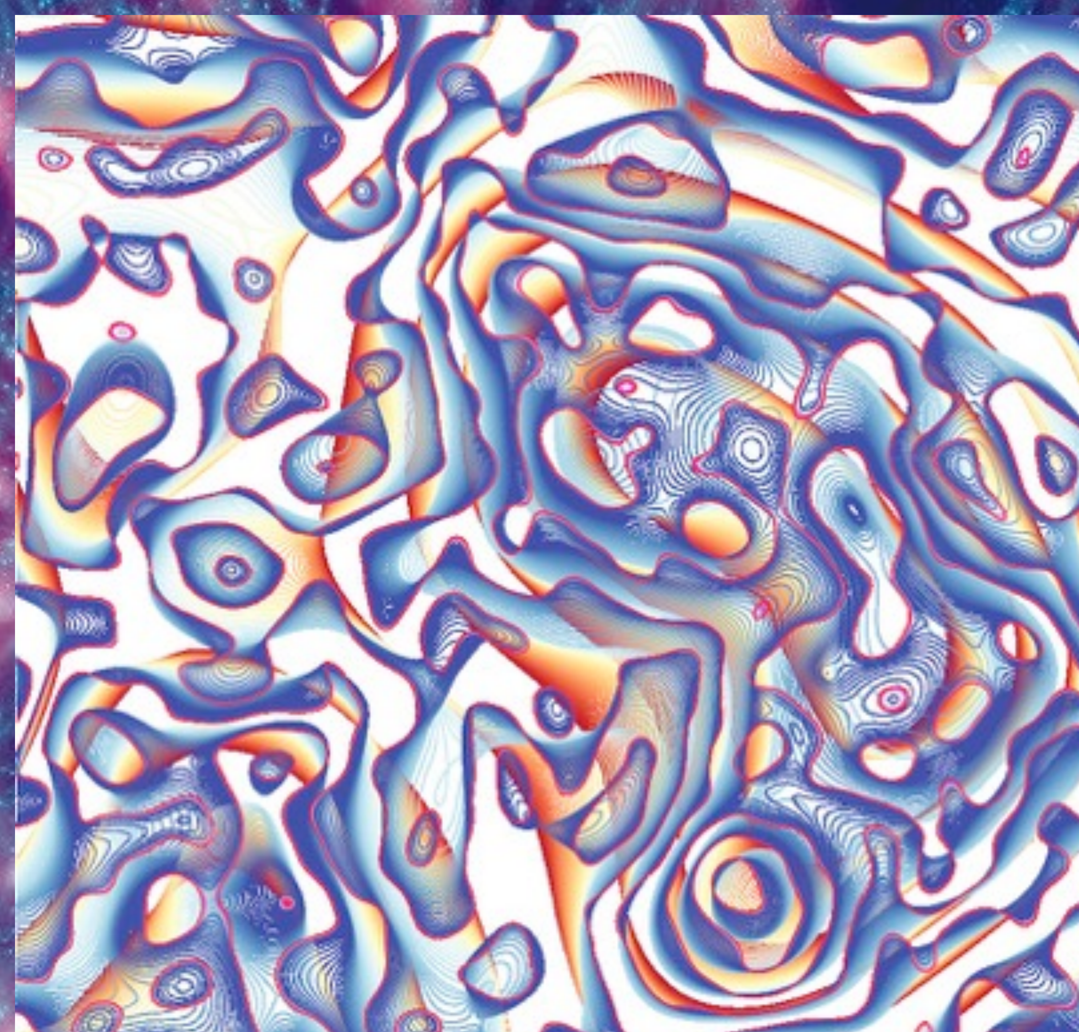


Unbiased geometric Dark Energy estimate up to σ ~ 0.2

How is the cosmic web woven?

- Context
- Random fields, Peak theory, critical events
- Cosmic connectivity
- Cosmic multiplicity
- Application: AGN in groups

horizon-AGN



$$\left.\frac{\partial^2 n}{\partial R \partial \nu}\right|_{\pm} = \frac{3\sqrt{3}(1-\tilde{\gamma}^2)(25\gamma^4+30\gamma^2(2\nu^2-1)-27)R}{20\sqrt{10}\pi^{5/2}(9-5\gamma^2)^{5/2}R_*^3\tilde{R}^2} e^{-\frac{9\nu^2}{2(9-5\gamma^2)}}.$$