LSS as a probe of physics

I. Consistency relations

II. Lyman-alpha power spectrum

Kyoto PTchat
2019

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I- CONSISTENCY RELATIONS

Go beyond PT and phenomenological models by deriving exact results without explicitly solving the dynamics.

- BBGKY: use the explicit equation of motion $$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

relation between n and (n+1) correlations

$$\frac{\partial}{\partial t}\langle f_1 ... f_n \rangle + \frac{\partial}{\partial \mathbf{x}}\langle f_1 ... f_n \mathbf{v} \rangle + \int d^3x d^3v \frac{\mathbf{x}}{x^3} \cdot \frac{\partial}{\partial \mathbf{v}} \langle f_1 ... f_{n+1} \rangle = 0$$

- Consistency relations: use symmetries of the system

More general

- lose details of the dynamics
- also applies to biased tracers
- remains valid whatever baryonic effects

test of general physical principles.

constrain models.

Kinematic consistency relations

Kehagias & Riotto (2013), Kehagias et al(2013), Peloso & Pietroni (2013a,b),
Creminelli et al. (2013a,b,c), P.V. (2013), P.V., Taruya and Nishimichi (2017)

A) Correlation and response functions

1311.1236

A general property for systems parameterized by a Gaussian field:

1) a Gaussian field: $\varphi(x)$

2) nonlinear functionals: $\rho_1, \quad \rho_2, ..., \rho_n$

We consider the mixed correlation:

$$C^{\ell,n} = \langle \varphi(x_1) \dots \varphi(x_\ell)\, \rho_1 \dots \rho_n \rangle = \int \mathcal{D}\varphi \; e^{-(1/2)\varphi \cdot C_0^{-1} \cdot \varphi} \; \varphi(x_1) \dots \varphi(x_\ell) \; \rho_1 \dots \rho_n$$

integrations by parts

$$C^{\ell,n} = C_0(x_1, x'_1) \dots C_0(x_\ell, x'_\ell) \cdot R^{\ell,n}(x'_1, .., x'_\ell)$$

Response function:

$$R^{\ell,n}(x_1, .., x_\ell) = \langle \frac{\mathcal{D}^\ell [\rho_1 \dots \rho_n]}{\mathcal{D}\varphi(x_1) .. \mathcal{D}\varphi(x_\ell)} \rangle$$

In the cosmological case, we consider the density field:

$$\langle \tilde{\delta}_{L0}(\mathbf{k}'_1)..\tilde{\delta}_{L0}(\mathbf{k}'_\ell)\,\tilde{\delta}(\mathbf{k}_1,t_1)..\tilde{\delta}(\mathbf{k}_n,t_n)\rangle = P_{L0}(k'_1)..P_{L0}(k'_\ell)\ \left\langle \frac{\mathcal{D}^\ell[\tilde{\delta}(\mathbf{k}_1,t_1)..\tilde{\delta}(\mathbf{k}_n,t_n)]}{\mathcal{D}\tilde{\delta}_{L0}(-\mathbf{k}'_1)..\mathcal{D}\tilde{\delta}_{L0}(-\mathbf{k}'_\ell)} \right\rangle$$

On large scales, or at early times, we recover the linear regime

$$k'_i \ll k_L : \qquad \langle \tilde{\delta}_{L0}(\mathbf{k}'_1)..\tilde{\delta}_{L0}(\mathbf{k}'_\ell)\,\tilde{\delta}(\mathbf{k}_1,t_1)..\tilde{\delta}(\mathbf{k}_n,t_n)\rangle \to \langle \tilde{\delta}(\mathbf{k}'_1,t'_1)..\tilde{\delta}(\mathbf{k}'_\ell,t'_\ell)\,\tilde{\delta}(\mathbf{k}_1,t_1)..\tilde{\delta}(\mathbf{k}_n,t_n)\rangle$$

We obtain the squeezed density correlation if we can evaluate the response function

$\Longrightarrow$ "CONSISTENCY RELATIONS"

B) Derivation of the kinematic consistency relations

A consequence of a symmetry of the system associated with the equivalence principle:

all particles/structures fall in the same fashion in a gravitational potential.

From a solution $\{\delta(\mathbf{x},t), \mathbf{v}(\mathbf{x},t), \Phi(\mathbf{x},t)\}$

we can build a new solution that corresponds to a uniform time-dependent translation,

$$\mathbf{x}' = \mathbf{x} - \mathbf{n}(\tau), \quad \mathbf{v}' = \mathbf{v} - \dot{\mathbf{n}}(\tau), \quad \delta' = \delta, \quad \Phi' = \Phi + (\ddot{\mathbf{n}} + \mathcal{H}\dot{\mathbf{n}}) \cdot \mathbf{x}'$$

We can absorb in this fashion, through a change of variable, the impact of a large-scale gravitational potential, which has a constant gradient at lowest order.

$k' \to 0 \qquad \delta(\mathbf{x},t) \to \delta(\mathbf{x}+\Delta\mathbf{x},t) \qquad$ with $\qquad \Delta\mathbf{x} = D_+(t) \int d\mathbf{k}' \Delta\tilde{\delta}_{L0}(\mathbf{k}') \frac{i\mathbf{k}'}{k'^2}$

$\tilde{\delta}(\mathbf{k}) \to \tilde{\delta}(\mathbf{k}) e^{-i\mathbf{k}\cdot\Delta\mathbf{x}}$

$$R^{1,n}_{k'\to 0} = \langle \tilde{\delta}(\mathbf{k}_1,t_1)..\tilde{\delta}(\mathbf{k}_n,t_n)\rangle \sum_{i=1}^{n} \frac{\mathbf{k}_i \cdot \mathbf{k}'}{k'^2} \bar{D}_+(t_i)$$

From the response function, we obtain the consistency relations:

$$\langle\tilde{\delta}(\mathbf{k}'_1,t'_1)..\tilde{\delta}(\mathbf{k}'_\ell,t'_\ell)\,\tilde{\delta}(\mathbf{k}_1,t_1)..\tilde{\delta}(\mathbf{k}_n,t_n)\rangle'_{k'_j\to 0} = P_L(k'_1,t'_1)...P_L(k'_\ell,t'_\ell)\langle\tilde{\delta}(\mathbf{k}_1,t_1)..\tilde{\delta}(\mathbf{k}_n,t_n)\rangle' \;\times\prod_{j=1}^{\ell}\left(-\sum_{i=1}^{n}\frac{\mathbf{k}_i\cdot\mathbf{k}'_j}{k'^2_j}\frac{\bar{D}_+(t_i)}{\bar{D}_+(t'_j)}\right)$$

Lowest-order case, bispectrum,

$$\lim_{k'\to 0} B(k',t';k_1,t_1;k_2,t_2) = -P_L(k',t')P(k_1;t_1,t_2)\left(\frac{\mathbf{k}_1\cdot\mathbf{k}'}{k'^2}\frac{\bar{D}_+(t_1)}{\bar{D}_+(t')}+\frac{\mathbf{k}_2\cdot\mathbf{k}'}{k'^2}\frac{\bar{D}_+(t_2)}{\bar{D}_+(t')}\right)$$

These relations vanish at equal times, because they merely express how small scales are uniformly transported by large-scale modes

These exact relations can be generalized to multi-fluid cases.

They remain valid for baryons, galaxies, ..., independently of small-scale physics.

These consistency relations rely on the following conditions:

- Gaussian initial conditions
- equivalence principle
- separation of scales

Exact results $\Rightarrow$ test of Gaussianity, of General Relativity, constraints on models

Null test: 0 = 0 at equal times, if Gaussian initial conditions and GR.

C) Non-Gaussian initial conditions

If the initial density field is a nonlinear function of a Gaussian field:

$$\delta_{L0}(\mathbf{k}) = \chi_0(\mathbf{k}) + \sum_{n=2}^{\infty} \int \prod_{i=1}^{n} d\mathbf{k}_i \delta_D\left(\mathbf{k} - \sum_{i=1}^{n} \mathbf{k}_i\right) \times f_{\mathrm{NL}\,0}^{(n)}(\mathbf{k}_1, \ldots, \mathbf{k}_n) \prod_{i=1}^{n} \chi_0(\mathbf{k}_i),$$

or its PDF is non-Gaussian:

$$\mathcal{P}(\delta_{L0}) = e^{-\int d\mathbf{k}\,\delta_{L0}(\mathbf{k})\delta_{L0}(-\mathbf{k})/2P_{\chi_0}(k)} \left[1 + \sum_{n=2}^{\infty} \int \prod_{i=1}^{n} d\mathbf{k}_i \times \delta_D\left(\sum_{i=1}^{n} \mathbf{k}_i\right) S_n(\mathbf{k}_1, \ldots, \mathbf{k}_n) \prod_{i=1}^{n} \delta_{L0}(\mathbf{k}_i)\right]$$

the consistency relations take a more complicated form:

$$\langle \delta_{L0}(\mathbf{k}') \prod_{j=1}^{m} \delta(\mathbf{k}_j, \tau_j) \rangle'_{k' \to 0} = -P_{\chi_0}(k') \left\langle \prod_{j=1}^{m} \delta(\mathbf{k}_j, \tau_j) \right\rangle' \sum_{j=1}^{m} D_+(\tau_j) \frac{\mathbf{k}_j \cdot \mathbf{k}'}{k'^2} + P_{\chi_0}(k') \sum_{n=2}^{\infty} n \int \prod_{i=1}^{n-1} d\mathbf{k}'_i$$
$$\times\, \delta_D\left(\mathbf{k}' - \sum_{i=1}^{n-1} \mathbf{k}'_i\right) S_n(-\mathbf{k}', \mathbf{k}'_1, \ldots, \mathbf{k}'_{n-1}) \left\langle \prod_{i=1}^{n-1} \delta_{L0}(\mathbf{k}'_i) \prod_{j=1}^{m} \delta(\mathbf{k}_j, \tau_j) \right\rangle'.$$

new terms that do not vanish at equal times

Angular-averaged consistency relations

P.V. (2013), Kehagias et al.(2013), T. Nishimichi & P.V. (2014,2015)

A) Approximate symmetry

1311.4286

To go beyond the leading-order relations, or to obtain new relations, we must find additional symmetries.

If we make the change of variables $\eta = \ln D_+, \quad \mathbf{v} = \dot{a} f \mathbf{u}, \quad \Phi = (\dot{a} f)^2 \varphi$ where $f = \frac{d \ln D_+}{d \ln a}$

the equations of motion read as

$$\frac{\partial \delta}{\partial \eta} + \nabla \cdot [(1+\delta)\mathbf{u}] = 0, \qquad \frac{\partial \mathbf{u}}{\partial \eta} + \left(\frac{3\Omega_{\rm m}}{2f^2} - 1 \right) \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla \varphi, \qquad \nabla^2 \varphi = \frac{3\Omega_{\rm m}}{2f^2} \, \delta,$$

Within the approximation $\Omega_{\rm m}/f^2 \simeq 1$ all explicit dependence on cosmology disappears.

→ approximate symmetry

This remains true beyond shell crossing. The equation of motion of the particle trajectories reads as:

$$\frac{\partial^2 \mathbf{x}}{\partial \eta^2} + \left(\frac{3\Omega_{\rm m}}{2f^2} - 1 \right) \frac{\partial \mathbf{x}}{\partial \eta} = -\nabla \varphi$$

B) Angular averaging

To get rid of the leading-order kinematic effect, associated with the uniform motion of small-scale structures, we integrate over the angles of the soft modes.

$$\tilde{C}_W^n = \int d\mathbf{k}' \tilde{W}(k') \, \langle \tilde{\delta}_{L0}(\mathbf{k}') \tilde{\delta}(\mathbf{k}_1, t_1)..\tilde{\delta}(\mathbf{k}_n, t_n) \rangle = \int d\mathbf{k}' \tilde{W}(k') \, P_{L0}(k') \langle \frac{\mathcal{D}[\tilde{\delta}(\mathbf{k}_1, t_1)..\tilde{\delta}(\mathbf{k}_n, t_n)]}{\mathcal{D}\tilde{\delta}_{L0}(-\mathbf{k}')} \rangle$$

This means that we need to compute the impact of a large-scale spherical overdensity onto small-scale fluctuations.

This corresponds to a change of the background matter density: change of the cosmological parameter $\Omega_{\rm m}$

➡ The approximate symmetry $\Omega_{\rm m}/f^2 \simeq 1$ allows us to describe the effect of a change of cosmological background

$$\tilde{\delta}_{\epsilon_0}(\mathbf{k}, t) = \tilde{\delta}[(1-\epsilon)\mathbf{k}, D_{+\epsilon_0}] + 3\epsilon\delta_D(\mathbf{k})$$

$$\left. \frac{\partial \tilde{\delta}(\mathbf{k}, t)}{\partial \epsilon_0} \right|_{\epsilon_0 = 0} = D_+(t) \left[\frac{13}{7} \frac{\partial \tilde{\delta}}{\partial \ln D_+} - \mathbf{k} \cdot \frac{\partial \tilde{\delta}}{\partial \mathbf{k}} \right]$$

This eventually leads to the angular-averaged consistency relations:

$$\int \frac{d\mathbf{\Omega}_{\mathbf{k}'}}{4\pi} \langle \tilde{\delta}(\mathbf{k}',t)\tilde{\delta}(\mathbf{k}_1,t)..\tilde{\delta}(\mathbf{k}_n,t)\rangle'_{k'\to 0} = P_L(k',t)\left[1+\frac{13}{21}\frac{\partial}{\partial \ln D_+} - \frac{1}{3}\sum_{i=1}^{n}\frac{\partial}{\partial \ln k_i}\right] \langle \tilde{\delta}(\mathbf{k}_1,t)..\tilde{\delta}(\mathbf{k}_n,t)\rangle'$$

These relations no longer vanish at equal times.

The lowest-order relation, for the bispectrum, reads as:

$$\int \frac{d\mathbf{\Omega}_{\mathbf{k}'}}{4\pi}\; B\left(\mathbf{k}',\mathbf{k}-\frac{\mathbf{k}'}{2},-\mathbf{k}-\frac{\mathbf{k}'}{2};t\right)_{k'\to 0} = P_L(k',t)\left[1+\frac{13}{21}\frac{\partial}{\partial \ln D_+}-\frac{1}{3}\frac{\partial}{\partial \ln k}\right]P(k,t)$$

Numerical check:

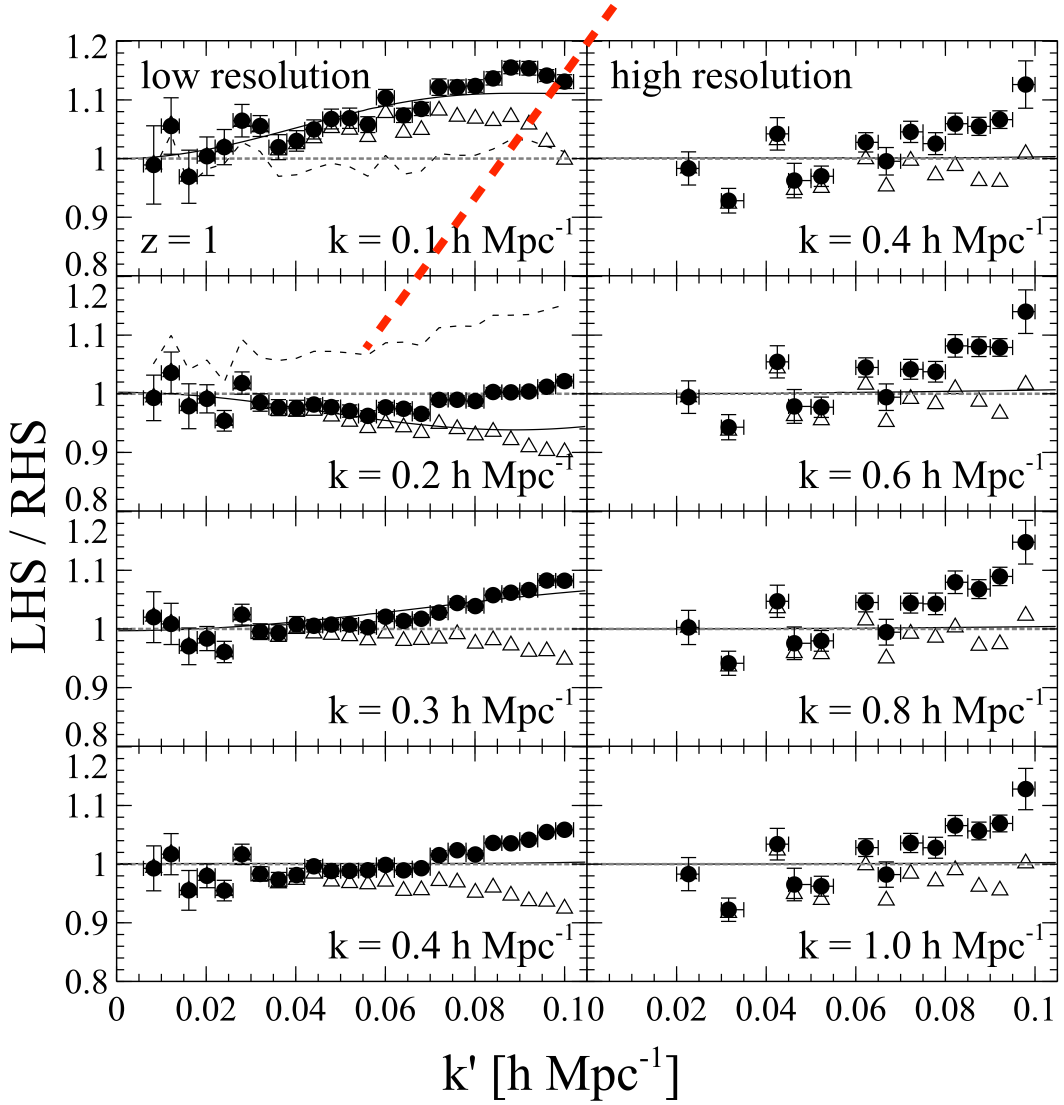
1402.3293

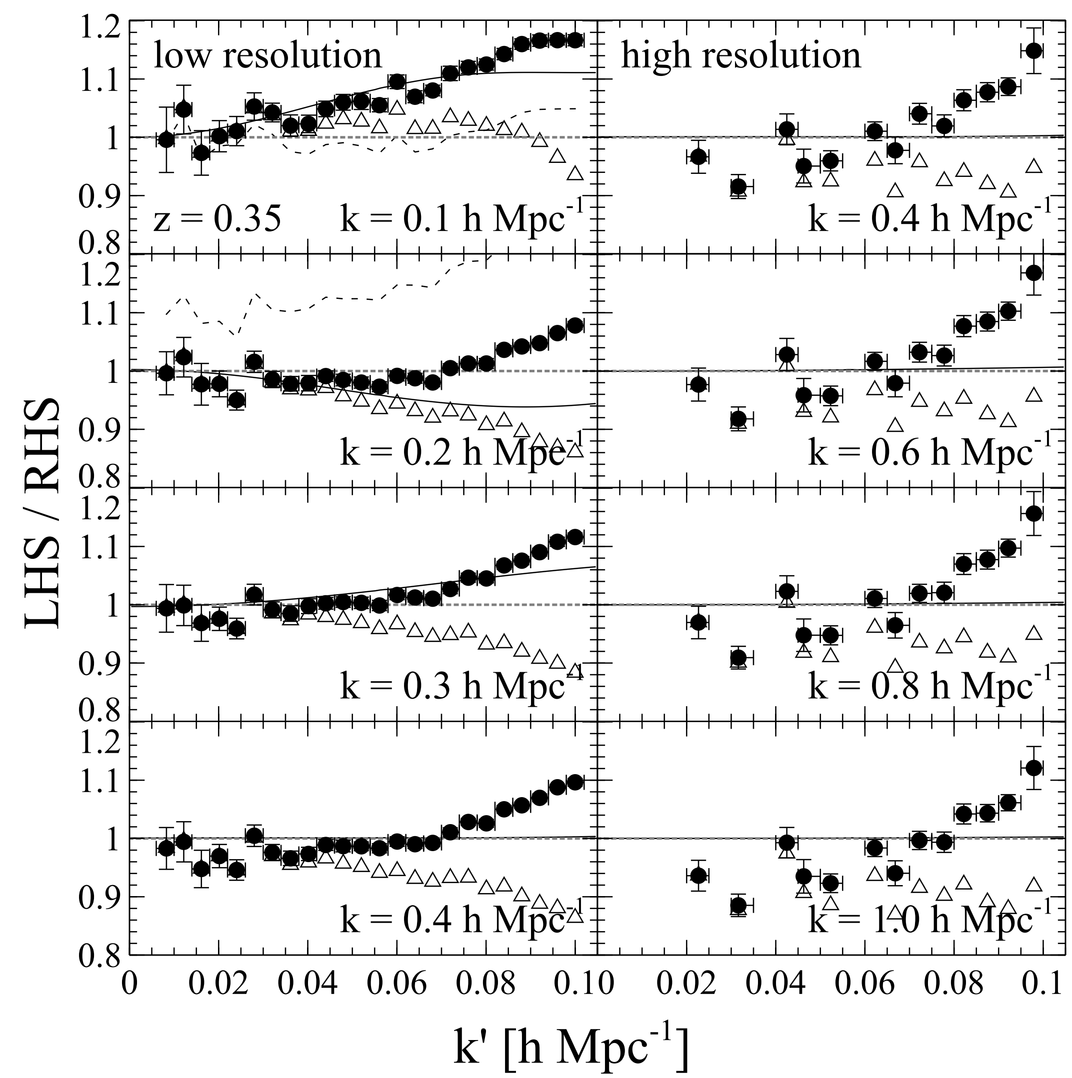
ratio of the nonlinear bispectrum to the consistency relation result, given by a product of one linear power spectrum and one nonlinear power spectrum.

consistency relation = 1

lowest-order PT result (out of the plot otherwise)

this approximate consistency relation significantly improves over lowest-order PT result, and goes up to k~1 h/Mpc





C) Redshift space

In redshift space the relations are more intricate:

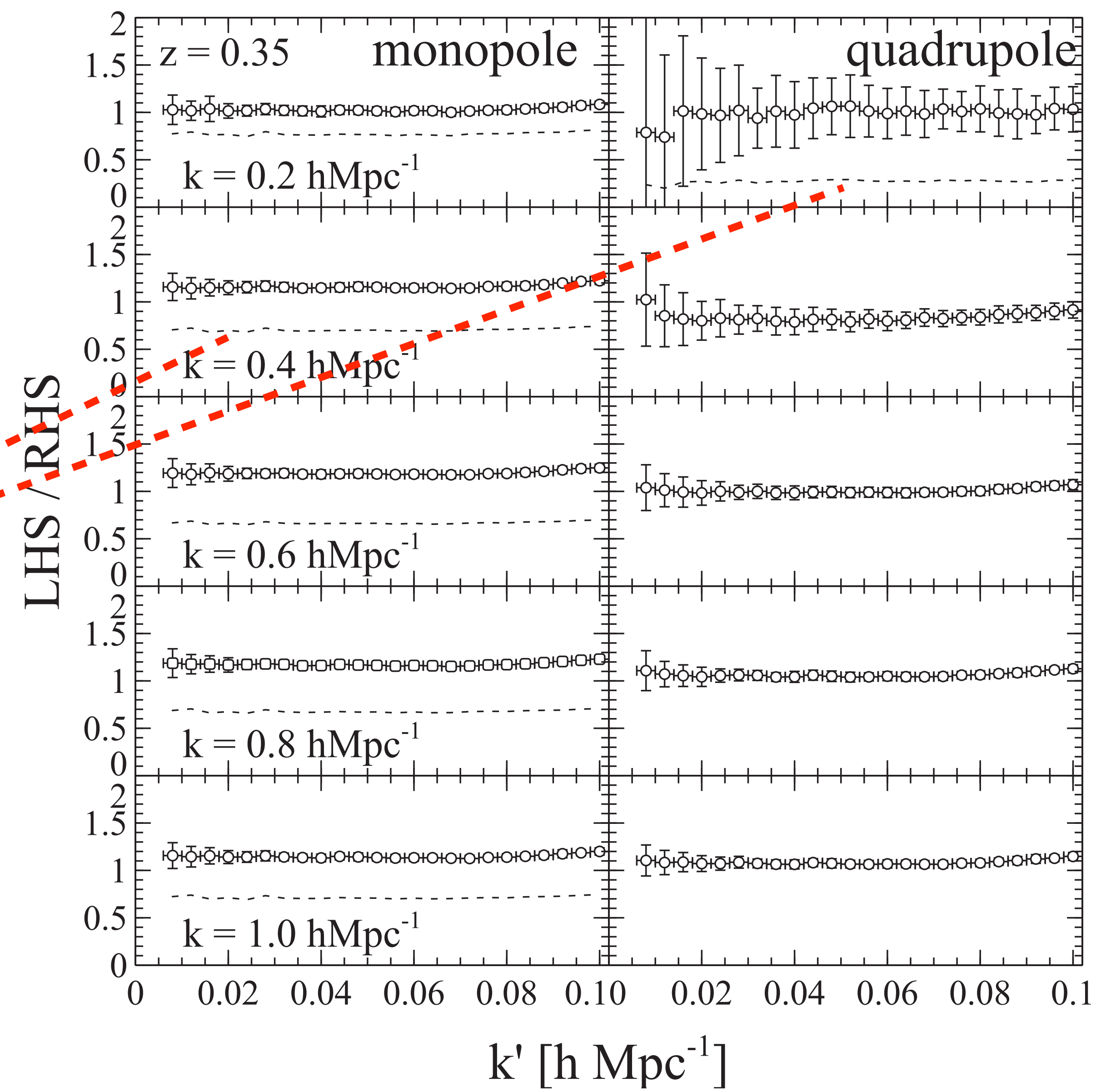
$$\int \frac{d\Omega_{k'}}{4\pi} \left\langle \frac{\tilde{\delta}^s(\boldsymbol{k}')}{1+f\mu'^2} \tilde{\delta}^s(\boldsymbol{k}_1)\ldots\tilde{\delta}^s(\boldsymbol{k}_n) \right\rangle'_{k'\to 0} = P_L(k') \left[1 + \frac{f}{3} + \frac{13}{21}\frac{\partial}{\partial \ln D_+} + \left(\frac{13}{7}+f\right)\frac{f}{3}\frac{\partial}{\partial f} - \sum_{i=1}^{n} \frac{k_i}{3}\frac{\partial}{\partial k_i} - f\sum_{i=1}^{n}\frac{k_{ri}}{3}\frac{\partial}{\partial k_{ri}} \right] \langle \tilde{\delta}^s(\boldsymbol{k}_1)\ldots\tilde{\delta}^s(\boldsymbol{k}_n)\rangle'.$$

Bispectrum monopole:

$$\int_{-1}^{1}\frac{d\mu}{2}\int\frac{d\Omega_{k'}}{4\pi}\frac{B^s_{k'\to 0}}{1+f\mu'^2} = P_L(k')\left\{ \left[1+\frac{f}{3}+\frac{13}{21}\frac{\partial}{\partial \ln D_+}+\left(\frac{13}{7}+f\right)\frac{f}{3}\frac{\partial}{\partial f}-\frac{1}{3}\frac{\partial}{\partial \ln k}\right]P^s_0(k) - \frac{2f}{15}P^s_2(k) - \frac{f}{3}\frac{\partial}{\partial \ln k}\left[\frac{1}{3}P^s_0(k)+\frac{2}{15}P^s_2(k)\right]\right\}$$

consistency relation = 1

lowest-order PT result



this approximate consistency relation significantly improves over lowest-order PT result, and goes up to k~1 h/Mpc

Density-velocity consistency relations

L. Rizzo, D. Mota and P.V. (2016,2017)

A) Non-zero equal-time relation

Let us go back to the exact kinematic consistency relations.

For a long-wavelength perturbation, we had the transformation:

$$\mathbf{x}(\mathbf{q},\tau) \rightarrow \hat{\mathbf{x}}(\mathbf{q},\tau) = \mathbf{x}(\mathbf{q},\tau) + D_+(\tau)\Delta\Psi_{L0}(\mathbf{q}),$$

This gave us for the density contrast:

$$\tilde{\delta}(\mathbf{k},\tau) \rightarrow \hat{\tilde{\delta}}(\mathbf{k},\tau) = \tilde{\delta}(\mathbf{k},\tau)e^{-i\mathbf{k}\cdot D_+\Delta\Psi_{L0}} = \tilde{\delta}(\mathbf{k},\tau) - iD_+(\mathbf{k}\cdot\Delta\Psi_{L0})\tilde{\delta}(\mathbf{k},\tau),$$

This also gives for the velocity field:

$$\tilde{\mathbf{v}}(\mathbf{k},\tau) \rightarrow \hat{\tilde{\mathbf{v}}}(\mathbf{k},\tau) = \tilde{\mathbf{v}}(\mathbf{k},\tau) - iD_+(\mathbf{k}\cdot\Delta\Psi_{L0})\tilde{\mathbf{v}}(\mathbf{k},\tau) + \frac{dD_+}{d\tau}\Delta\Psi_{L0}\delta_D(\mathbf{k}),$$

uniform translation

change of the velocity amplitude

this effect will not disappear in equal-time statistics !

This Dirac term at k=0 will be relevant in composite operators:

$$\mathbf{p} = (1+\delta)\mathbf{v}, \qquad \tilde{\mathbf{p}}(\mathbf{k}) = \tilde{\mathbf{v}}(\mathbf{k}) + \int d\mathbf{k}_1 d\mathbf{k}_2 \delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\tilde{\delta}(\mathbf{k}_1)\tilde{\mathbf{v}}(\mathbf{k}_2).$$

This leads to:

$$\boxed{k'\rightarrow 0: \ \frac{\mathcal{D}\tilde{\mathbf{p}}(\mathbf{k})}{\mathcal{D}\tilde{\delta}_{L0}(\mathbf{k}')} = D_+\frac{\mathbf{k}\cdot\mathbf{k}'}{k'^2}\tilde{\mathbf{p}}(\mathbf{k}) + \frac{dD_+}{d\tau}i\frac{\mathbf{k}'}{k'^2}[\delta_D(\mathbf{k})+\tilde{\delta}(\mathbf{k})].}$$

Nonzero consistency relation at equal times:

$$\left\langle \tilde{\delta}(\mathbf{k}') \prod_{j=1}^{n} \tilde{\delta}(\mathbf{k}_j) \prod_{j=n+1}^{n+m} \tilde{\mathbf{p}}(\mathbf{k}_j) \right\rangle'_{k'\to 0} = -iP_L(k') \frac{d\ln D_+}{d\tau} \sum_{i=n+1}^{n+m} \left\langle \prod_{j=1}^{n} \tilde{\delta}(\mathbf{k}_j) \prod_{j=n+1}^{i-1} \tilde{\mathbf{p}}(\mathbf{k}_j) \left(\frac{\mathbf{k}'}{k'^2} [\delta_D(\mathbf{k}_i) + \tilde{\delta}(\mathbf{k}_i)] \right) \prod_{j=i+1}^{n+m} \tilde{\mathbf{p}}(\mathbf{k}_j) \right\rangle',$$

For the bispectrum:

$$\langle \tilde{\delta}(\mathbf{k}')\tilde{\delta}(\mathbf{k})\tilde{\mathbf{p}}(-\mathbf{k})\rangle'_{k'\to 0} = -i\frac{\mathbf{k}'}{k'^2}\frac{d\ln D_+}{d\tau}P_L(k')P(k),$$

Again, it also applies to biased tracers, independently of baryonic physics etc.....

$$\langle \tilde{\delta}(\mathbf{k}')\tilde{\delta}_g(\mathbf{k})\tilde{\mathbf{p}}_g(-\mathbf{k})\rangle'_{k'\to 0} = -i\frac{\mathbf{k}'}{k'^2}\frac{d\ln D_+}{d\tau}P_L(k')P_{\delta_g\delta_g}(k),$$

We also obtain for the divergence of the momentum field:

$$\lambda \equiv \nabla \cdot [(1+\delta)\mathbf{v}], \qquad \tilde{\lambda}(\mathbf{k}) = i\mathbf{k}\cdot\tilde{\mathbf{p}}(\mathbf{k}).$$

$$\langle \tilde{\delta}(\mathbf{k}')\tilde{\delta}_g(\mathbf{k})\tilde{\lambda}_g(-\mathbf{k})\rangle'_{k'\to 0} = -\frac{\mathbf{k}\cdot\mathbf{k}'}{k'^2}\frac{d\ln D_+}{d\tau}P_L(k')P_{\delta_g\delta_g}(k),$$

B) Link with observable quantities

1) ISW

Secondary CMB anisotropy due to the Integrated Sachs-Wolfe effect:

$$\Delta_{\mathrm{ISW}}(\boldsymbol{\theta}) = 2\int \mathrm{d}\eta\, \mathrm{e}^{-\tau(\eta)} \frac{\partial \Psi}{\partial \eta}[r, r\boldsymbol{\theta}; \eta],$$

This can be expressed in terms of the density field and its time derivative through the Poisson equation:

$$\frac{\partial \tilde{\Psi}}{\partial \eta} = \frac{4\pi \mathcal{G}_{\mathrm{N}} \bar{\rho}_0}{k^2 a}(\tilde{\lambda} + \mathcal{H}\tilde{\delta}),$$

with:

$$\lambda \equiv \nabla \cdot [(1+\delta)\boldsymbol{v}] = -\frac{\partial \delta}{\partial \eta}.$$

2) kSZ

Secondary CMB anisotropy due to the kinematic SZ effect:

$$\Delta_{\mathrm{kSZ}}(\boldsymbol{\theta}) = -\int \mathrm{d}\boldsymbol{l} \cdot \boldsymbol{v}_{\mathrm{e}} \sigma_{\mathrm{T}} n_{\mathrm{e}} \mathrm{e}^{-\tau} = \int \mathrm{d}\eta\, I_{\mathrm{kSZ}}(\eta) \boldsymbol{n}(\boldsymbol{\theta}) \cdot \boldsymbol{p}_{\mathrm{e}},$$

with: $I_{\mathrm{kSZ}}(\eta) = -\sigma_{\mathrm{T}} \bar{n}_{\mathrm{e}} a \mathrm{e}^{-\tau},$ $\quad n_{\mathrm{e}} \boldsymbol{v}_{\mathrm{e}} = \bar{n}_{\mathrm{e}}(1+\delta_{\mathrm{e}})\boldsymbol{v}_{\mathrm{e}} = \bar{n}_{\mathrm{e}} \boldsymbol{p}_{\mathrm{e}}.$

C) ISW consistency relations for 3-pt correlations

I) Galaxy-galaxy-ISW correlation

$$\xi_3(\delta^{\rm s}_{\rm g}, \delta^{\rm s}_{\rm g_1}, \Delta^{\rm s}_{\rm ISW_2}) = \langle \delta^{\rm s}_{\rm g}(\boldsymbol{\theta})\, \delta^{\rm s}_{\rm g_1}(\boldsymbol{\theta}_1)\, \Delta^{\rm s}_{\rm ISW_2}(\boldsymbol{\theta}_2)\rangle.$$

$$\Theta \gg \Theta_{\rm L}, \quad \Theta \gg \Theta_j, \quad |\boldsymbol{\theta} - \boldsymbol{\theta}_j| \gg |\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|. \quad \Rightarrow \quad k \ll k_{\rm L}, \quad k \ll k_j,$$

$$\xi_3 = \frac{(\boldsymbol{\theta} - \boldsymbol{\theta}_2)\cdot(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2)}{|\boldsymbol{\theta} - \boldsymbol{\theta}_2||\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|}(2\pi)^4 \int {\rm d}\eta\, b_{\rm g} I_{\rm g} I_{\rm g_1} I_{\rm ISW_2} \frac{{\rm d}\ln D}{{\rm d}\eta} \int_0^\infty {\rm d}k_\perp {\rm d}k_{1\perp}\, \tilde{W}_\Theta(k_\perp r)\tilde{W}_{\Theta_1}(k_{1\perp} r)\tilde{W}_{\Theta_2}(k_{1\perp} r)$$

$$\times P_{\rm L}(k_\perp, \eta) P_{\rm g_1,m}(k_{1\perp}, \eta) J_1(k_\perp r|\boldsymbol{\theta} - \boldsymbol{\theta}_2|)\; J_1(k_{1\perp} r|\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|),$$

- This is an explicit expression of the form: $\langle \delta_g \delta_g \Delta_{\rm ISW}\rangle = \langle \delta_g \delta\rangle_L \, \langle \delta_g \delta\rangle$

- Specific angular dependence (can be understood from symmetry).

2) Lensing-lensing-ISW correlation

Three-point correlation with the lensing convergence:

$$\xi_3(\kappa^{\rm s}, \kappa_1^{\rm s}, \Delta_{\rm ISW_2}^{\rm s}) = \langle \kappa^{\rm s}(\boldsymbol{\theta})\, \kappa_1^{\rm s}(\boldsymbol{\theta}_1)\, \Delta_{\rm ISW_2}^{\rm s}(\boldsymbol{\theta}_2)\rangle,$$

$$\xi_3 \;=\; \frac{(\boldsymbol{\theta}-\boldsymbol{\theta}_2)\cdot(\boldsymbol{\theta}_1-\boldsymbol{\theta}_2)}{|\boldsymbol{\theta}-\boldsymbol{\theta}_2||\boldsymbol{\theta}_1-\boldsymbol{\theta}_2|}(2\pi)^4 \int {\rm d}\eta\, I_\kappa I_{\kappa_1} I_{\rm ISW_2} \frac{{\rm d}\ln D}{{\rm d}\eta} \quad \int_0^\infty {\rm d}k_\perp {\rm d}k_{1\perp}\, \tilde{W}_\Theta(k_\perp r)\tilde{W}_{\Theta_1}(k_{1\perp} r)\tilde{W}_{\Theta_2}(k_{1\perp} r)$$

$$\times\, P_{\rm L}(k_\perp, \eta) P(k_{1\perp}, \eta) J_1(k_\perp r|\boldsymbol{\theta}-\boldsymbol{\theta}_2|)\, J_1(k_{1\perp} r|\boldsymbol{\theta}_1-\boldsymbol{\theta}_2|).$$

- This is an explicit expression of the form: $\langle \kappa\kappa\Delta_{\rm ISW}\rangle = P_L\; P$

- Specific angular dependence

C) kSZ consistency relations for 3-pt correlations

1) Galaxy-galaxy-kSZ correlation

$$\xi_3(\delta_{\rm g}^{\rm s}, \delta_{\rm g_1}^{\rm s}, \Delta_{\rm kSZ_2}^{\rm s}) = \langle \delta_{\rm g}^{\rm s}(\boldsymbol{\theta})\, \delta_{\rm g_1}^{\rm s}(\boldsymbol{\theta}_1)\, \Delta_{\rm kSZ_2}^{\rm s}(\boldsymbol{\theta}_2)\rangle,$$

$$\xi_{3\|}^{\|} = -(2\pi)^4 \int {\rm d}\eta \, \frac{\rm d}{{\rm d}\eta}\left[b_{\rm g} I_{\rm g} D\right] I_{\rm g_1} I_{\rm kSZ_2} \frac{{\rm d}D}{{\rm d}\eta} \int_0^\infty {\rm d}k_\perp {\rm d}k_{1\perp}\, \tilde{W}_\Theta(k_\perp r) \tilde{W}_{\Theta_1}(k_{1\perp} r) \tilde{W}_{\Theta_2}(k_{1\perp} r)$$

$$\times \frac{k_{1\perp}}{k_\perp} P_{L0}(k_\perp) P_{\rm g_1,e}(k_{1\perp}, \eta) J_0(k_\perp r|\boldsymbol{\theta} - \boldsymbol{\theta}_2|)\, J_0(k_{1\perp} r|\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|),$$

+....

This is an explicit expression of the form:

$$\langle \delta_g \delta_g \Delta_{\rm kSZ} \rangle = \langle \delta_g \delta \rangle_L \, \langle \delta_g \delta_e \rangle$$

Not as convenient as the ISW correlations, because of the mixed galaxy-free electrons power spectrum.

II- LYMAN-ALPHA POWER SPECTRUM AS A PROBE OF MODIFIED GRAVITY

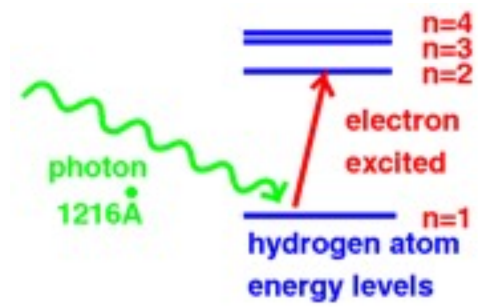
Ph. Brax, P. Valageas

JCAP 01 (2019) 049
arXiv:1810.06661

A. LYMAN-ALPHA FOREST PHYSICS

J. Cohn webpage

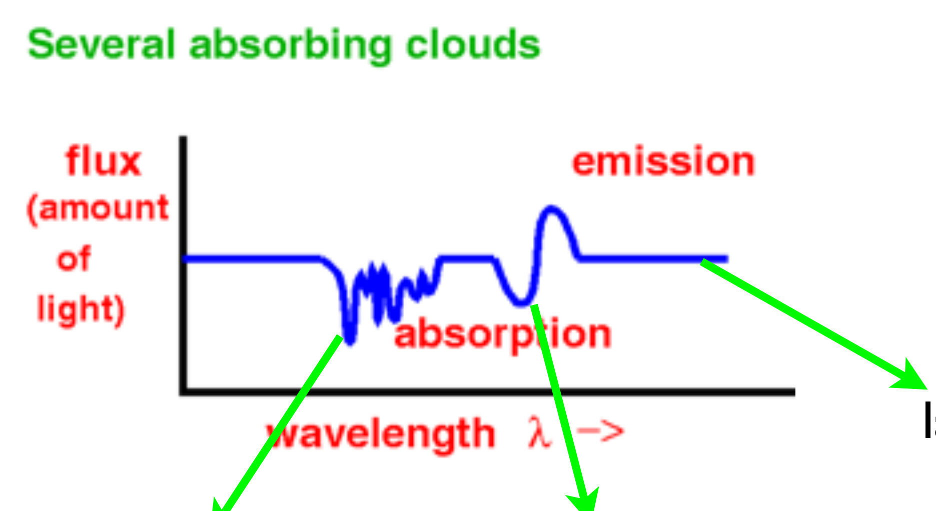
Absorption of light by neutral hydrogen



Absorption along the line of sight by clouds (or density fluctuations)

Because of the Hubble expansion, the wavelength of the light ray from the source grows with time

The absorption takes place at the position along the line of sight where the photon was at 1216 A



absorbed close to us

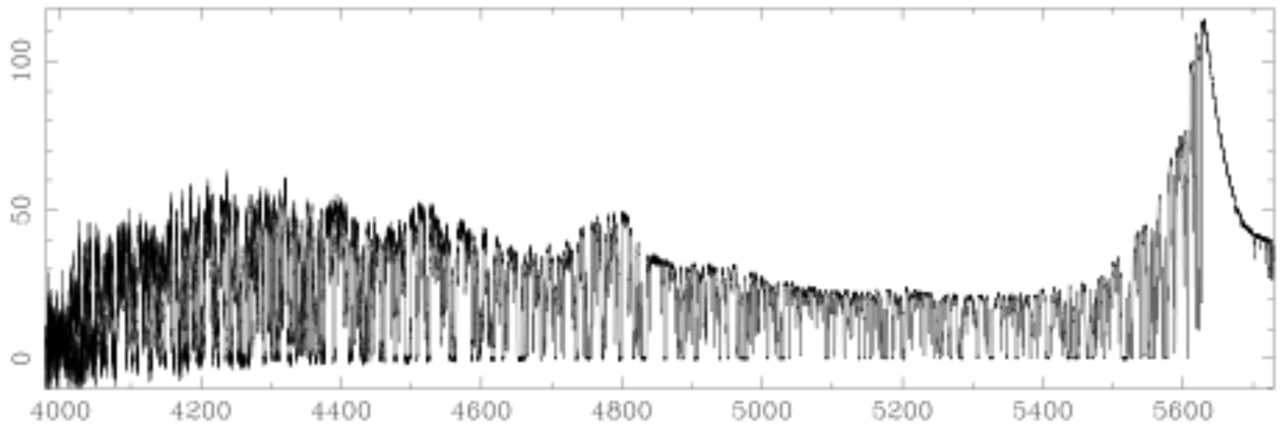
absorbed close to the source

large λ, never absorbed

the measured spectrum is a map of the density along the line of sight !

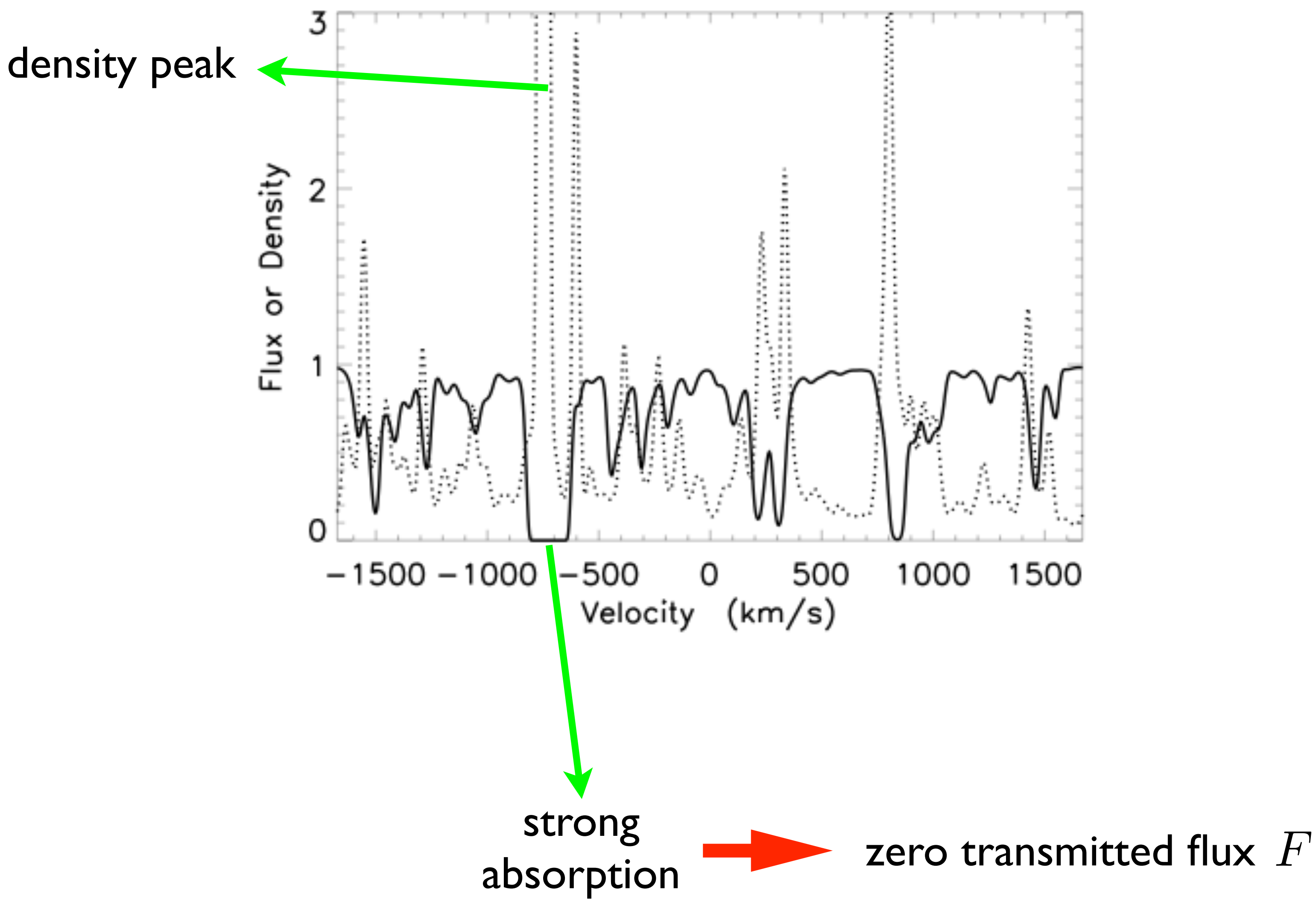
M. White

Spectrum of the light received from a distant quasar



QSO 1422+23

Spectrum '=' density



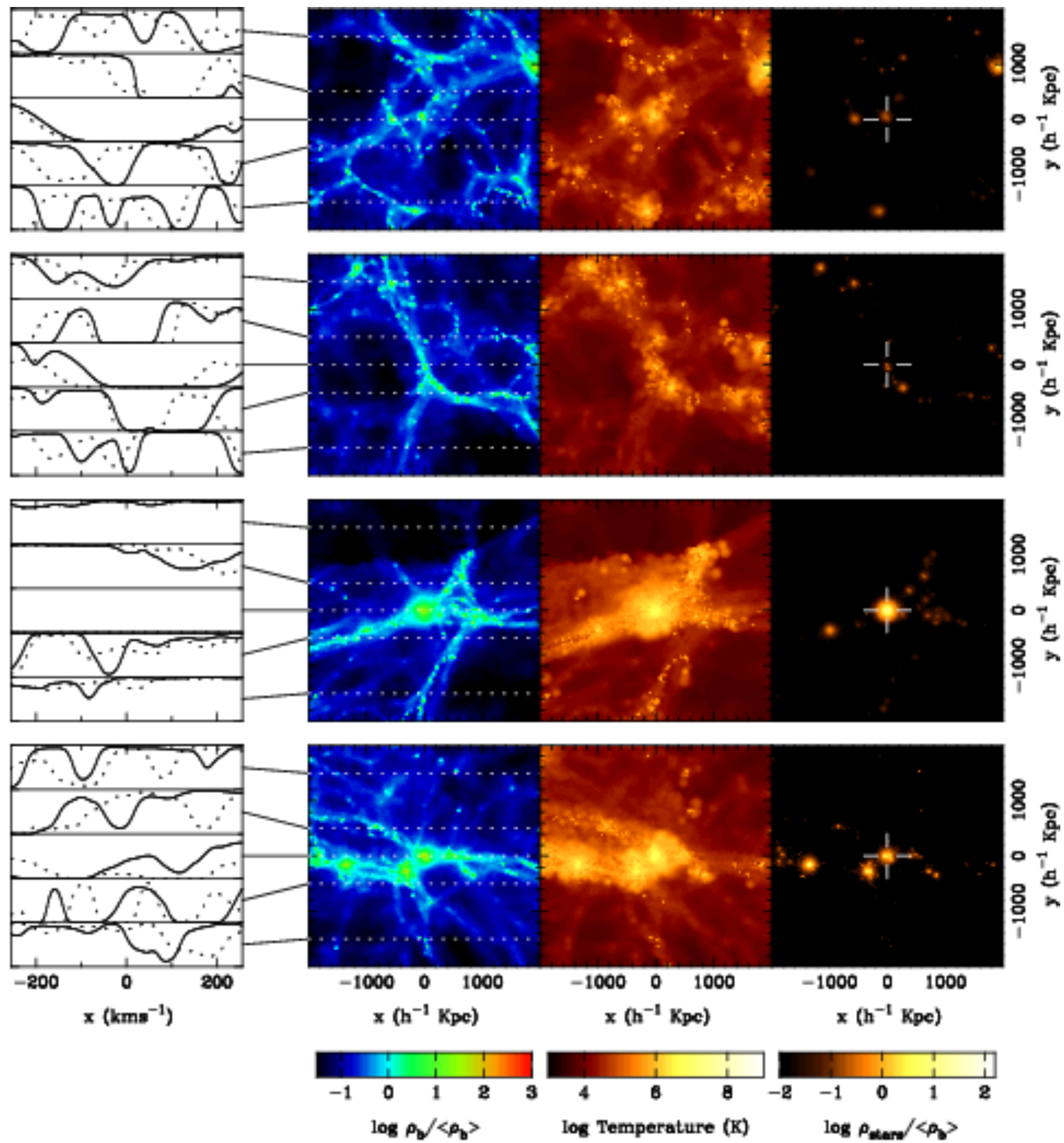
M. White

Galaxy-IGM connection

Probe of the IGM

Matter distribution on large scales

Moderate density fluctuations in filaments and pancakes, outside of galaxies (which correspond to Lyman-limit or damped systems)



B- MODIFIED-GRAVITY THEORIES

1) 5th force

Scalar-tensor theories: add a new scalar field φ

5th force $\rightarrow$ typically amplifies gravity and the growth of perturbations.

On small scales, quasi-static approximation, linear regime:

$$\tilde{\delta}'' + \mathcal{H}\tilde{\delta}' - \frac{3\Omega_{\rm m}}{2}\mathcal{H}^2\mu(k,a)\tilde{\delta} = 0,$$

$$\mu(k,a) = 1 + \epsilon(k,a) \qquad \epsilon(k,a) = \frac{2\beta^2(a)}{1+\frac{m^2(a)a^2}{k^2}}.$$

coupling strength

mass (1/range) of the scalar field

2) f(R) theories

$$S_{f(R)} = \frac{1}{16\pi\mathcal{G}_{\rm N}}\int d^4x\sqrt{-g}\, f(R)$$

$$f(R) = R - 2\Lambda^2 - f_{R_0}\frac{R_0^2}{R},$$

$$f_{R_0} = -10^{-4},\ -10^{-5}\ \text{and}\ -10^{-6}.$$

Strong lensing, dynamics in dwarfs, equiv. principle in solar system:

$$|f_{R_0}| \lesssim 10^{-6}$$

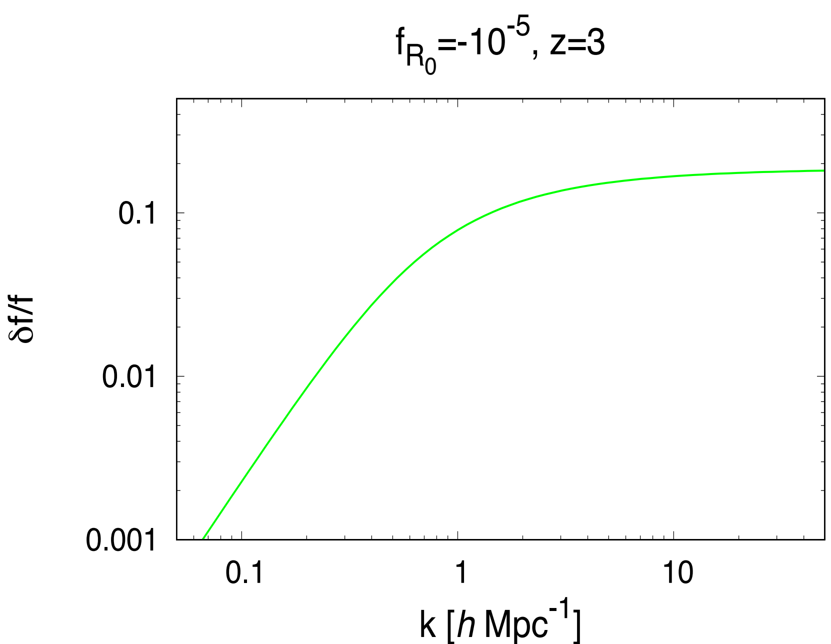
Background=LCDM, perturbations slightly amplified.

$$\boxed{\beta = 1/\sqrt{6}}$$

$$\boxed{m_0 = \frac{H_0}{c}\sqrt{\frac{\Omega_{m0}+4\Omega_{\Lambda 0}}{(n+1)|f_{R_0}|}} \sim 1\mathrm{Mpc}^{-1}}$$

Relative deviation of the linear growth-rate from LCDM:

$$f(k,a) = \frac{\partial \ln D_+}{\partial \ln a}(k,a).$$



3) K-mouflage model

$$S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{\tilde{M}_{\rm Pl}^2}{2}\tilde{R} + \tilde{\mathcal{L}}_\varphi(\varphi)\right] + \int d^4x\sqrt{-g}\mathcal{L}_{\rm m}(\psi_{\rm m}^{(i)}, g_{\mu\nu})$$

coupling of matter to the scalar field: $g_{\mu\nu} = A^2(\varphi)\tilde{g}_{\mu\nu}$. $A(\varphi) = 1 + \frac{\beta\varphi}{\tilde{M}_{\rm Pl}} + \ldots,$

nonlinear kinetic function (screening): $\tilde{\mathcal{L}}_\varphi(\varphi) = \mathcal{M}^4 K(\tilde{\chi})$ with $\tilde{\chi} = -\frac{1}{2\mathcal{M}^4}\tilde{\nabla}^\mu\varphi\tilde{\nabla}_\mu\varphi.$

linear (unscreened) regime: $\tilde{\chi} \to 0:\ K(\tilde{\chi}) \simeq -1 + \tilde{\chi} + \ldots,$

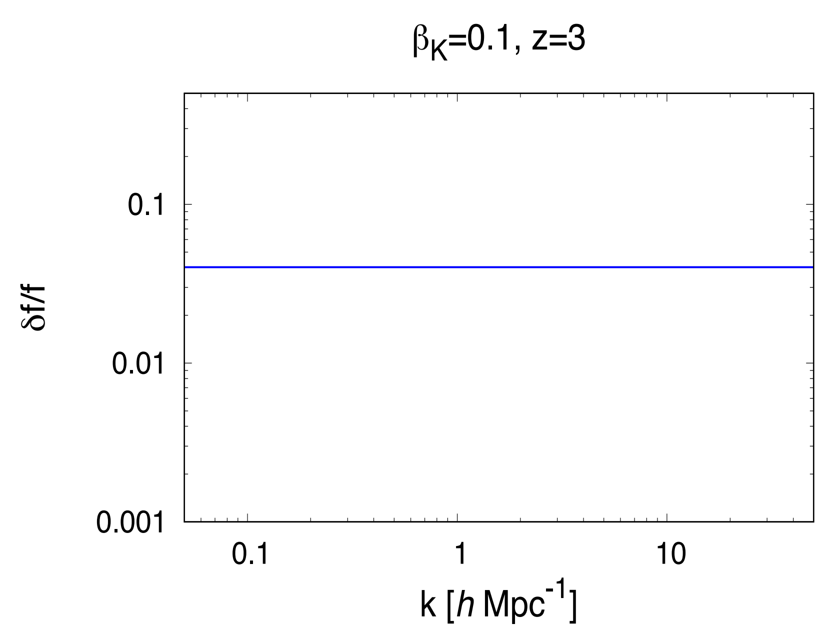
no potential → zero mass → long range, scale-independent

Both the background and the perturbations are slightly perturbed.

CMB, Solar System: $\beta \lesssim 0.1$

Relative deviation of the linear growth-rate from LCDM:

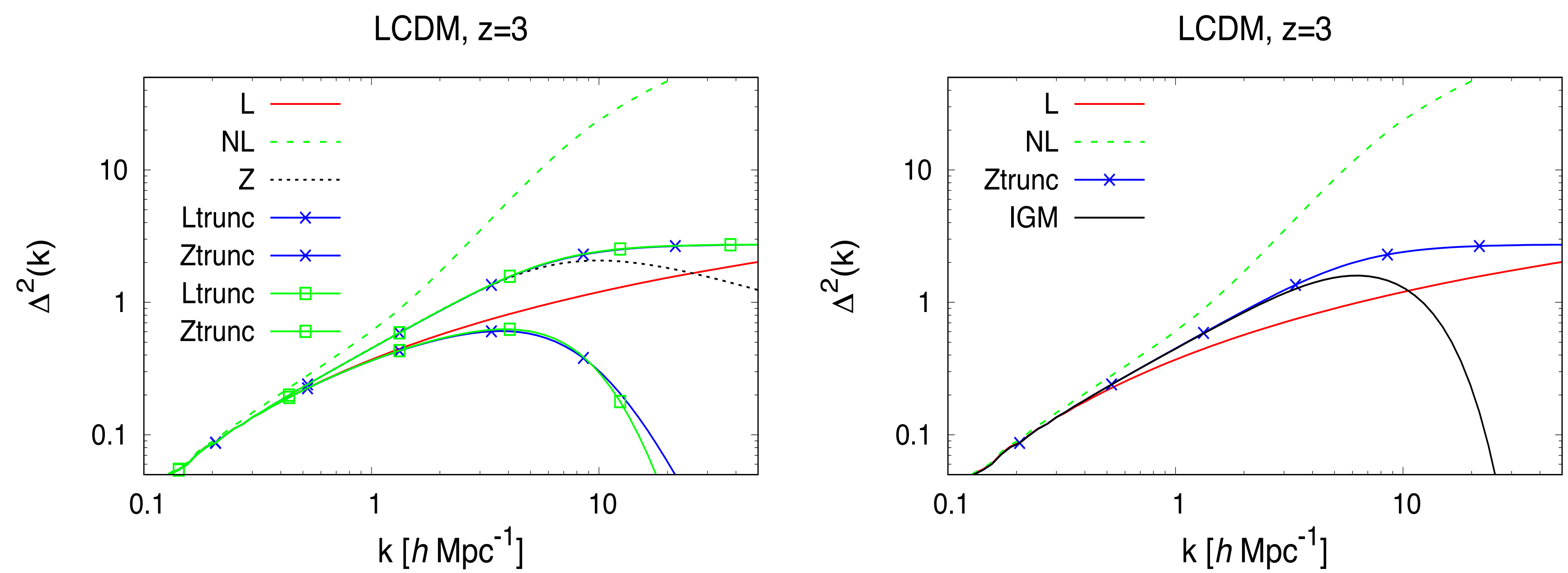
$$f(k,a) = \frac{\partial \ln D_+}{\partial \ln a}(k,a).$$



C- IGM power spectrum

Use a truncated Zeldovich approximation: $P_{\rm IGM}(k) = P_{\rm Ztrunc}(k)\; e^{-(k/k_s)^2}$

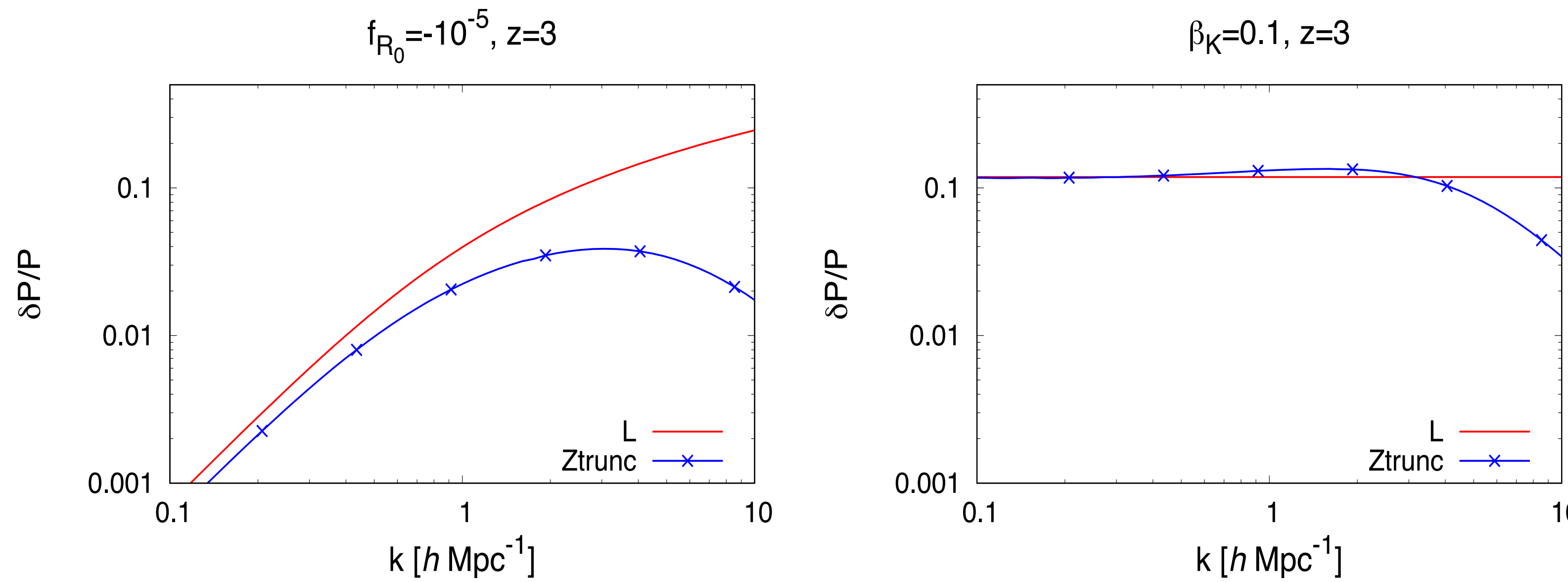
$$P_{\rm Ztrunc} = \max_{k_{\rm trunc}} P_{\rm Z}[P_{\rm Ltrunc}] \quad \text{with} \quad P_{\rm Ltrunc}(k) = P_{\rm L}(k)/(1+k^2/k_{\rm trunc}^2)^2.$$



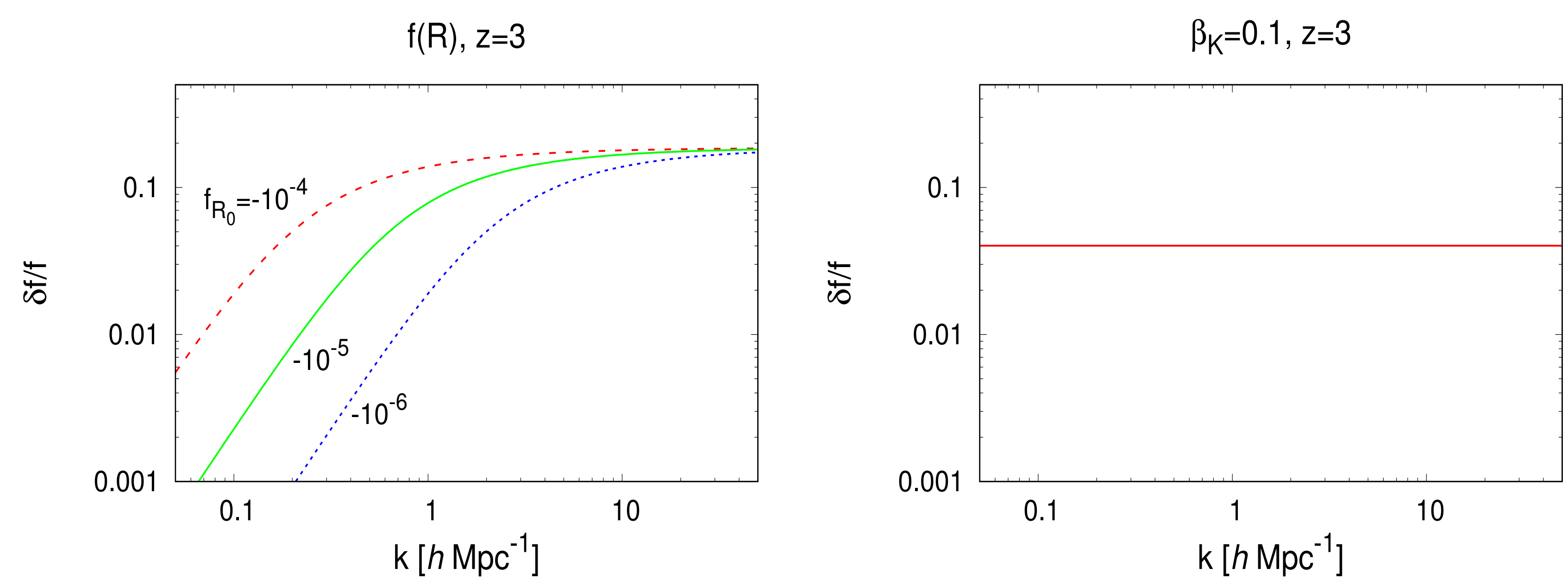
Recovers the large-scale cosmic web, associated with moderate density fluctuations.
Highly nonlinear virialized halos do not contribute to the Lyman-alpha forest.

Modified-gravity:

Relative deviation of the matter power spectrum



Relative deviation of the linear growth rate



D- Lyman-alpha power spectrum

1) 3D power spectrum

$$P_{\delta_F}(\mathbf{k}, z) = b^2_{\delta_{\mathrm{F}}} (1 + \beta\mu^2)^2 \, P_{\mathrm{IGM}}(k)/(1 + f|k\mu|/k_{\mathrm{NL}}) \, e^{-(k\mu/k_{\mathrm{th}})^2},$$

free parameter

Kaiser effect

velocity dispersion in collapsed structures

thermal broadening

$$k_{\mathrm{th}} = \frac{aH}{b_{\mathrm{th}}}, \quad b_{\mathrm{th}} = \sqrt{\frac{k_{\mathrm{B}}T}{2m_p}},$$

$$\mathbf{s} = \mathbf{x} + \frac{v_\parallel}{aH}\mathbf{e}_z,$$

- We noticed that if we use the nonlinear matter power spectrum instead of P_IGM we get a steep growth for the Lyman-alpha power spectrum at high k.

- We take $\beta = 1.3f$ (simulations)

In principle, we could define (Seljak 2012): $$\beta = f\frac{b_{\delta_F,\eta}}{b_{\delta_F,\delta}}$$

$$\eta = -\frac{\frac{\partial v_\parallel}{\partial x_\parallel}}{aH}$$

$$b_{\delta_F,\delta} = \frac{\partial \delta_F}{\partial \delta} \qquad b_{\delta_F,\eta} = \frac{\partial \delta_F}{\partial \eta}$$

However, analytical models do not work very well, especially because of the velocity part (Cieplak & Slosar 2016).

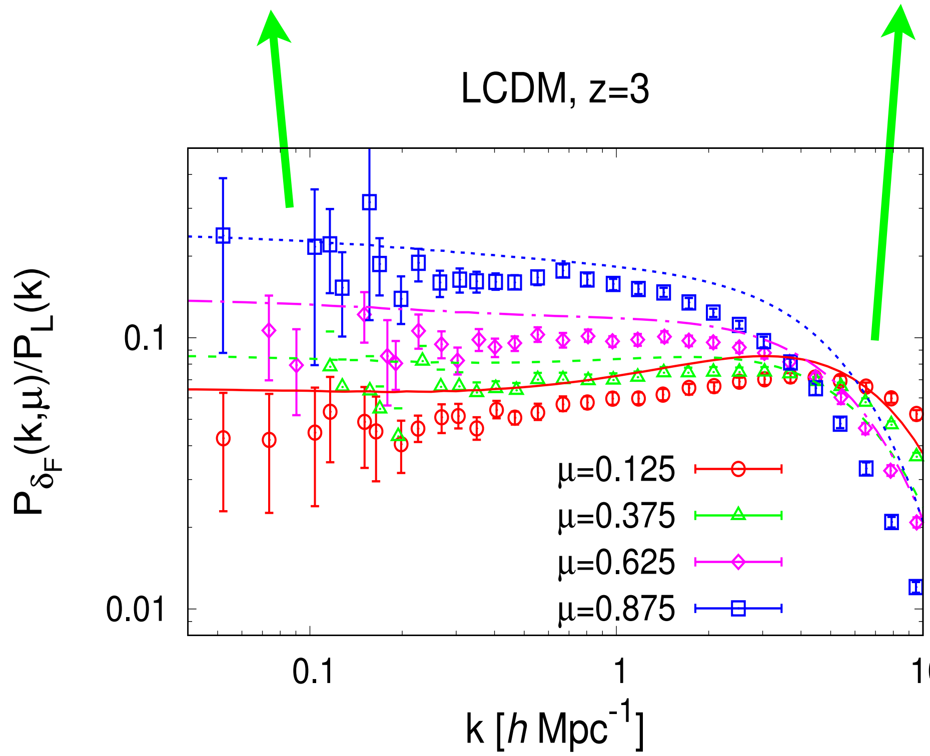
3D Lyman-alpha power spectrum:

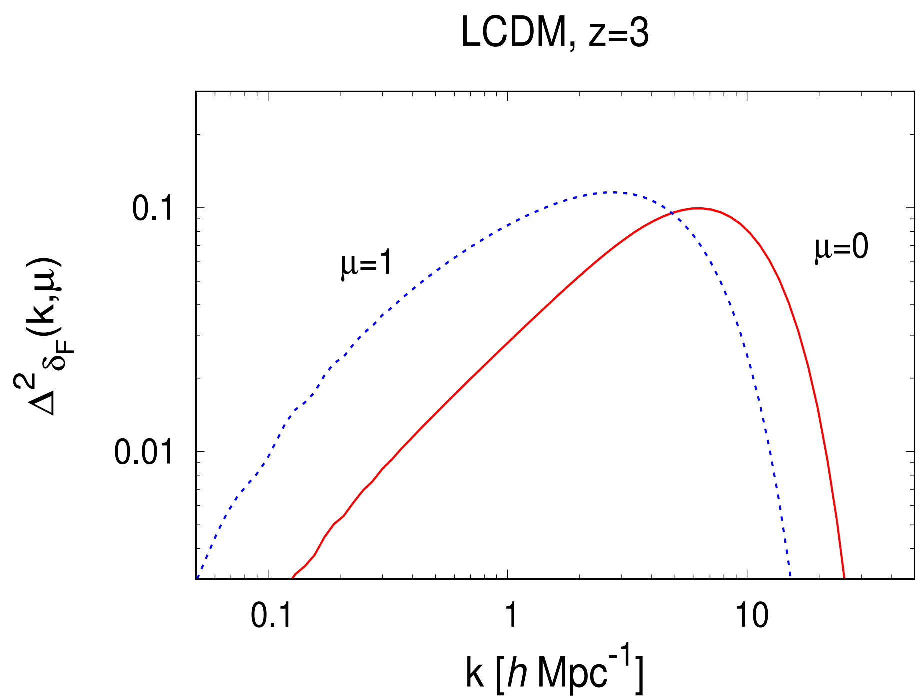
ratio

$P_F(k,\mu)/P_L$

Kaiser effect

velocity dispersion

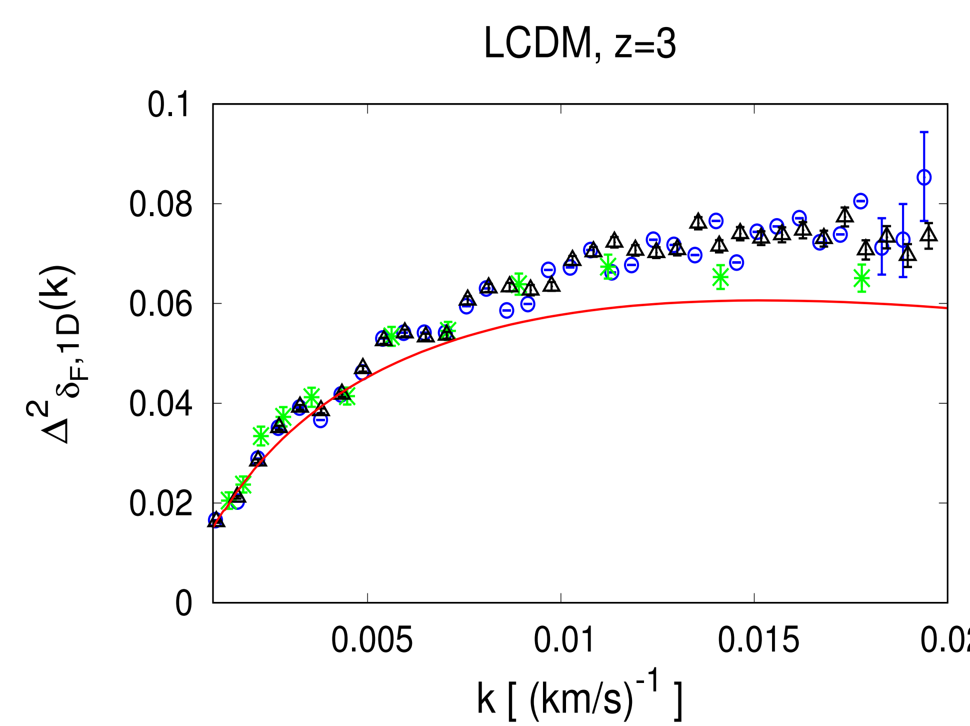




2) 1D power spectrum

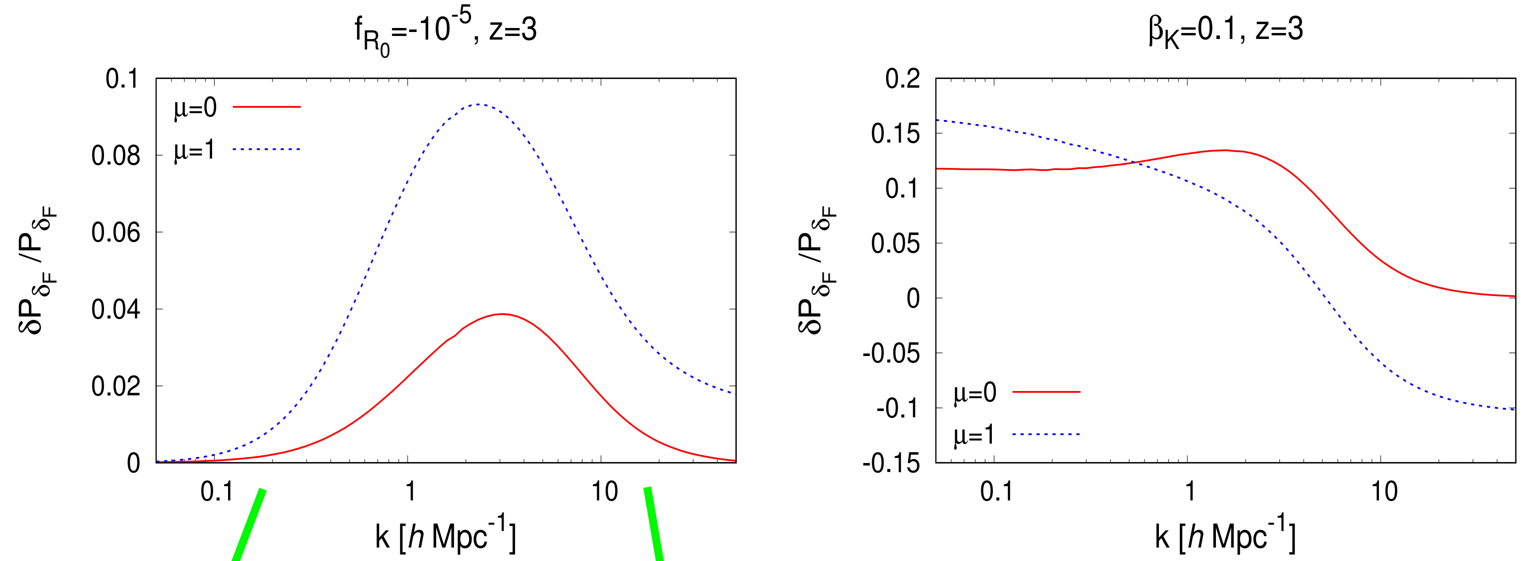
1D Lyman-alpha power spectrum along the line of sight

$$P_{\delta_F,1\mathrm{D}}(k_z) = \int_{-\infty}^{\infty} dk_x dk_y P_{\delta_F}(\mathbf{k}) = 2\pi \int_{k_z}^{\infty} dk\, k P_{\delta_F}(k, \mu = k_z/k).$$



Modified-gravity:

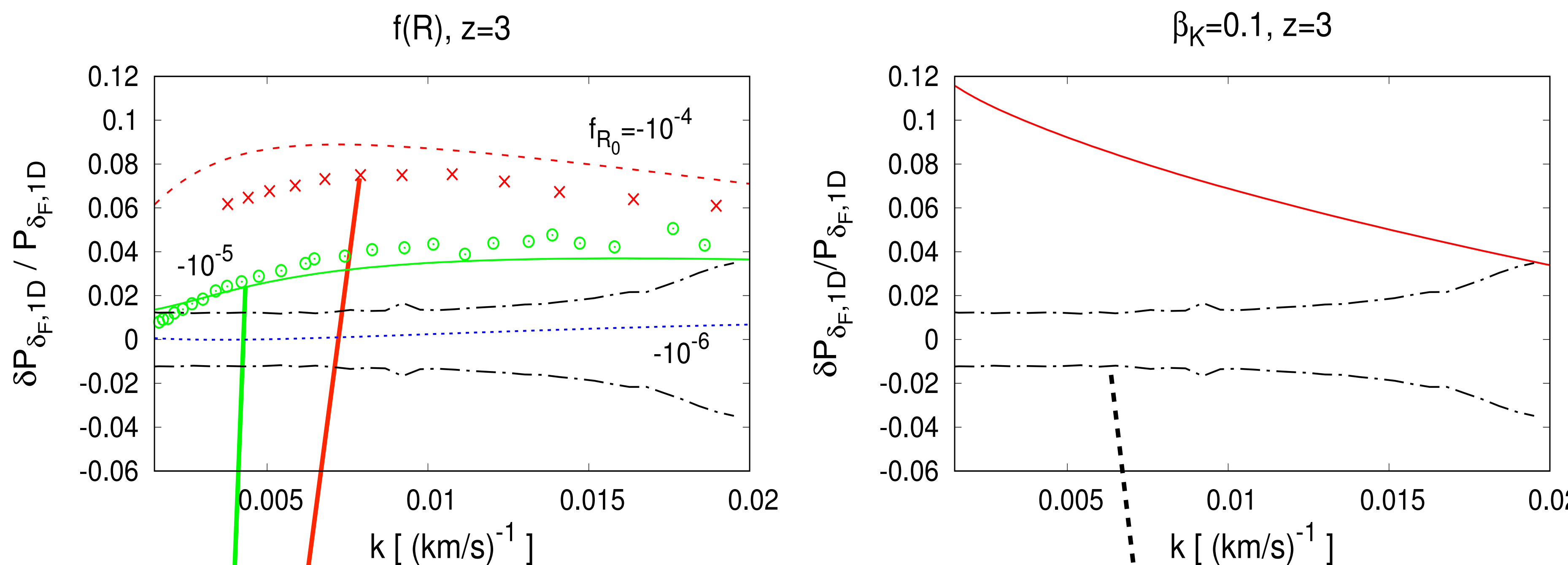
Relative deviation of the 3D Lyman-alpha power spectrum:



Kaiser effect

velocity dispersion

Relative deviation of the 1D Lyman-alpha power spectrum



simulations

Arnold et al. (2015)

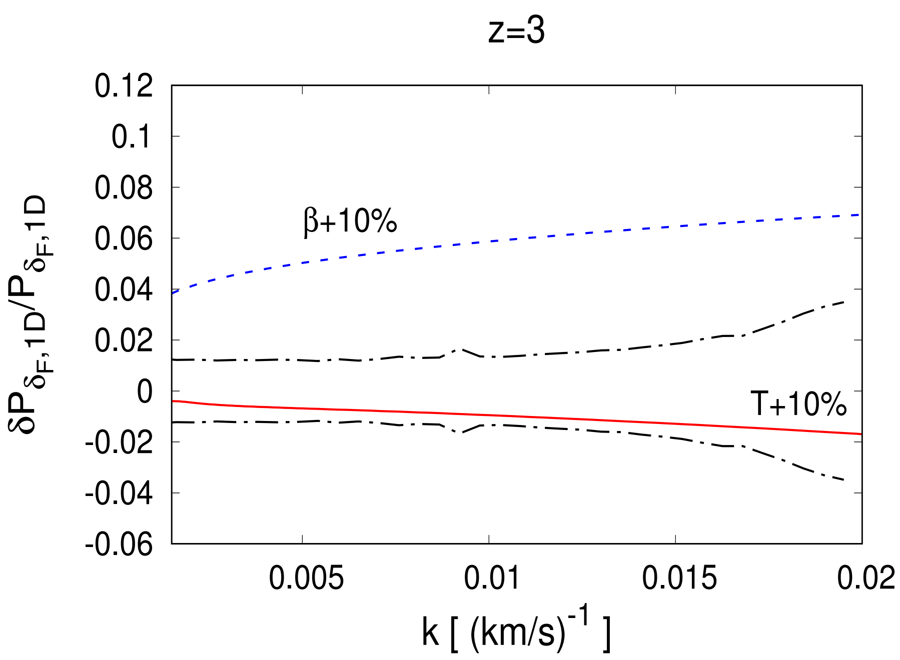
1σ obs. errorbar

➡ reasonable agreement with simulations for f(R)

➡ significant effect for K-mouflage

Degeneracies with physical parameters

Effects of a 10% increase of the IGM temperature or of the bias ratio β



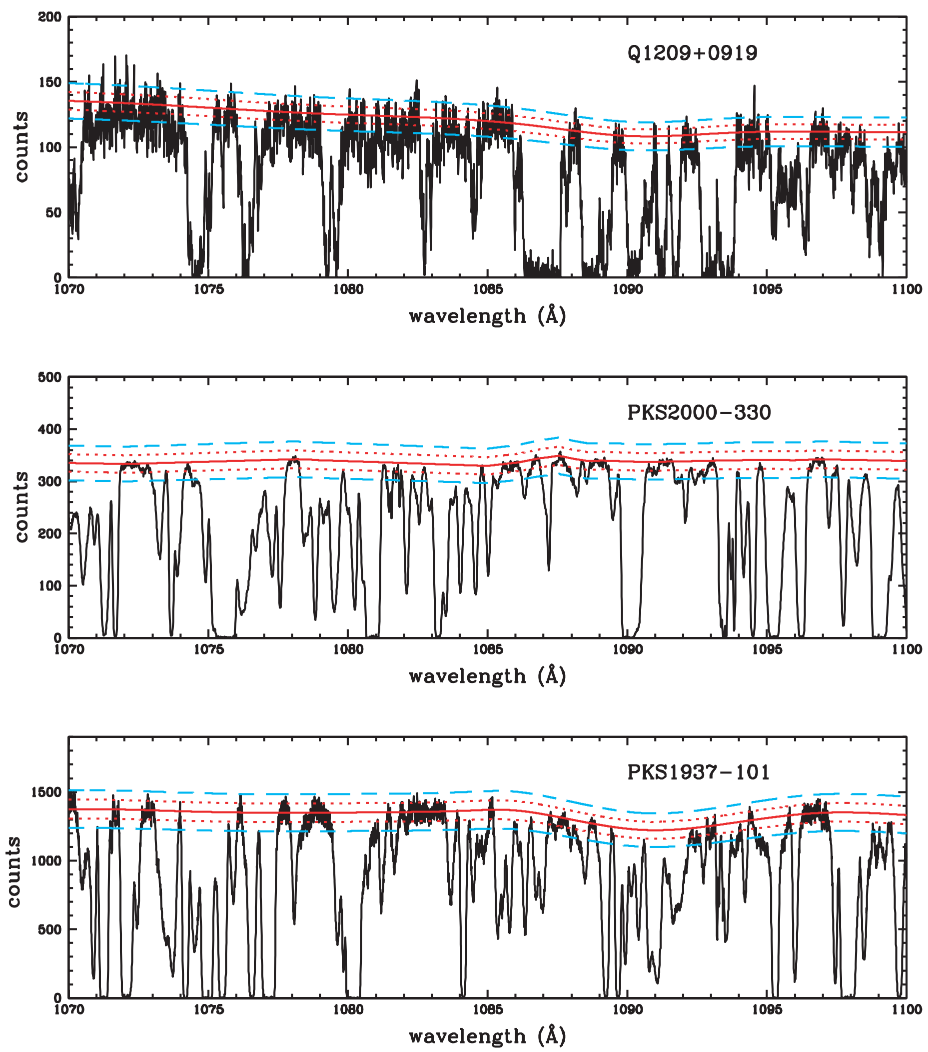
These effects are modest and show a different scale dependence than the modified-gravity effects.

➡ it should be possible to break degeneracies.

E- FLUX PROBABILITY DISTRIBUTION

Transmitted flux $F = e^{-\tau}$ $\tau \propto n_{HI}$

Probability distribution function $\mathcal{P}(F)$



data

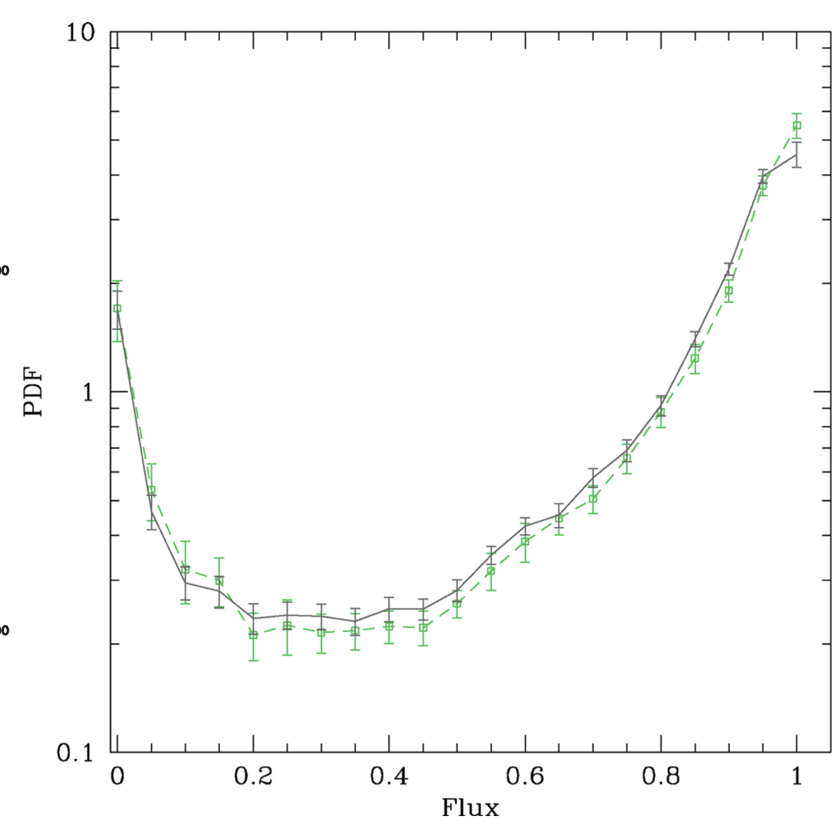
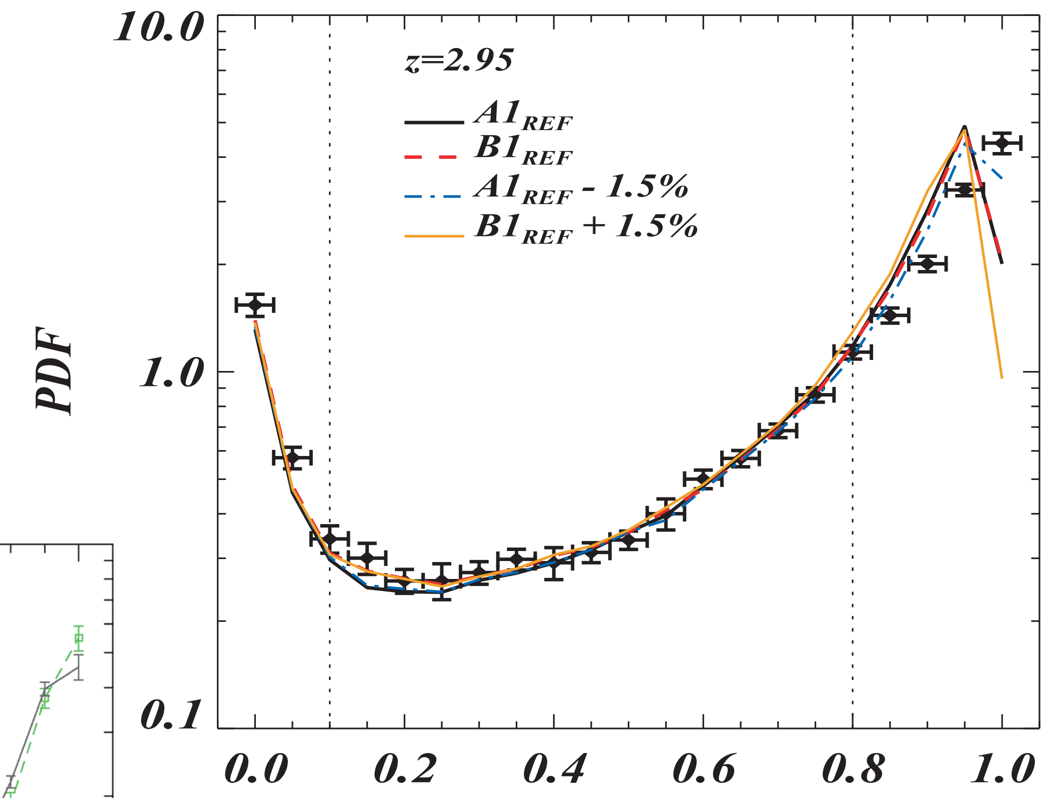


Figure 12. The flux PDF measured by K07 at $\langle z \rangle = 2.94$ (dark-grey curve) plotted with error bars compared to the PDF measured from the two spectra in our sample, Q0055−269 and PKS 2126−158, used in the K07 measurement (open squares and long dashed curve). This comparison uses pixels in the same wavelength range as adopted by K07.



hydro simulations

Calura et al. (2012)

Equilibrium between photo-ionisation and recombination:

éq. de photo-ionisation : $\Gamma_H \, n_{HI} = \alpha_H \, n_{HII} \, n_{e-}$

$\alpha_H(T_0) \simeq 4{,}36 \cdot 10^{-10} \, T_0^{-0{,}75} \, s^{-1} \, cm^3$ à $T_0 \geq 5.000$ K : taux de recombinaison

Γ_H : en s^{-1} : taux d'ionisation

$\Gamma_H = \int_{\nu_0}^{\infty} d\nu \, 4\pi \, \frac{J(\nu)}{h\nu} \, \sigma_{HI}(\nu)$ ν_0 : seuil d'ionisation : 13,6 eV = 912 Å

$\sigma_{HI} = 6{,}3 \cdot 10^{-18} \left(\frac{\nu}{\nu_0}\right)^{-3}$ cm^2 : section efficace

$$n_{HI} = \frac{\alpha(T_0)}{G_1 J_{21}} (1-Y)\left(1-\frac{Y}{2}\right) \left[\frac{\Omega_b}{\Omega_0} \frac{\rho}{m_p}\right]^2$$

$$\tau \propto n_{HI} \propto (1+\delta)^2 T^{-0.7}$$

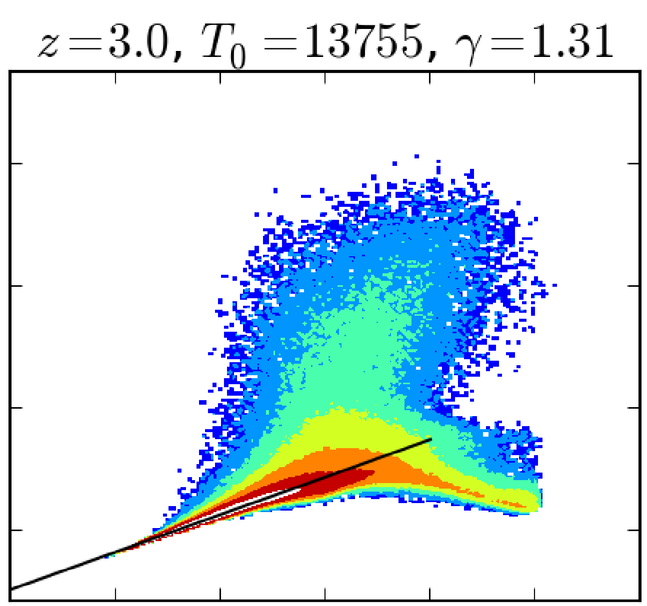
Temperature of the IGM:

$$\frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{2}{3}\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} + \frac{1}{t_{10}}\frac{\rho}{\rho_J}\left(\frac{T}{T_J}\right)^{-\nu}$$

adiabatic cooling

photo-ionization heating

$$T \propto \rho^{\gamma-1}$$



Borde et al. (2014)

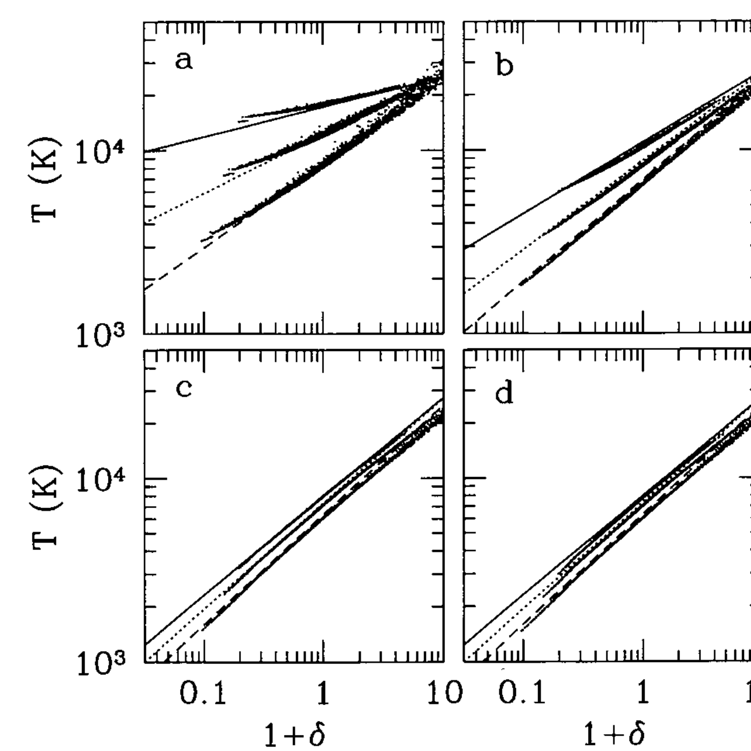


Figure 3. The temperature–density relation for 4 different sudden-reionization models: sudden reionization (see equation 6) at $z=5$ (a), $z=7$ (b), $z=10$ (c) and $z=10$ (d). For each reionization

Hui & Gnedin (1997)

Fluctuating Gunn-Peterson approximation:

$$\tau \propto \rho^2 T^{-0.7} \propto (1+\delta)^\alpha \quad \text{with} \quad \alpha = 2 - 0.7(\gamma - 1), \qquad T \sim 10^4\mathrm{K}, \quad \gamma \sim 1.3$$

$$F = e^{-\tau} = e^{-A(1+\delta)^\alpha}.$$

Factor A related to the photo-ionizing flux. In practice, it is set by the matching with data of the mean flux: $\langle F \rangle$

Smoothing scale:

$$k_s = 2.2\, k_\mathrm{J} \quad \text{with} \quad k_\mathrm{J} = \frac{a}{c_s}\sqrt{4\pi \mathcal{G}_\mathrm{N} \bar{\rho}}, \quad c_s = \sqrt{\frac{5 k_\mathrm{B} T}{3 \mu m_p}},$$

Jeans scale was smaller in the past

Gnedin & Hui (1998)

Jeans wave number

PDF of the flux in terms of the PDF of the matter density:

$$\mathcal{P}(F) = \mathcal{P}(\delta_s) \left| \frac{d\delta_s}{dF} \right|.$$

$$\mathcal{P}(\delta_s) = \int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{dy}{2\pi \mathrm{i} \sigma_s^2} e^{[y\delta_s - \varphi_s(y)]/\sigma_s^2} \quad \text{with} \quad \varphi_s(y) = -\sum_{n=2}^{\infty} \frac{(-y)^2}{n!} \frac{\langle \delta_s^n \rangle_c}{\sigma_s^{2(n-1)}},$$

(explained in previous talks by C. Uhlemann / F. Bernardeau)

LCDM:

mildly nonlinear scale:

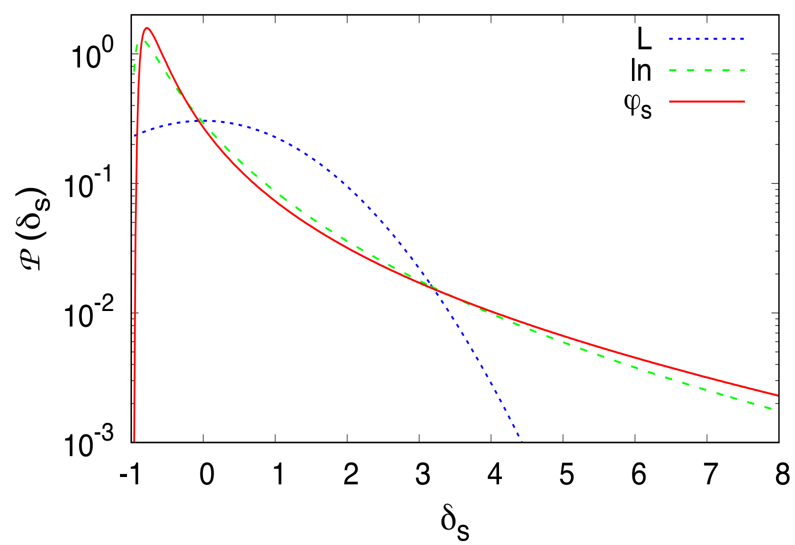
$$x_s \sim 250\,\mathrm{kpc}$$

$$\sigma_L^2 \simeq 1.7$$

$$\xi \simeq 2.7$$

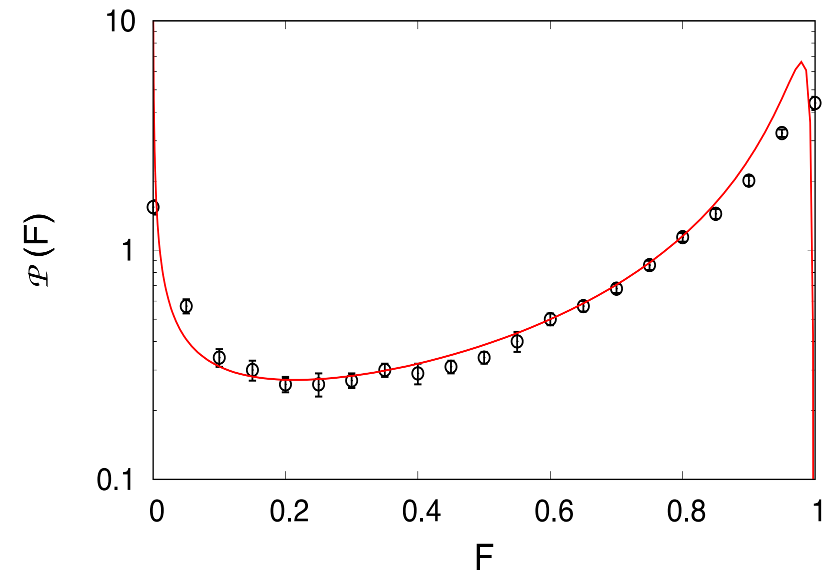
density contrast

LCDM, z=3



Flux

LCDM, z=3



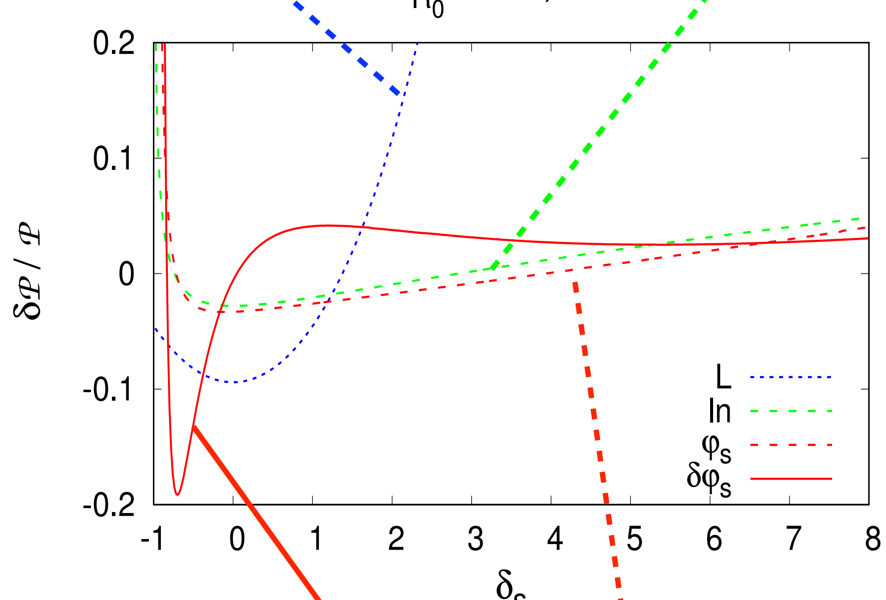
Modified-gravity:

Amplify the growth of density perturbations ➡ the variance and the tails are greater

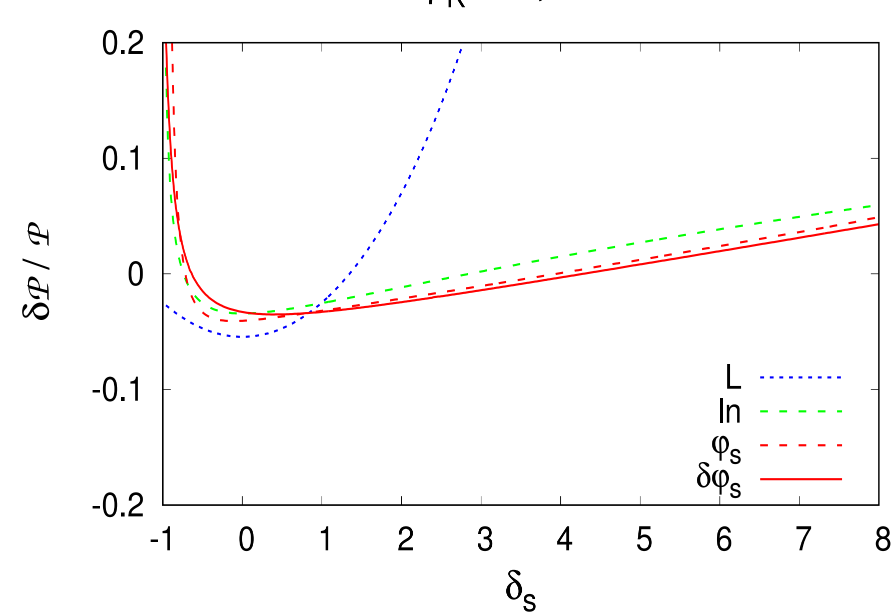
linear (Gaussian)

lognormal

f_{R_0}=-10^{-4}, z=3



β_K=0.1, z=3



relative deviation for PDF of the density contrast

(spherical collapse) $\Delta\varphi$

φ

K-mouflage is closer to LCDM than f(R), because it does not generate a new scale dependence (in the unscreened regime).

Perturbative bias and skewness:

$\delta_s = (1 + \nu_2 \delta_{Ll}) \delta_{Ls}$ small-scale perturbations are enhanced by large-scale modes

EdS: $\nu_2 = 34/21$

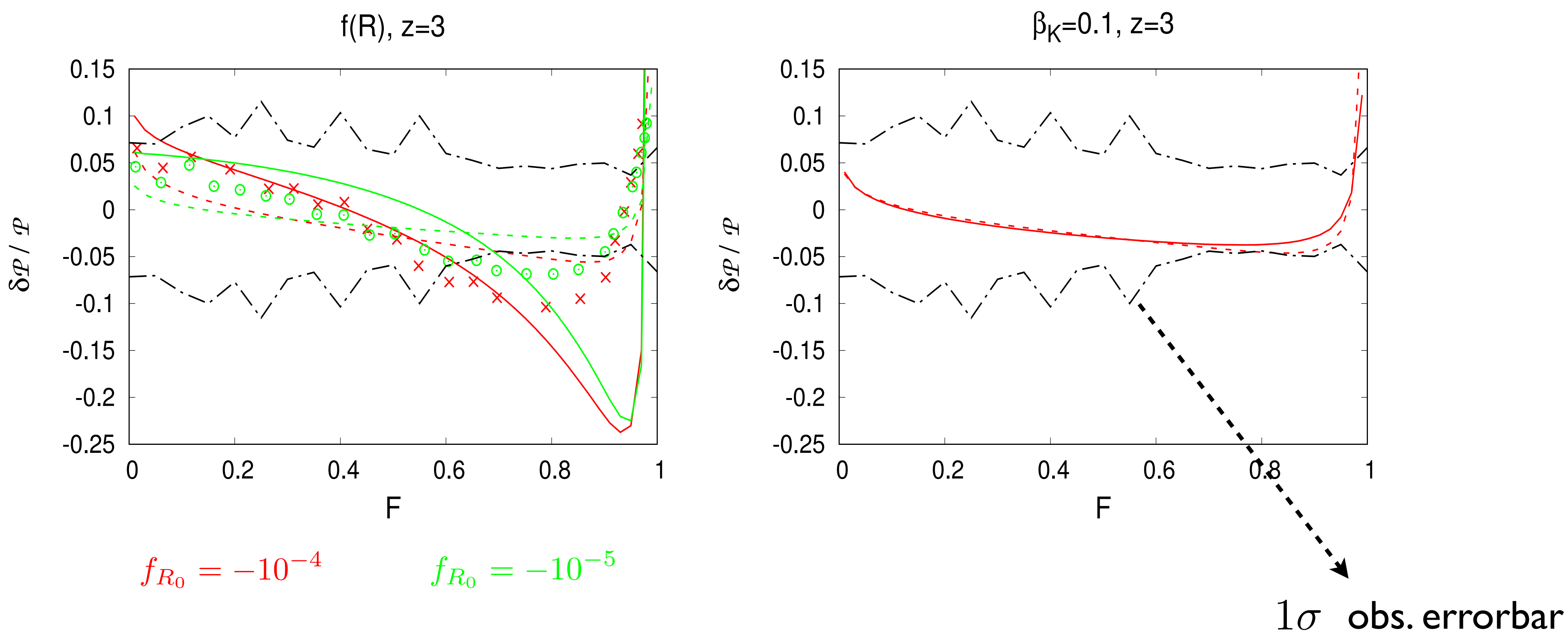
$S_3 = 3\nu_2 + d \ln \sigma^2 / d \ln x$ skewness of spherical cells

model	LCDM	$f_{R_0} = -10^{-4}$	$f_{R_0} = -10^{-5}$	$f_{R_0} = -10^{-6}$	$\beta_K = 0.1$
$\nu_2(k_s, 1h\mathrm{Mpc}^{-1})$	1.62	1.65	1.65	1.64	1.62
$\nu_2(k_s, k_s)$	1.62	1.62	1.63	1.63	1.62

scale dependence for f(R)

The tails are again enhanced for the PDF of the Lyman-alpha flux.

relative deviation for PDF of the flux



➡ The model is not very good for f(R) and shows larger uncertainties than for K-mouflage.

➡ But this is not a competitive probe.

V. CONCLUSION

- The deviations from LCDM of the Lyman-alpha statistics strongly depend on the details of the modified-gravity model.
In particular, on whether it leads to a new scale dependence.

- For models with new scale dependence, the PDF of the flux is difficult to predict accurately. This is probably related to the fact that Lyman-alpha clouds are already in the mildly nonlinear regime and to redshift-space distortions.

- However, the PDF of the flux is unlikely to be a competitive probe.

- The power spectrum of the Lyman-alpha flux is easier to model.
Its behavior also depends on the properties of the modified-gravity model.

- Whereas it is not competitive for f(R) models, it could be a useful probe for models such as K-mouflage.