Higher-order corrections to the peak clustering

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Introduction
Motivation 1

- Density peaks: formation sites of the structure formation in the universe

Linear evolution

nonlinear evolution

structures formed (strongly nonlinear)
Motivation 2

- Clustering properties of primordial black holes (PBH)

Radiation dominated phase:

Jeans length ~ Horizon radius
~ Schwarzschild radius of horizon mass
=> PBH

- So far, PBH abundances are much discussed, but little is known about clustering properties
  - clustering of peaks?
Initial clustering of PBH?


FIG. 1. Schematic representation of qualitatively different small-scale spatial distribution of PBHs at formation. On the left, PBHs are in dense clusters, as predicted in Ref. [22]. On the right, PBHs are distributed approximately randomly. In this work, we show that the latter distribution is what is expected for PBHs forming from large density fluctuations.
Simple model of peak clustering
A simple model of peak clustering

- Thresholded regions are proxy for peaks

\[
\delta_{\text{th}} = \nu \sigma
\]

\[
\rho_{\text{pk}}(\mathbf{x}) \sim \rho_{\text{th}}(\mathbf{x}) = \bar{\rho}_{\text{tot}} \Theta(\delta(\mathbf{x}) - \delta_{\text{th}})
\]

\[
\nu \equiv \frac{\delta_{\text{th}}}{\sigma}, \quad \sigma^2 \equiv \langle \delta^2 \rangle
\]

\[
P(\delta) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\delta^2/\sigma^2} \quad \beta \equiv \frac{\bar{\rho}_{\text{th}}}{\bar{\rho}_{\text{tot}}} = \int d\delta P(\delta) \Theta(\delta - \delta_{\text{th}}) = \frac{1}{2} \text{erfc} \left( \frac{\nu}{\sqrt{2}} \right)
\]

\[
1 + \xi_{\text{th}}(r) = \frac{\langle \rho_{\text{th}}(\mathbf{x}_1) \rho_{\text{th}}(\mathbf{x}_2) \rangle}{\bar{\rho}_{\text{th}}^2} = \frac{1}{\beta^2} \int_\nu^{\infty} \frac{dv_1 dv_2}{2\pi} \frac{1}{\sqrt{1 - w^2(r)}} \exp \left[ -\frac{\nu_1^2 + \nu_2^2 - 2w(r)\nu_1\nu_2}{2(1 - w^2(r))} \right]
\]

\[
\begin{aligned}
\beta &= \int d\delta P(\delta) \Theta(\delta - \delta_{\text{th}}) = \frac{1}{2} \text{erfc} \left( \frac{\nu}{\sqrt{2}} \right) \\
1 + \xi_{\text{th}}(r) &= \frac{\rho_{\text{th}}(\mathbf{x}_1) \rho_{\text{th}}(\mathbf{x}_2)}{\bar{\rho}_{\text{th}}^2} = \frac{1}{\beta^2} \int_\nu^{\infty} \frac{dv_1 dv_2}{2\pi} \frac{1}{\sqrt{1 - w^2(r)}} \exp \left[ -\frac{\nu_1^2 + \nu_2^2 - 2w(r)\nu_1\nu_2}{2(1 - w^2(r))} \right]
\end{aligned}
\]

Limiting cases

- large-separation limit

\[ 1 + \xi_{pk}(r) \approx e^{w(r)}\nu^2, \quad w(r) \ll \nu^{-1} \ll 1 \]


\[ \xi_{pk}(r) \approx \nu^2 w(r) = \left( \frac{\nu}{\sigma} \right)^2 \xi(r), \quad w(r) \ll \nu^{-2} \ll 1 \]

- (enhancement of correlation function)

Enhancement of correlation function

Borgani and Guzzo, Nature 409, 39 (2001)
Higher-order theory of peak clustering
Peak theory

**BBKS:**

- Thresholded regions ≠ Peaks

\[ n_{pk} = \Theta(\alpha - \nu) \delta_{ij}^3(\alpha, i) \Theta(\lambda_3) | \text{det}(\alpha, ij) | \]

\[ \frac{\nu - \gamma x}{\sqrt{2(1 - \gamma^2)}} \]

\[ f(x) = \frac{x}{2} (x^2 - 3) \left( \text{erf} \left( \frac{1}{2} \sqrt{\frac{5}{2}} x \right) + \text{erf} \left( \sqrt{\frac{5}{2}} x \right) \right) + \frac{2}{5\pi} \left[ \left( \frac{x^2}{2} - \frac{8}{5} \right) e^{-5x^2/2} + \left( \frac{31}{4} x^2 + \frac{8}{5} \right) e^{-5x^2/8} \right] \]
Peak clustering

- Exact evaluation of the peak clustering is difficult
  - BBKS derived approximate peak clustering in a large-separation limit (w/o exclusion effect etc.)

- Correlation function of peaks:

\[
\langle n_{pk}(\mathbf{x}_1)n_{pk}(\mathbf{x}_2) \rangle = \\
\langle \Theta(\alpha(\mathbf{x}_1) - \nu)\delta_D^3(\alpha, i(\mathbf{x}_1))\Theta(\lambda_3(\mathbf{x}_1)|\det(\alpha, ij(\mathbf{x}_1))\rangle\Theta(\alpha(\mathbf{x}_2) - \nu)\delta_D^3(\alpha, i(\mathbf{x}_2))\Theta(\lambda_3(\mathbf{x}_2)|\det(\alpha, ij(\mathbf{x}_2))\rangle
\]

- \( \Rightarrow \) difficult !!

- Naively 20-dimensional integrations !!
Peak clustering

- iPT formalism of bias

\[
P_{pk}(k) = \sum_{n=1}^{\infty} \frac{1}{n!} \int_{k_1 \cdots k_n = \mathbf{k}} \left[ \hat{c}_n(k_1, \ldots, k_n) \right]^2 W^2(k_1 R) \cdots W^2(k_n R) P(k_1) \cdots P(k_n)
\]

\[
\hat{c}_1(k) = b_{10} + b_{01} k^2
\]

\[
\hat{c}_2(k_1, k_2) = b_{20} + b_{21} \left( k_1^2 + k_2^2 \right) + b_{02} k_1^2 k_2^2 - 2\chi_1(k_1 \cdot k_2) + \omega_{10} \left[ 3(k_1 \cdot k_2)^2 - k_1^2 k_2^2 \right]
\]

\[
\hat{c}_3(k_1, k_2, k_3) = b_{30} + b_{21} \left( k_1^2 + k_2^2 + k_3^2 \right) + b_{12} \left( k_1^2 k_2^2 + \text{cyc.} \right) + b_{03} k_1^2 k_2^2 k_3^2 - 2b_{01} \chi_1(k_1 \cdot k_2 + \text{cyc.})
\]

\[
\int_{k_1 \cdots k_n = \mathbf{k}} \cdots \equiv \int \frac{d^3k_1}{(2\pi)^3} \cdots \frac{d^3k_n}{(2\pi)^3} (2\pi)^3 \delta_D(k_1 \cdots k_n - \mathbf{k}) \cdots
\]

\[
k_{1 \cdots n} \equiv k_1 + \cdots + k_n
\]
Peak clustering

- Reducing multi-dimensional integrals
  - Rayleigh expansion & analytic angular integrations → combination of 1D FFT
- Schmittfull & Blah 2016, Schmittfull+ 2016
- Using Mathematica is critical
- 2nd order result agrees with a direct calculation by Desjacques et al. (2010)

\[ \xi^{(n)}_n(r) = \int \frac{k^2 dk}{2\pi^2} j_0(kr) P_L(k) W^2(kR) \]

\[ \xi^{(N)}_k(r) = \sum_{N=1}^{\infty} \frac{1}{N!} \xi^{(N)}_k(r), \]

\[ \xi^{(N)}_k(r) = \frac{1}{N!} \int \frac{k^2 dk}{2\pi^2} j_0(kr) P_L(k) W^2(kR) \]

\[ \xi^{(N)}_k(r) = \sum_{N=1}^{\infty} \frac{1}{N!} \xi^{(N)}_k(r), \]

\[ \xi^{(N)}_k(r) = \int \frac{k^2 dk}{2\pi^2} j_0(kr) P_L(k) W^2(kR) \]
Peak clustering: Large-scale structure (Preliminary)

- Correlation function of peaks (CDM+baryons spectrum)

\[ \nu = 2.5 \text{ peaks} \]
Peak clustering: Large-scale structure (Preliminary)

- Power spectrum of peaks (CDM+baryons spectrum)

\[ \nu = 2.5 \text{ peaks} \]
Peak exclusion effect?

negative correlation
~ below 3 x smoothing scale

Baldauf et al.
1D MC
Summary

- Higher-order perturbation theory of peak clustering
- With iPT formalism, higher-order terms are systematically derived
- We need only 1D FFT to evaluate higher-order terms (while the number of terms are huge for higher orders, use of Mathematica helps)