Francis Bernardeau
IAP Paris
and
IPhT Saclay

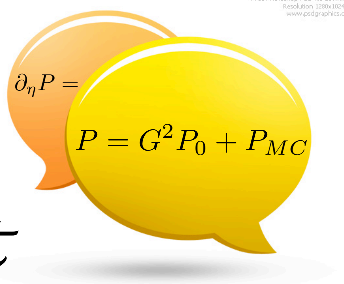
PTchat@Kyoto

8-12 April 2019 Yukawa Institute for Theoretical Physics

A Large-Deviation Principle at play in large-scale structure cosmology

Old ideas in a « new pot »

Institut de Physique Théorique
CEA-Saclay
Gif-sur-Yvette
20-22 September, 2011



PT *chat*

A Workshop on Resummation Methods in Cosmological Perturbation Theory

Convenors:
Francis Bernardeau
Martín Crocce
Román Scoccimarro
Emiliano Sefusatti

Participants:
Stefano Anselmi
Benjamin Audren
Tobias Baldauf
Guillermo Ballesteros
Philippe Brax
Ram Brustein
Vincent Desjacques
Julien Lesgourgues
Sabino Matarrese
Takahiko Matsubara
Takahiro Nishimichi
Massimo Pietroni
Robert Smith
Atsushi Taruya
Patrick Valageas
Nicolas Van de Rijt
Filippo Vernizzi
Zvonomir Vlah

The workshop aims at gathering active researchers in the development of efficient analytical methods for the computation of the statistical properties of the large-scale structure of the Universe. It will provide the opportunity for participants to present and discuss the merit ands scopes of the different Perturbation Theory approaches that have been put forward in recent years.

This meeting is supported by the French Programme National de Cosmologie et Galaxies

PTChat at Cargèse
April 30 – Mai 3, 2013

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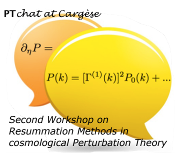


With the advent of a new generation of wide field cosmological surveys aiming at characterizing the mass and energy content of the universe, it becomes important to develop tools for predicting and computing cosmic field statistical properties, such as cosmic density spectra or bispectra beyond the linear regime. To achieve such an objective, besides N-body simulations, one can rely on Perturbation Theory techniques that allow to approach such quantities in a controlled way. Furthermore those methods could in principle be exploited for a variety of cosmological models that include non-standard effects such as massive neutrinos or modified gravity models.
In this context, this workshop aims at gathering active researchers in the development of efficient analytical methods for the computation of the statistical properties of the large-scale structure of the Universe. It will provide the opportunity for participants to present and discuss the merits and scopes of the different Perturbation Theory approaches that have been put forward in recent years.

Main topics will include
- hardcore methods of perturbation theory
- application to redshift-space distortions
- biasing mechanisms and properties of halos
- construction of modified gravity & dark energy models
- impact of massive neutrinos on the development of large-scale structure
- computations of covariances

Eminent scientists in the field will animate the school. These include:
Ph. Brax (IPhT Saclay, FR), V. Desjacques (Genève, CH), M. Peloso (U. Minesota, US), M. Pietroni (INFN Padua, IT), D. Pogosian (U. Alberta Edmonton, CA), L. Senatore (CERN and U. Standford, US), R. Scoccimarro (NYU New York, US), E. Sefusatti (ICTP, IT) and A. Taruya (RESCEU Tokyo, JP).
The scientific program will gradually be established, based on the proposals of accepted contributions

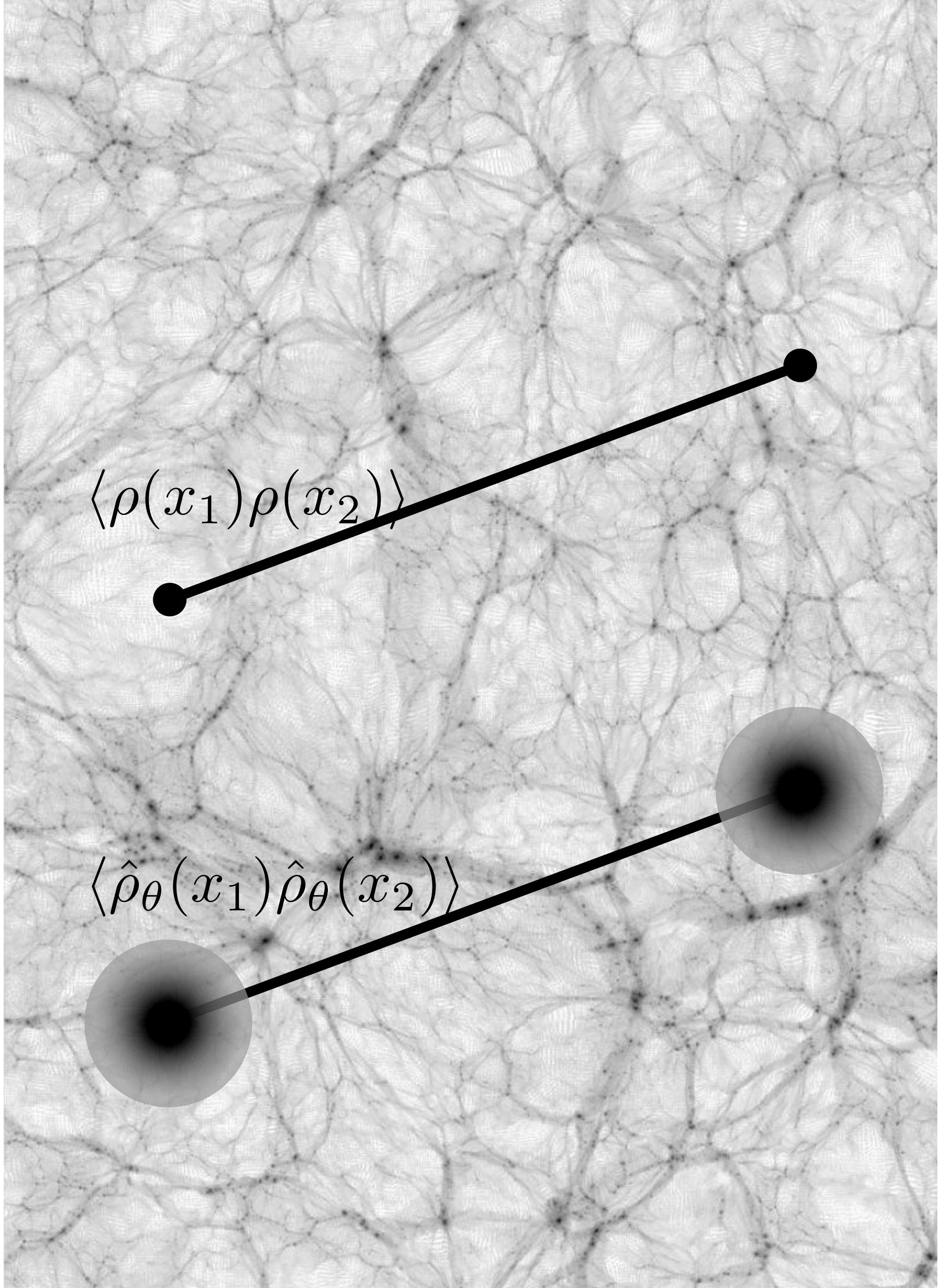
Organization Committee
Francis Bernardeau (IPhT Saclay FR), Takahiro Nishimichi (IPMU & IAP, Tokyo JP and Paris FR), Patrick Valegeas (IphT Saclay FR)

Application and registration
http://www.iesc.univ-corse.fr/index.php?id=81&L=0&tx_iescielecoles_pi4[idecole]=804

No Registration fees
Deadline for applications to April 7th, 2013

Moving from « naked » correlation functions to « dressed » or regularised correlation functions

e.g. clipped density field, or peaks, etc.

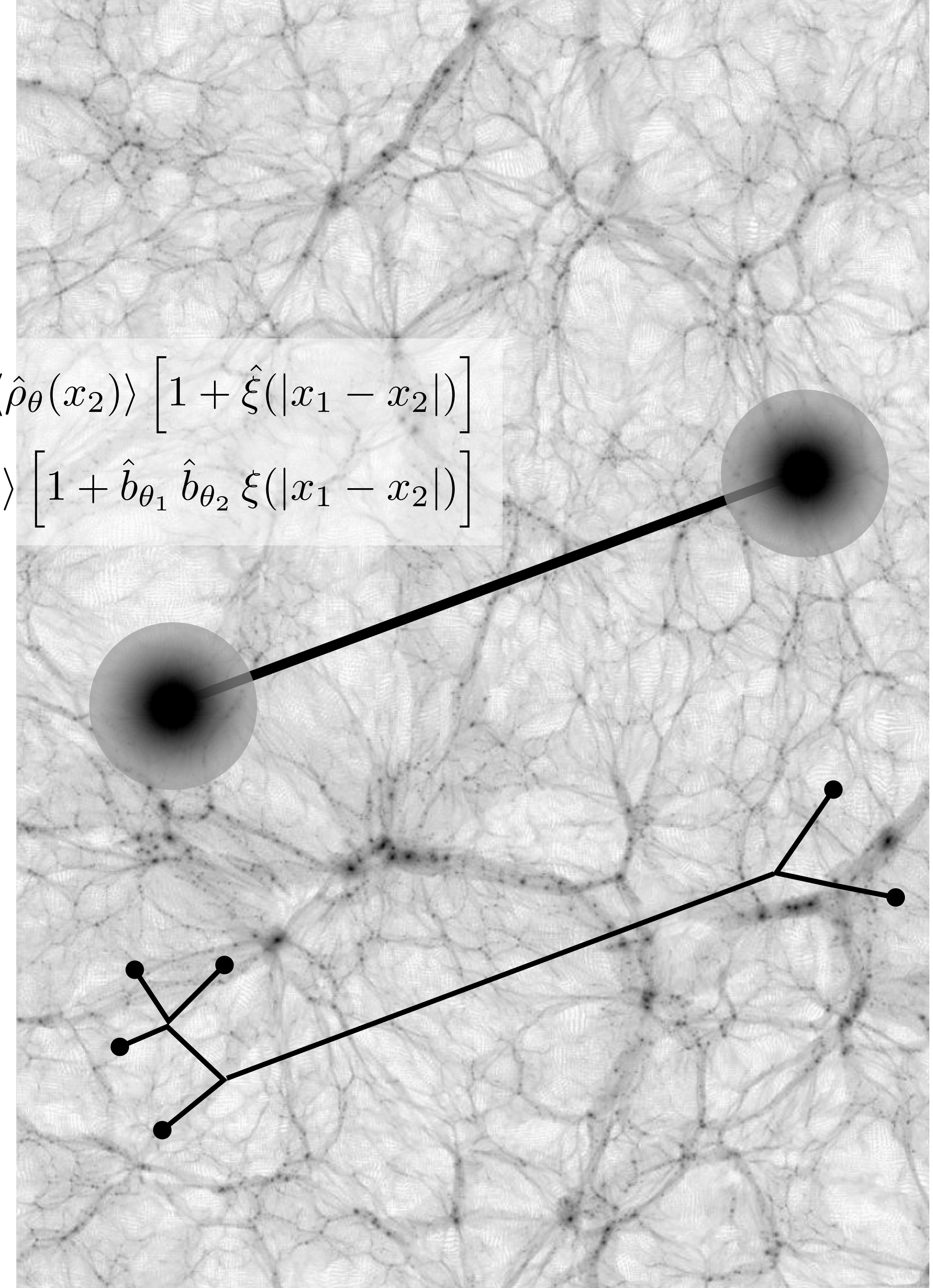


Building dressed correlation functions

$$\langle \rho(x_1)\rho(x_2)\rangle = 1 + \xi(|x_1 - x_2|)$$

$$\langle \hat\rho_\theta(x_1)\hat\rho_\theta(x_2)\rangle = \langle \hat\rho_\theta(x_1)\rangle\langle \hat\rho_\theta(x_2)\rangle \left[1 + \hat\xi(|x_1 - x_2|)\right]$$

$$= \langle \hat\rho_\theta(x_1)\rangle\langle \hat\rho_\theta(x_2)\rangle \left[1 + \hat b_{\theta_1}\,\hat b_{\theta_2}\,\xi(|x_1 - x_2|)\right]$$



This form assumes that the « dressed » density is defined locally (like the local density in a spherical region, from higher order derivative, etc.) and that the scale at which it is defined is much smaller than the separation.

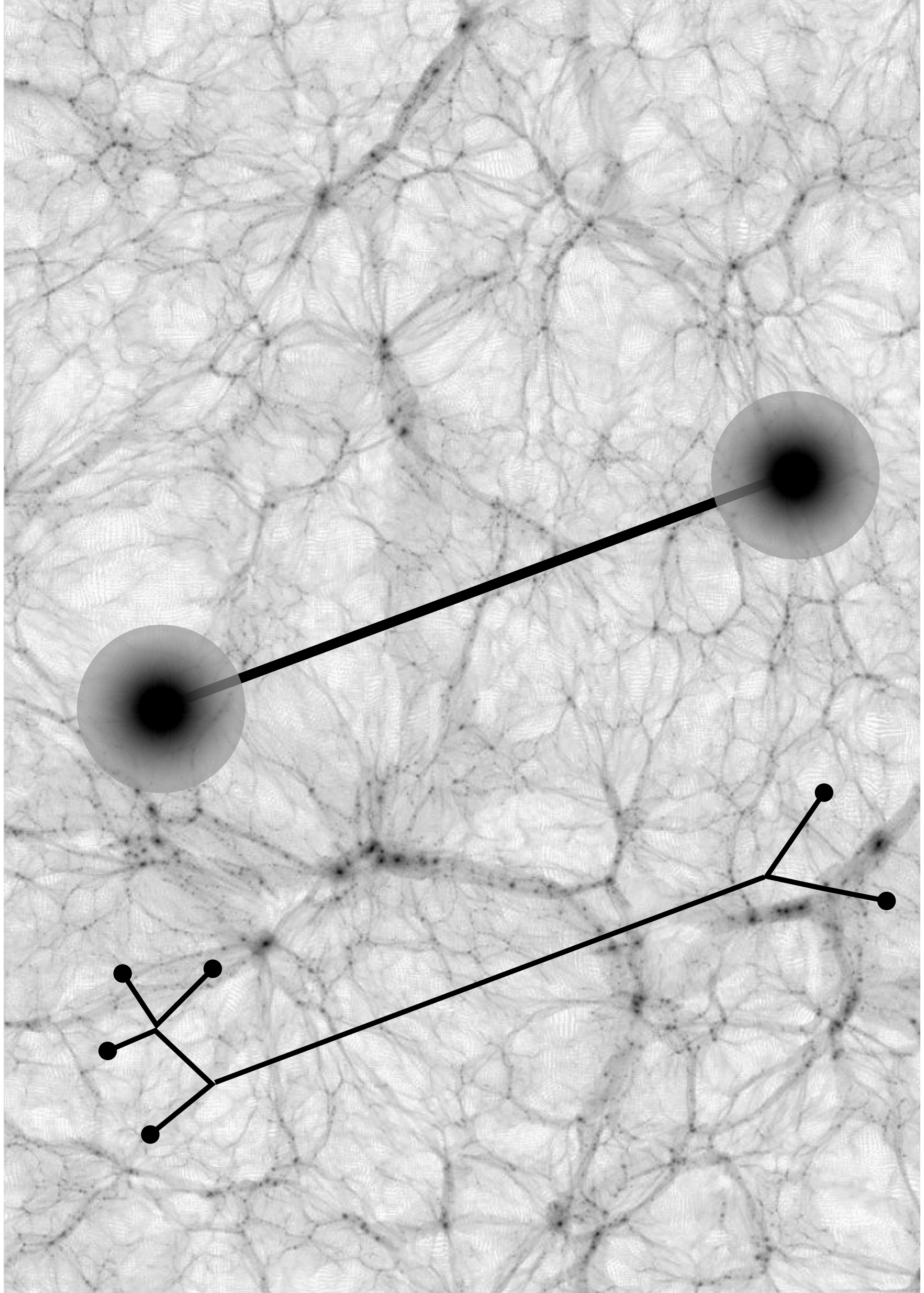
Perturbatively, it collects contributions coming from higher correlation function in the squeezed limit.

Building dressed correlation functions

If one knows how to compute the dressed density, then the bias factor is given by the linear response of the dressed density with a large-scale variation.

Are there quantities that are better suited for such calculations, taking into account non-linear evolution?

The new pot = the large deviation principle.



Large-deviation theory, one step beyond the central limit theorem.

It adresses the question: what is the most likely way for an unlikely event to happen?

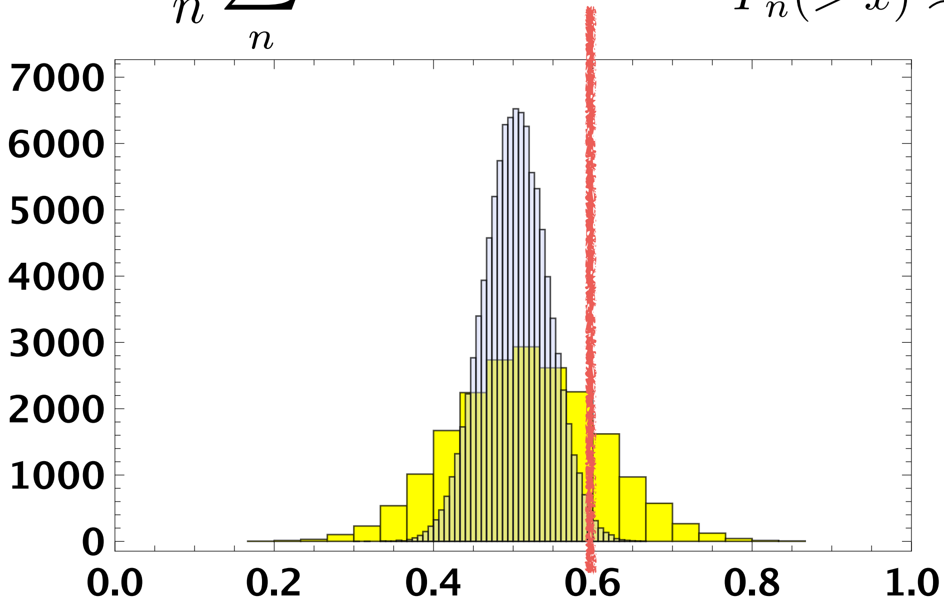
Can serve as a computational method and/or guideline for quantities of interest

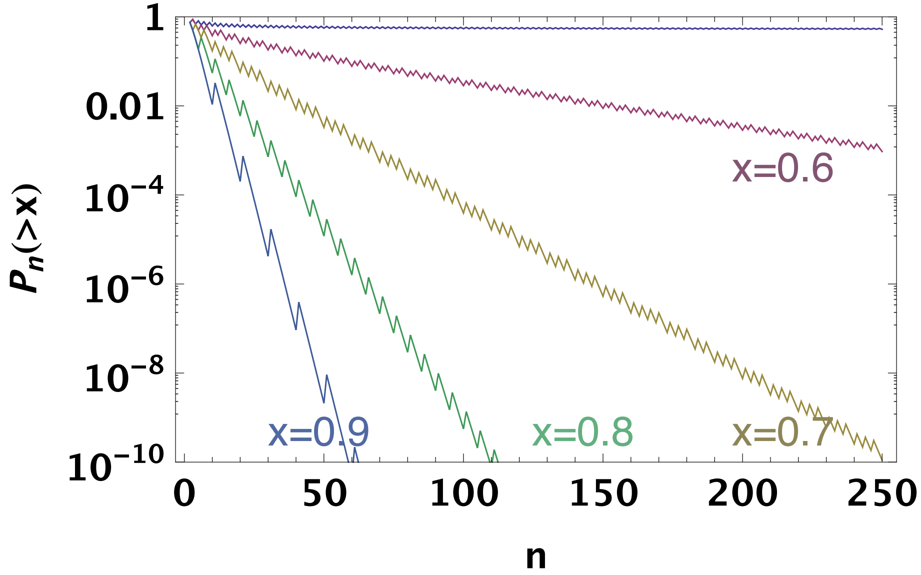
Basics of theory of large deviation functions

Review paper by Hugo Touchette, '09

One exemple : tossing coins and taking the average number of heads

$$x = \frac{1}{n}\sum_n t_n \qquad P_n(> x) \asymp \exp(-nI(x))$$





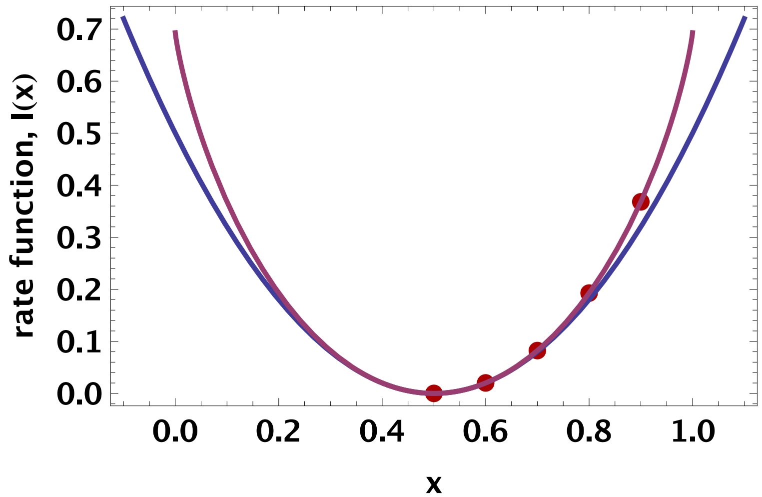
Put a threshold at a fixed position

Central limit theorem : $I(x) = 2(x-0.5)^2$

Exact result : $I(x) = x\log[x] + (1-x)\log[1-x] + \log[2]$

The cumulant generating function : $\varphi(\lambda) = \log\left(e^{\lambda}/2 + 1/2\right)$

Cramér's Theorem : both are Legendre transform of one-another



Key theorems: from rate function to scaled cumulant generating functions

The Contraction Principle: the rate function of an unlikely event is the rate function of the most likely configuration for it to happen.

For a mapping $x \rightarrow y$ we have , $I(y) = \inf_{x,\ x \rightarrow y} I(x)$

that is the rate function for y is the smallest rate function (the most probable) of the values (configurations) that lead to y.

*The **Gärtner-Ellis Theorem** (Cramér's Theorem for IID): the rate function is the Legendre-Fenchel transform of the (scaled) cumulant generating function*

$$I(\rho) = \sup_{\lambda} [\lambda \rho - \varphi(\lambda)]$$

Under some regularity conditions, this relation can be inverted in

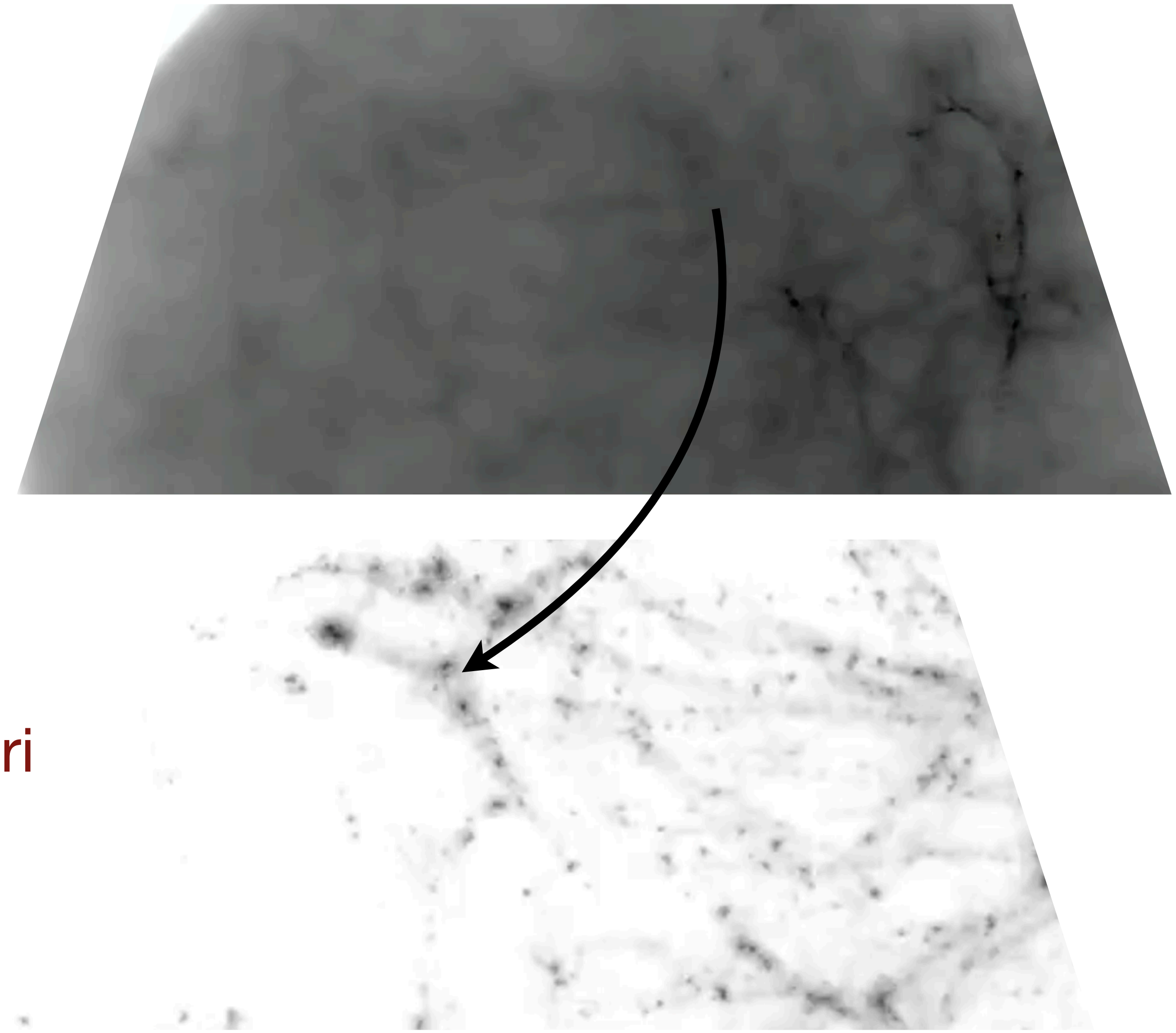
$$\varphi(\lambda) = \sup_{\rho} [\lambda \rho - I(\rho)]$$

The scaled cumulant generating function:

$$\varphi(\lambda) = \lim_{\langle \rho^2 \rangle_c \to 0} \langle \rho^2 \rangle_c \sum_{p=1}^{\infty} \frac{\langle \rho^p \rangle_c}{p!} \left(\frac{\lambda}{\langle \rho^2 \rangle_c} \right)^p = \lambda + \frac{\lambda^2}{2} + S_3 \frac{\lambda^3}{3!} + \dots$$

Large-Deviation Principle in the context of large-scale structure cosmology

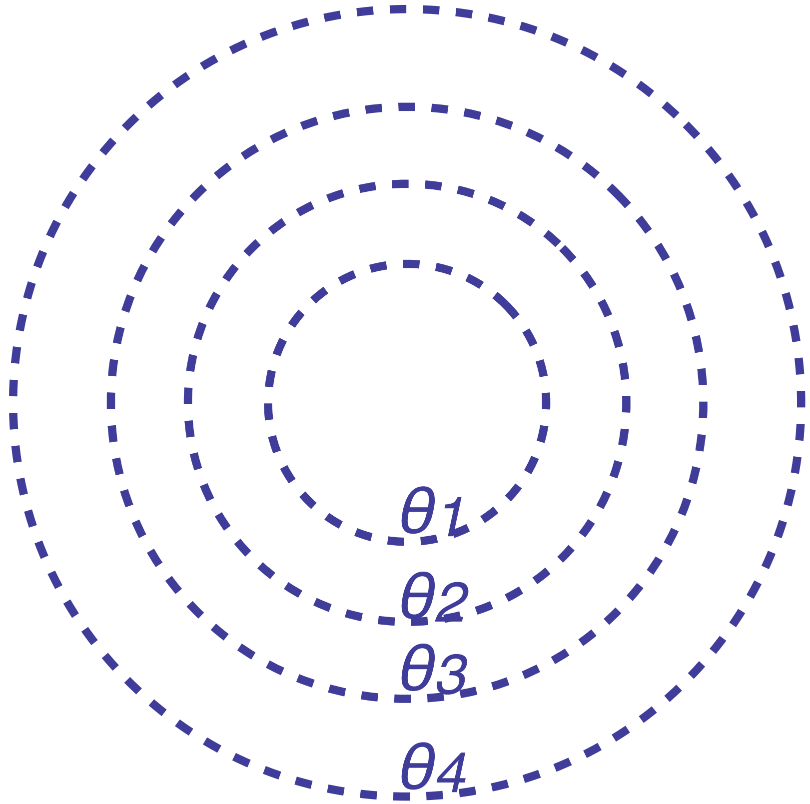
Discrete of continuous sets of Gaussian variables obey the Large Deviation Principle: their rate function is a simple quadratic form.



One needs a mapping… (a priori non-linear and non-local)

An explicit large-deviation regime

P. Reimberg, FB, '15



If one restricts the ensemble of realisations to spherically symmetric configurations, one can define a set of random variables - the densities in concentric shells - for which we know the rate function and the mapping into their nonlinear values.

The collection $\{\delta^{lin}(\theta_i)\}_{1\leq i\leq N}$ of correlated gaussian random variables obeys the LDP with rate function:

$$I(\delta_<^{lin}(\theta_1),\ldots,\delta_<^{lin}(\theta_N)) = \frac{\sigma^2(\theta_N)}{2}\sum_{ij}\Xi_{ij}\,\delta_<^{lin}(\theta_i)\,\delta_<^{lin}(\theta_j)$$

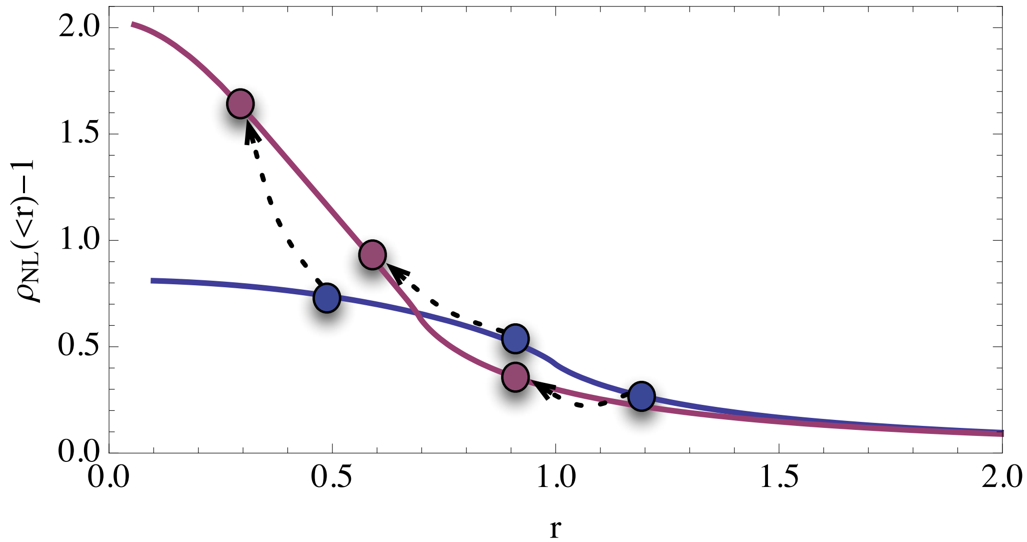
where $\Xi = \Sigma^{-1}$, and $\sigma^2(\theta_N) = \Sigma_{NN}$.

The spherical collapse: the solution for specific initial conditions (with adiab. modes)

The radius evolution

$$\frac{\mathrm{d}^2 R}{\mathrm{d}t^2} = -\frac{GM(< R)}{R^2}$$

The exact non-linear mapping for spherically symmetric initial profile (for growing mode setting)



Note that this mapping is independent on the small scale physics (with baryons, shell crossings, etc.) ;

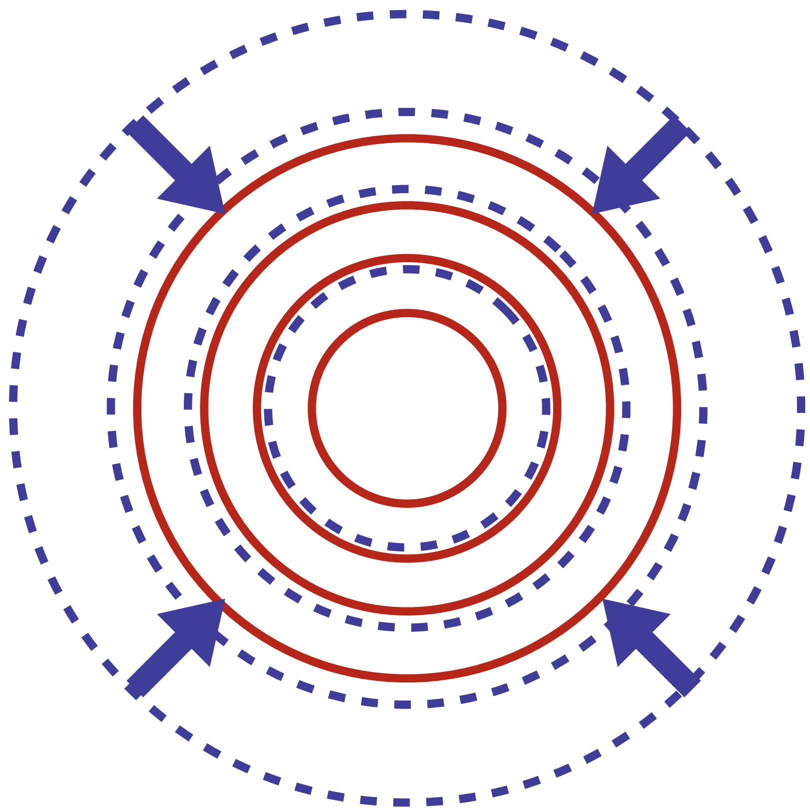
Is it good enough for spherically symmetric observables ? Not necessarily (e.g. Zel'dovich approximation, FB, Reimberg, in prep.)

There exists a mapping which maps the initial radii into the nonlinear ones

P. Reimberg, FB, '15

$$\delta_<(\vartheta) = \zeta(\delta_<^{\text{lin}}(\theta))$$

$$\vartheta = \theta\,\zeta^{-1/D}(\delta_<^{\text{lin}}(\theta))$$



The scaled cumulant generating function of ***any*** functional of the non-linear density profile is then given by,

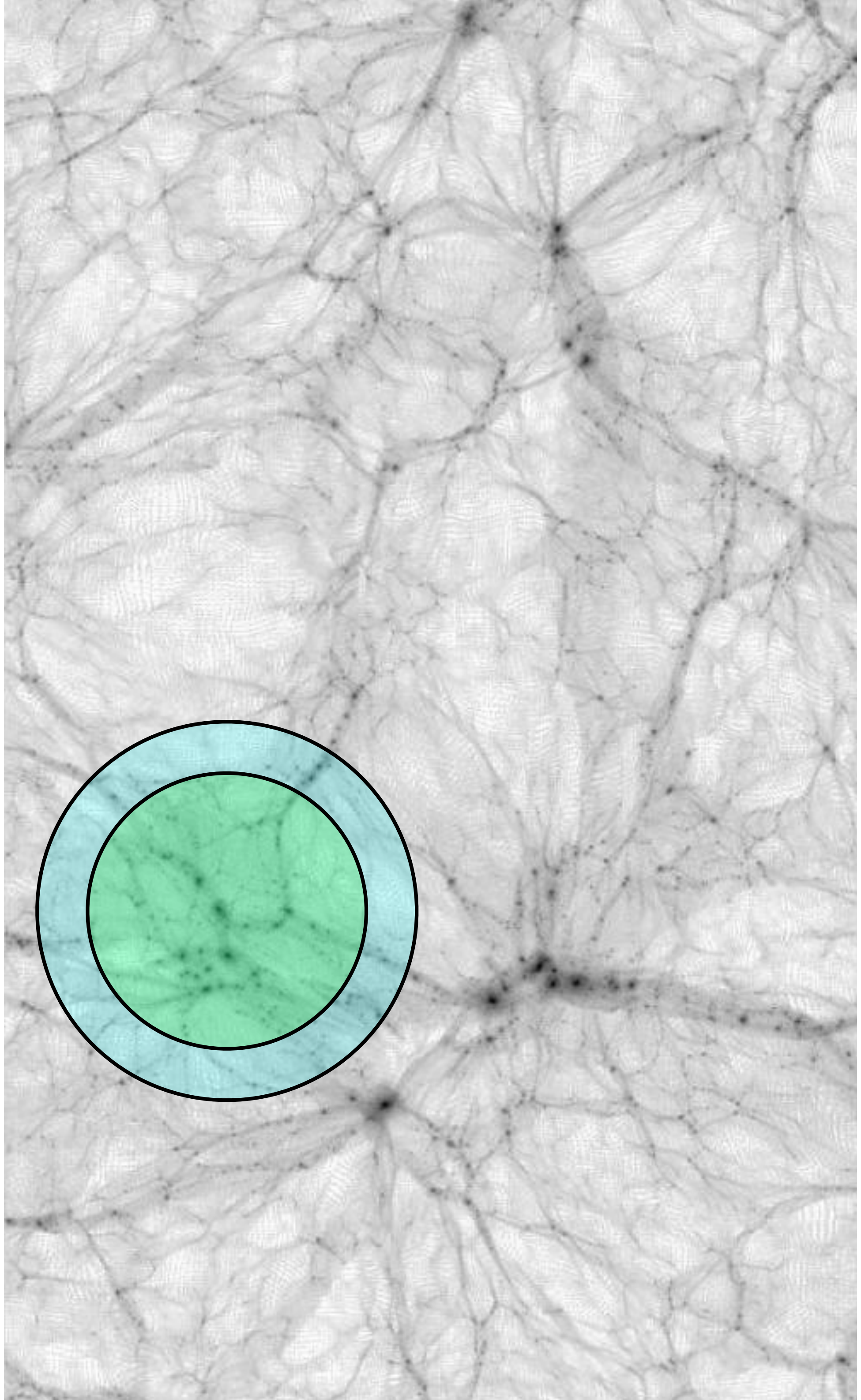
$$\varphi(\lambda) = \sup_{\delta_<^{\text{lin}}(\theta)} \left[\lambda\;\hat{\rho}\{\delta_<(\vartheta)\} - I(\delta_{\text{lin}}(\theta_1), \ldots, \delta_{\text{lin}}(\theta_N))\right]$$

$\hat{\rho}\{\delta_<(\vartheta)\}$ does not have to be local, linear or defined from a discrete number of shells.

Consequences in the context of LSS cosmology are at least 2 folds

- ***you do not need to impose $\delta(x)$ to be small everywhere, only the variance has to be small;***
- *you have a possible working procedure provided you can identify the most likely initial configuration and its probability (rate function).*

Such an identification can be done for configurations with enough symmetries: in practice with spherical (or cylindrical) symmetry.



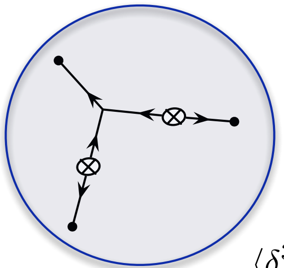
Standard result: the cumulants of the top-hat smoothed density

scaled cumulant GF is Legendre T. of rate function:

$$\varphi(\lambda) = \lim_{\langle\rho^2\rangle_c \to 0} \langle\rho^2\rangle_c \sum_{p=1}^{\infty} \frac{\langle\rho^p\rangle_c}{p!} \left(\frac{\lambda}{\langle\rho^2\rangle_c}\right)^p = \lambda + \frac{\lambda^2}{2} + S_3\frac{\lambda^3}{3!} + \dots$$

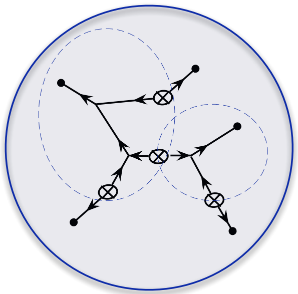
Average of (combination of) tree order expression of the p-point correlation functions in spherical cells.

Expression of $S_p = \lim_{\langle\delta^2\rangle_c \to 0} \frac{\langle\delta^p\rangle_c}{\langle\delta^2\rangle_c^{p-1}}$ *= tree order expr.*



$$\langle\delta^3\rangle = 6\int \frac{\mathrm{d}\mathbf{k}_1}{(2\pi)^3} P(k_1)P(k_2) \times F_2(\mathbf{k}_1,\mathbf{k}_2)W(k_1R)W(k_2R)W(|\mathbf{k}_1+\mathbf{k}_2|R) \propto \langle\delta^2\rangle^2$$

...



it has a non trivial dependence on the wave vectors through the functions F3 and F2

$$S_3 = \frac{34}{7} + \gamma_1,$$

$$S_4 = \frac{60712}{1323} + \frac{62\,\gamma_1}{3} + \frac{7\,\gamma_1^2}{3} + \frac{2\,\gamma_2}{3},$$

$$S_5 = \frac{200575880}{305613} + \frac{1847200\,\gamma_1}{3969} + \frac{6940\,\gamma_1^2}{63} + \frac{235\,\gamma_1^3}{27} + \frac{1490\,\gamma_2}{63} + \frac{50\,\gamma_1\,\gamma_2}{9} + \frac{10\,\gamma_3}{27},$$

$$\gamma_p = \frac{\mathrm{d}^p \log\sigma^2(R_0)}{\mathrm{d}\log^p R_0}.$$

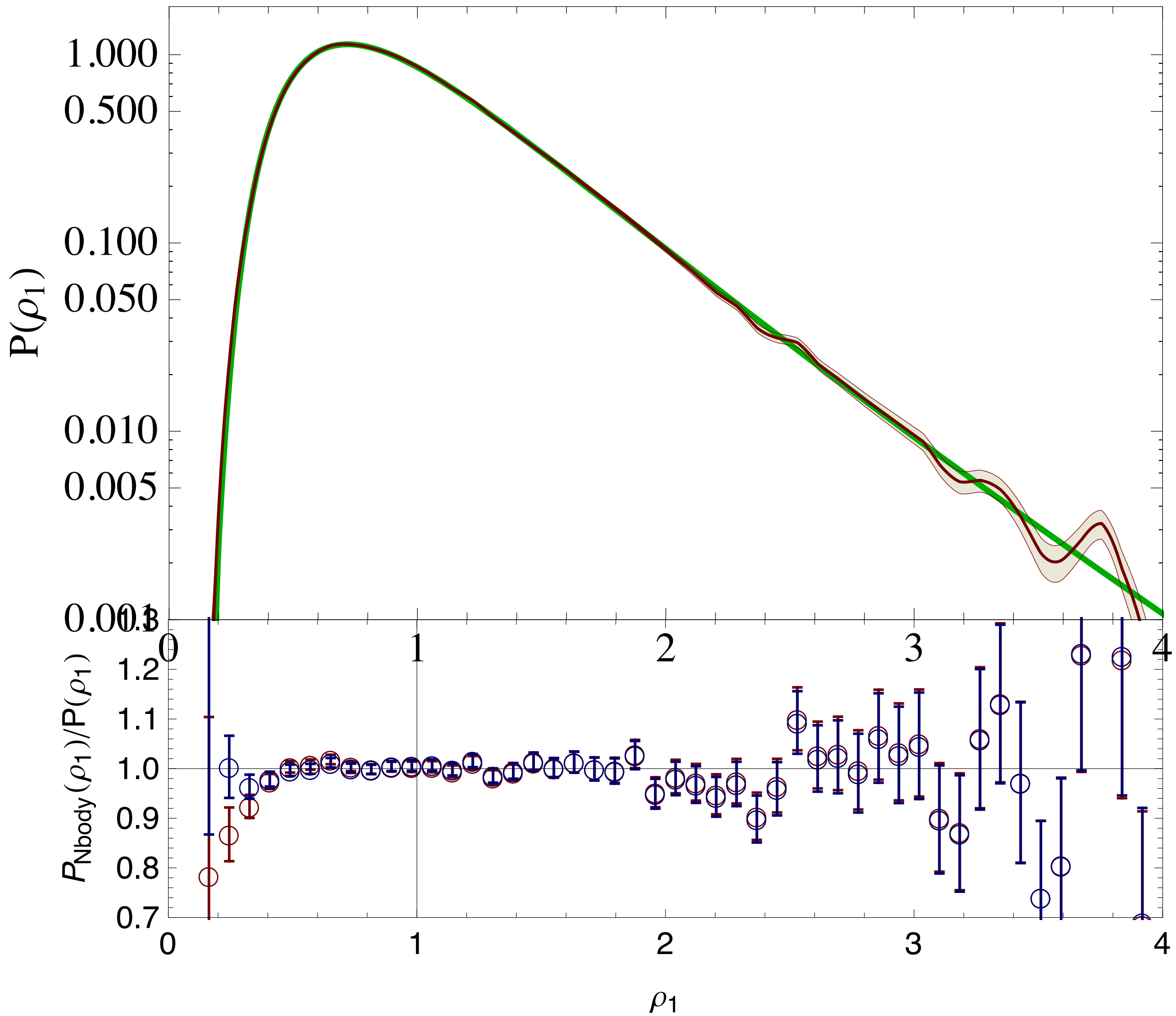
1-cell density cumulants (FB '94)

Application 1: 1-cell PDF and stats

FB Pichon, Codis '13

The inverse Laplace transform,

$$\mathcal{P}(\hat{\rho}_1) = \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \frac{\mathrm{d}\lambda_1}{2\pi \mathrm{i}} \exp(-\lambda_1 \hat{\rho}_1 + \varphi(\lambda_1))$$



R = 10 h^{-1} Mpc

Computation of the 1-cell density PDF, LDP applied to $\mu = \log\hat\rho$

Uhlemann, Codis, FB, Pichon, Reimberg '15

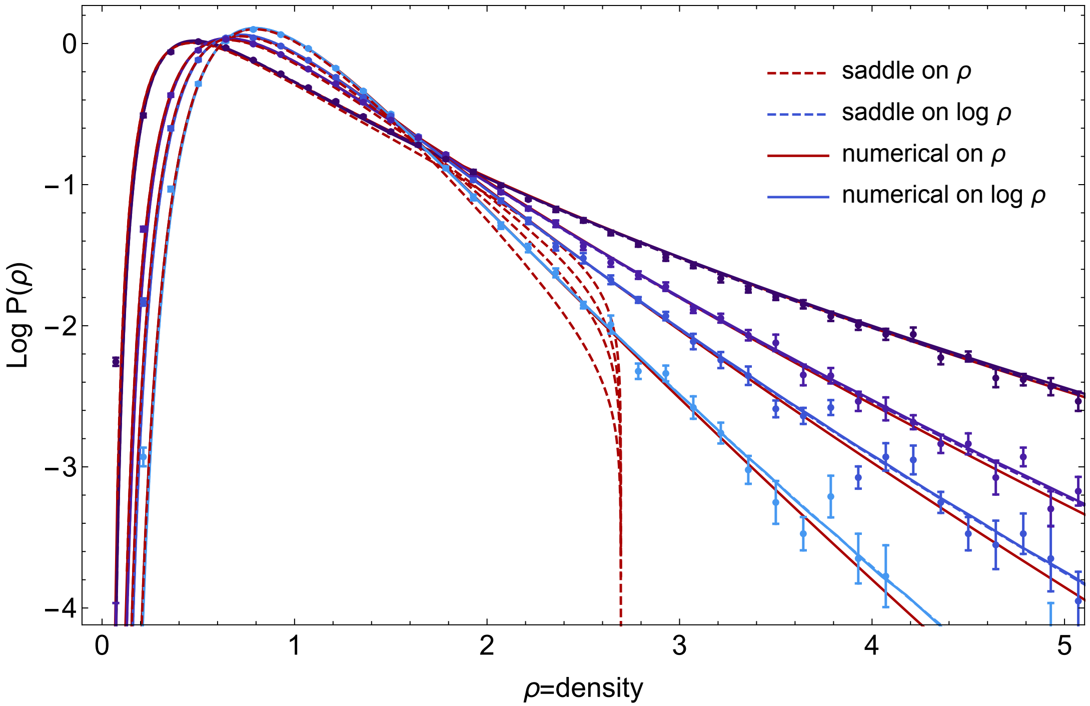
The general expression of the PDF

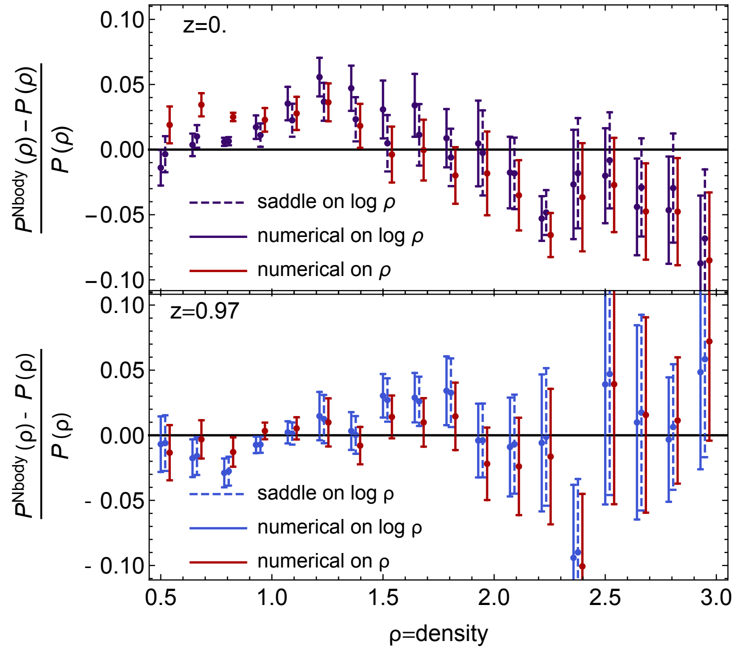
$$\mathcal{P}_{R,\mu}(\mu)\mathrm{d}\mu = \int_{-i\infty}^{+i\infty}\frac{\mathrm{d}\lambda}{2\pi i}\exp[-\lambda\mu+\phi_{R,\mu}(\lambda)]\,,$$

$$\mathcal{P}_R(\hat\rho)\mathrm{d}\hat\rho = \mathcal{P}_{R,\mu}(\log(\hat\rho))\frac{\mathrm{d}\hat\rho}{\hat\rho}\,,$$

The saddle point expression

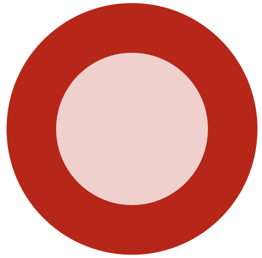
$$\mathcal{P}_R(\hat\rho) = \sqrt{\frac{\Psi_R''[\hat\rho]+\Psi_R'[\hat\rho]/\hat\rho}{2\pi}}\exp\left(-\Psi_R[\hat\rho]\right)$$

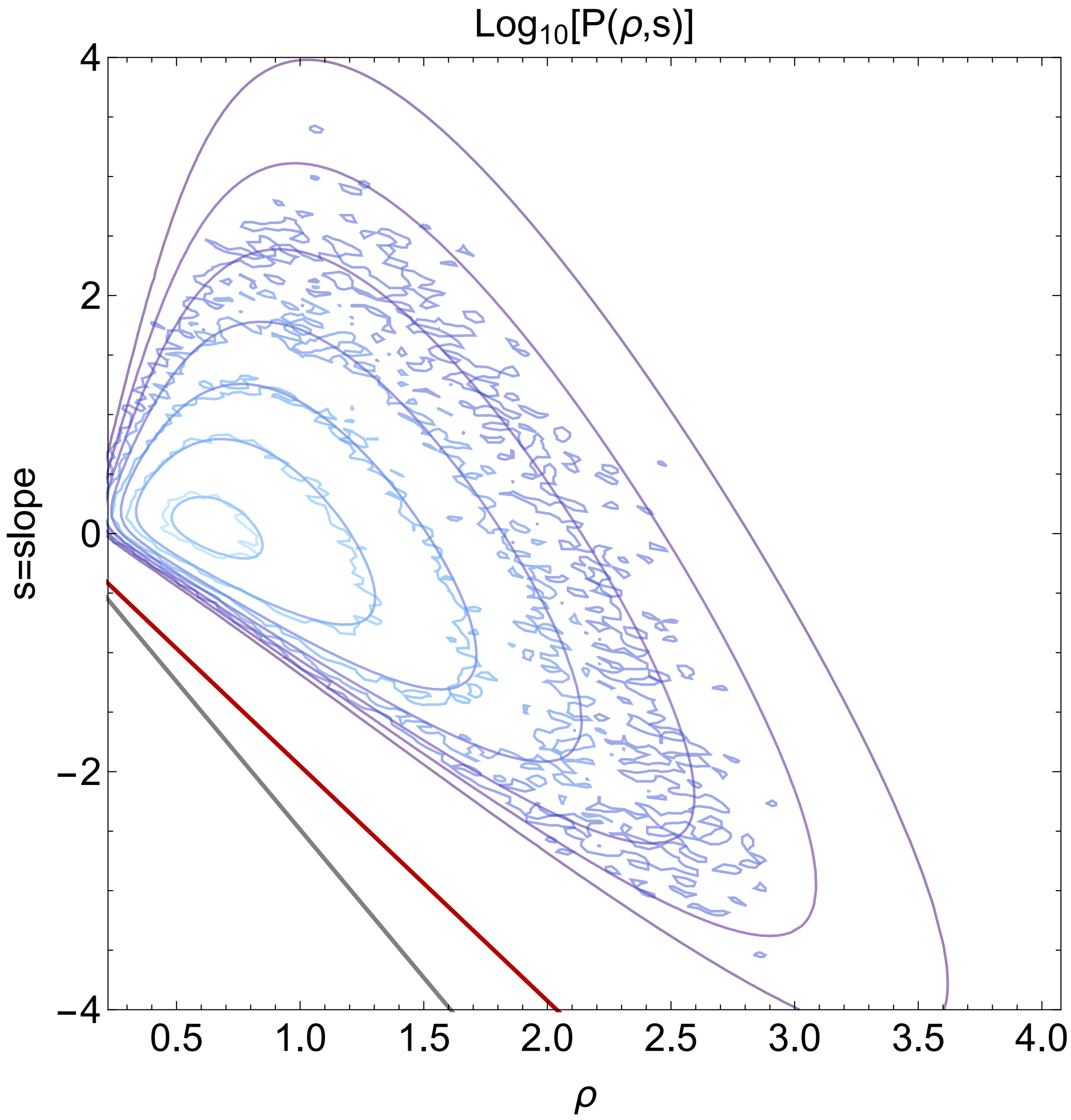




The 2-cell probability distribution function

FB, Pichon, Codis '13
Uhlemann, Codis, FB, Pichon, Reimberg '15





Choice of variables

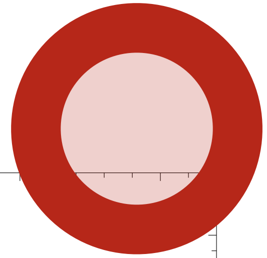
$$\mu_1 = \log\left(r^3\hat{\rho}_2 + \hat{\rho}_1\right)\ ,$$
$$\mu_2 = \log\left(r^3\hat{\rho}_2 - \hat{\rho}_1\right)\ ,$$

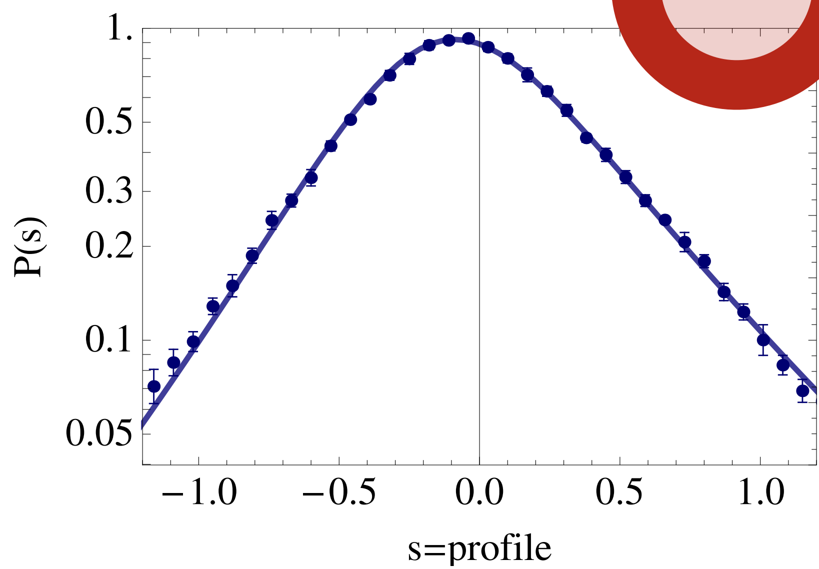
Saddle point expression

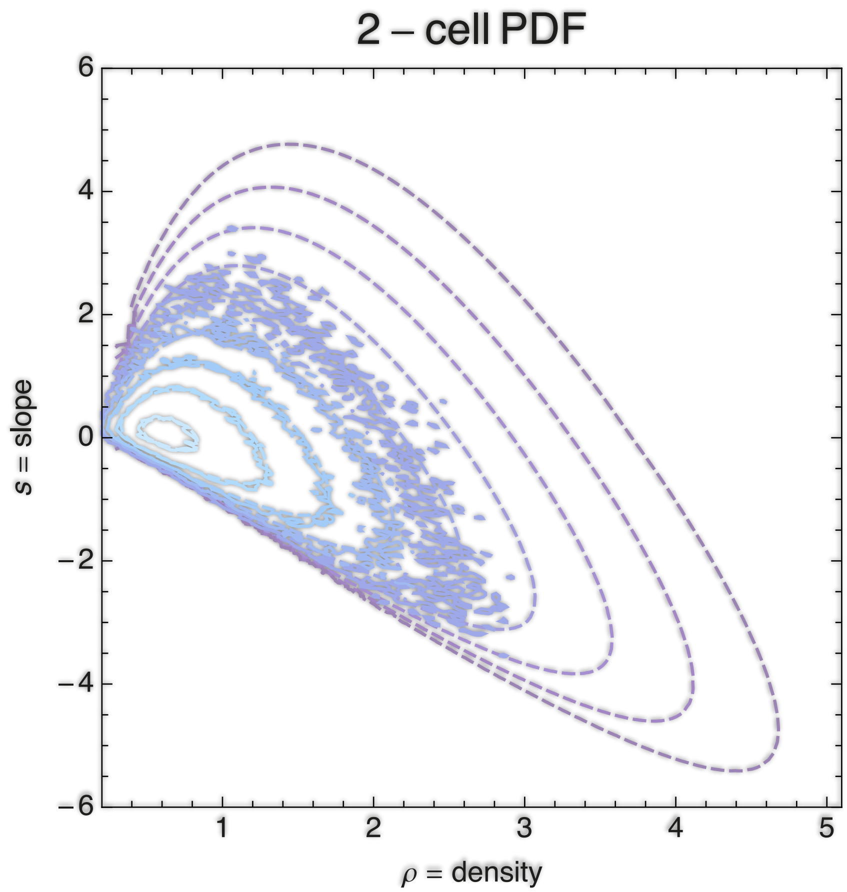
$$\mathcal{P}_{R_1,R_2}(\hat{\rho}_1,\hat{\rho}_2) = \frac{\exp\left[-\Psi_{R_1,R_2}(\hat{\rho}_1,\hat{\rho}_2)\right]}{2\pi} \times \sqrt{\det\left[\frac{\partial^2\Psi_{R_1,R_2}}{\partial\mu_i\partial\mu_j}\right]}\left|\det\left[\frac{\partial\mu_i}{\partial\hat{\rho}_j}\right]\right|$$

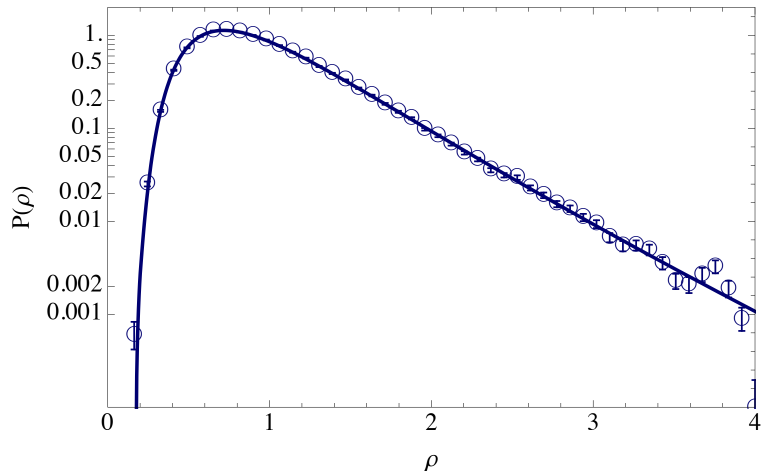
From cumulant to PDFs

FB, Pichon, Codis '15









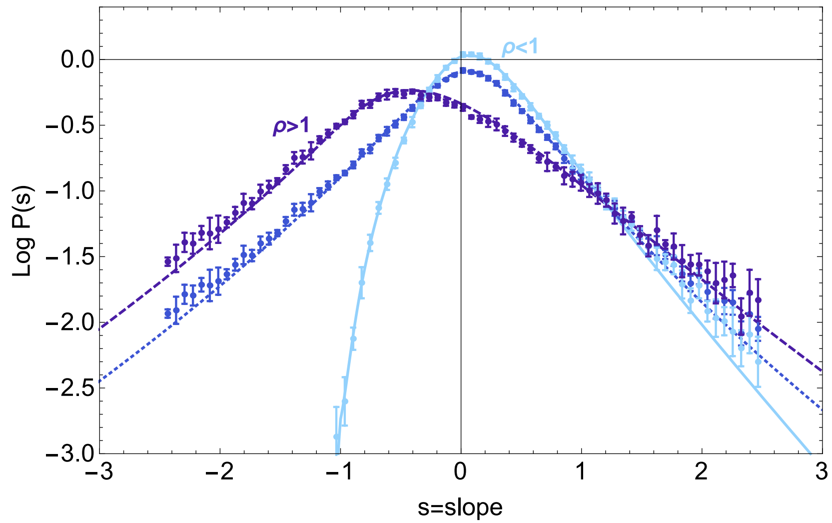
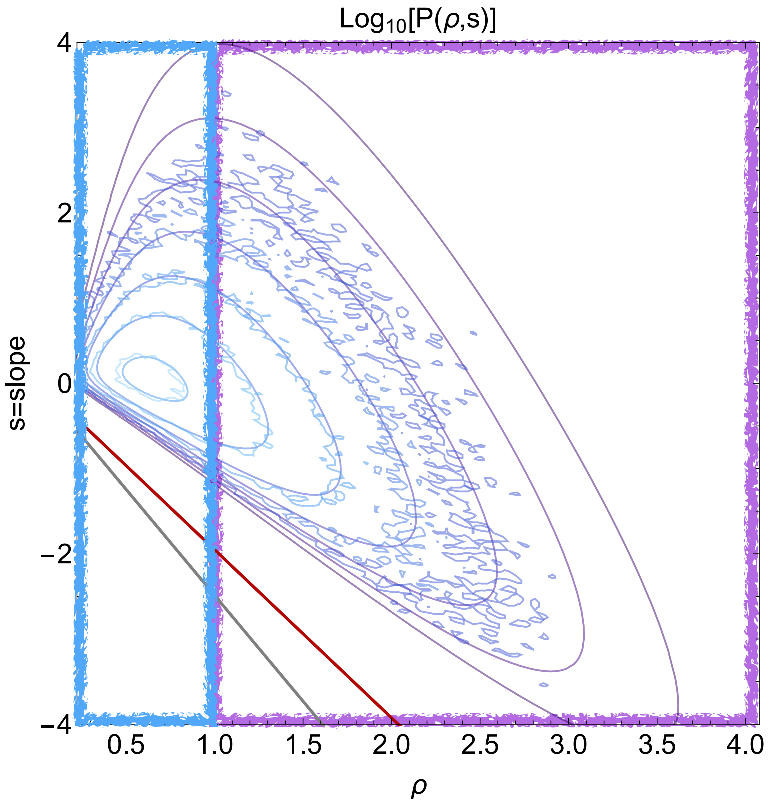


Figure 3. Density profiles in underdense (solid light blue), overdense (dashed purple) and all regions (dashed blue) for cells of radii R_1 = 10 Mpc/h and R_2 = 11 Mpc/h at redshift z = 0.97. Predictions are successfully compared to measurements in simulations (points with error bars).

It opens the way to build constrained PDFs



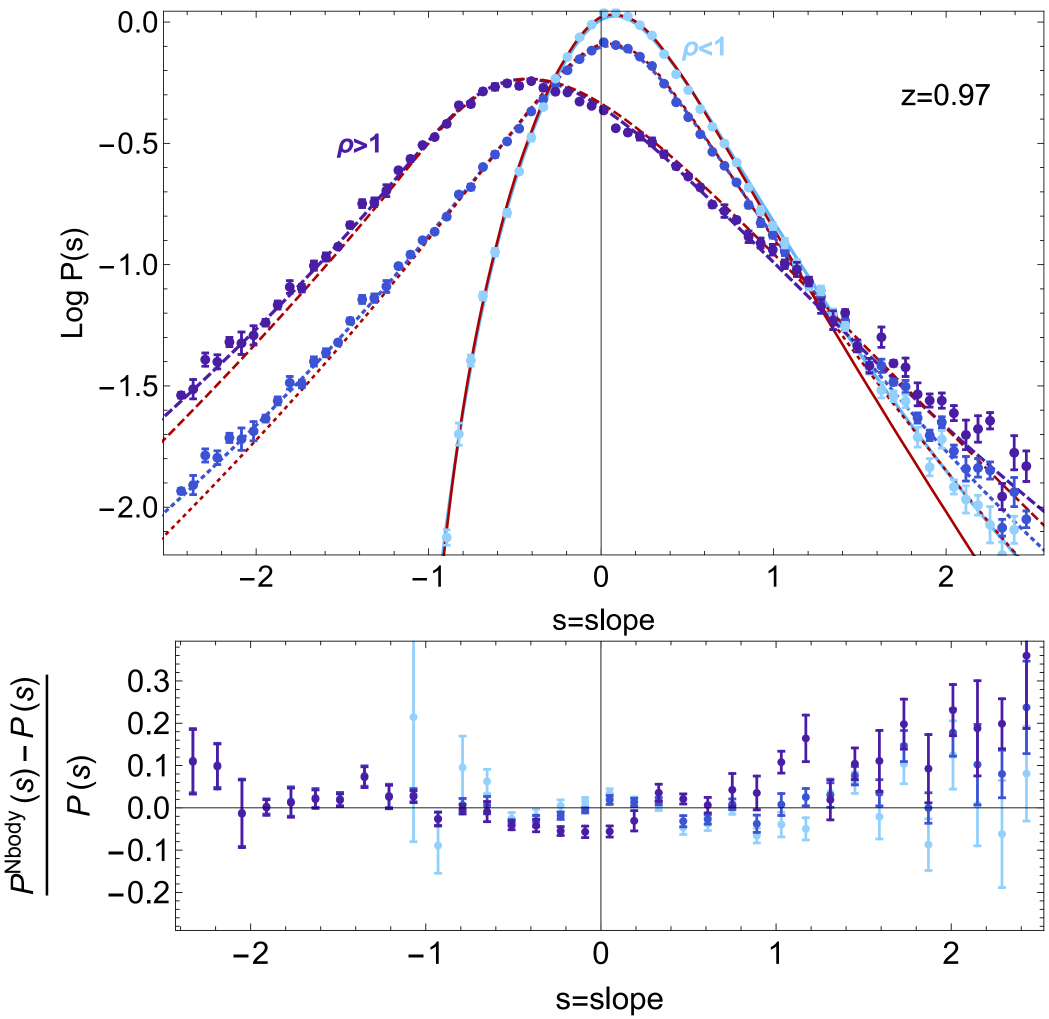
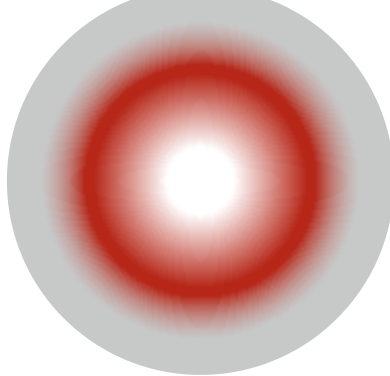


Figure 11. *Top panel:* PDF of the slope, of the slope when the inner density is below one and of the slope when the inner density is above one. Error bars represent the error on the mean as measured in our simulation, red lines represent the numerical integration while blue lines are the log-mass saddle approximation given by equation (30). The agreement is very good for the whole range of density and slope probed by the simulation. *Bottom panel:* residuals of measured slope PDFs compared to the log-mass saddle approximation corresponding to the blue lines in the top panel.

A realistic Mass-aperture statistics



P. Reimberg, FB, '17

$$\varphi(\lambda) = -\inf_{\delta_<^{\rm lin}} \left[\lambda\, \mathrm{M_{ap}}(\delta_<^{\rm lin}) + \frac{\sigma_{\rm F}^2}{2} \int \mathrm{d}\theta\, \mathrm{d}\theta'\, \delta_<^{\rm lin}(\theta)\, \delta_<^{\rm lin}(\theta')\, \xi(\theta,\theta') \right]$$

$$\mathrm{M_{ap}} = \int \mathrm{d}^2\vartheta\, \mathrm{W}(\vartheta)\, \frac{\gamma_{\rm t}}{1-\kappa}$$

- *Gaussian profile*
- *Taking into account the fact that what we measure is the reduced shear (i.e. a non-linear functional of the profile)*

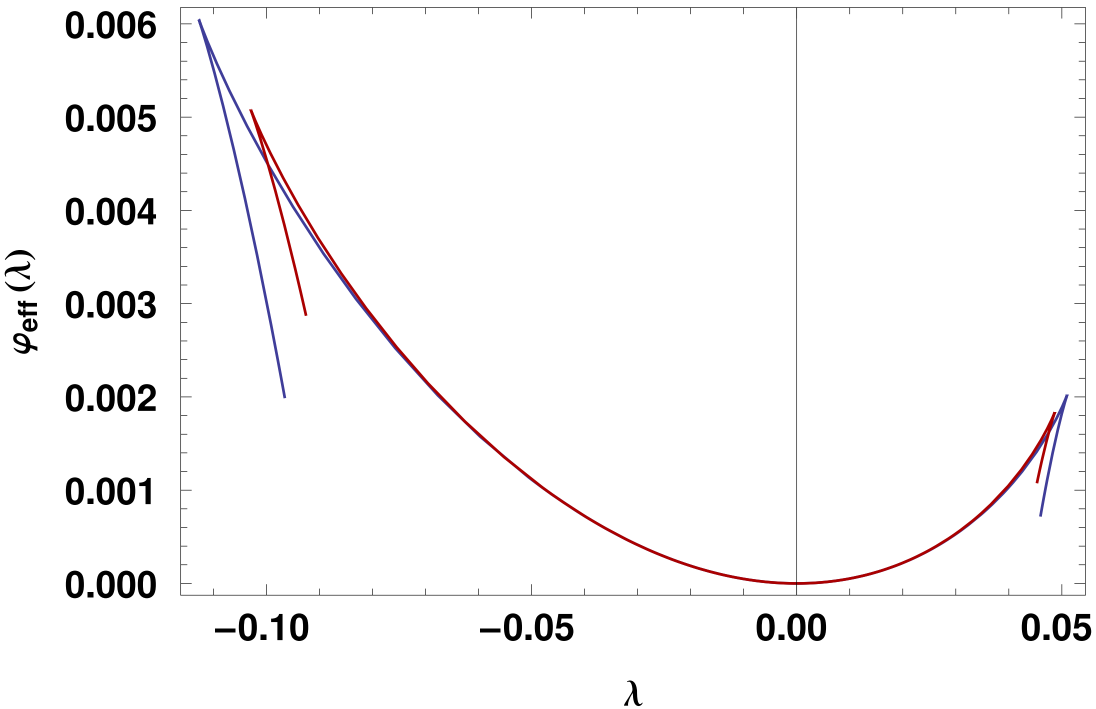


FIG. 5: The effective cumulant generating functions $\varphi^{\kappa}_{\rm eff}$ and $\varphi^{g}_{\rm eff}$ satisfying Eq. (51). The projection factor $w_{\rm eff} = 0.1$ is used on the $\varphi^{g}_{\rm eff}$ data.

Towards a complete theory of cell density statistics…

FB '99, FB, Pichon, Codis, '15

Joint PDFs read in the large-separation limit (no finite separation effects)

$$\mathcal{P}(\{\hat\rho_k\},\{\hat\rho'_k\};r_e) = \mathcal{P}(\{\hat\rho_k\})\mathcal{P}(\{\hat\rho'_k\})\left[1+\xi(r_e)b(\{\hat\rho_k\})b(\{\hat\rho'_k\})\right]$$

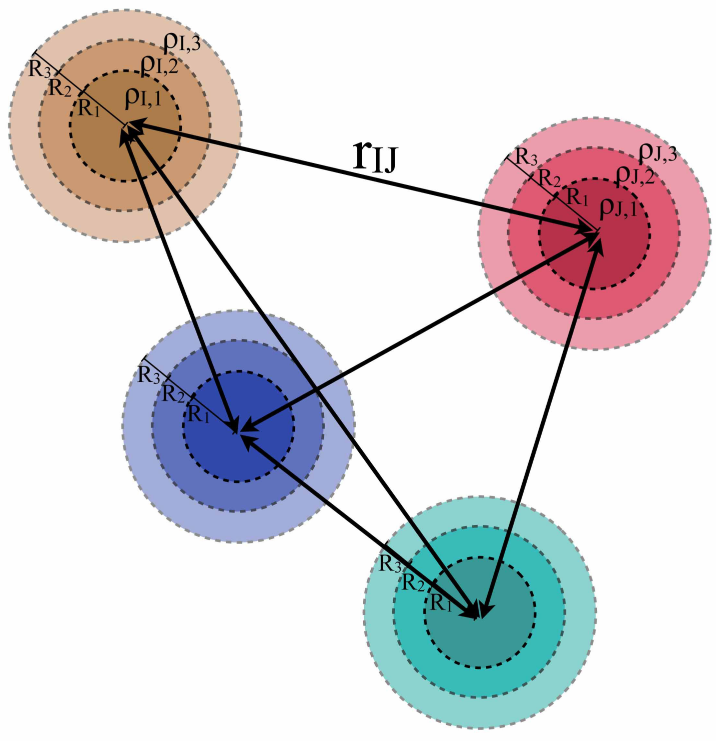


Figure 1. The configuration of spherical cells considered in this paper which is made of multiple sets of concentric spheres separated by distances $r_{\rm IJ}$. Their respective density, $\rho_{\rm I,i}$, corresponds to a set of n spheres of same radii $R_{\rm I,i} \equiv R_i$.

Correlation of measured density probabilities in different locations

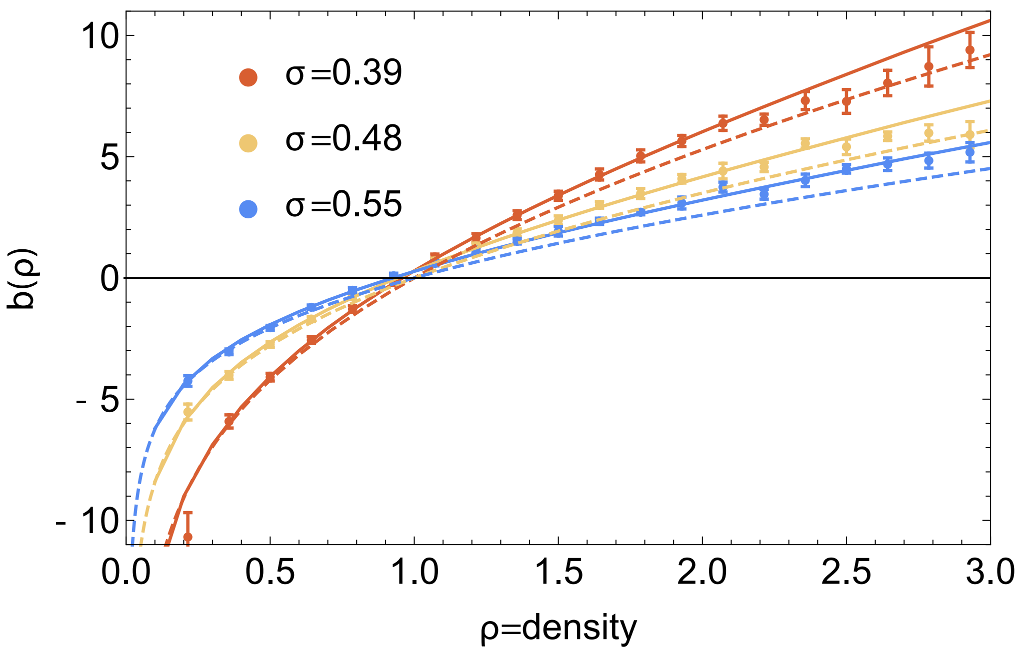
$$\left\langle \hat{\mathcal{P}}(\hat\rho_i)\hat{\mathcal{P}}(\hat\rho_j)\right\rangle = \bar{\mathcal{P}}(\hat\rho_i)\bar{\mathcal{P}}(\hat\rho_j)(1+\xi b_i b_j)$$

Results for the bias for the density and for the slope.

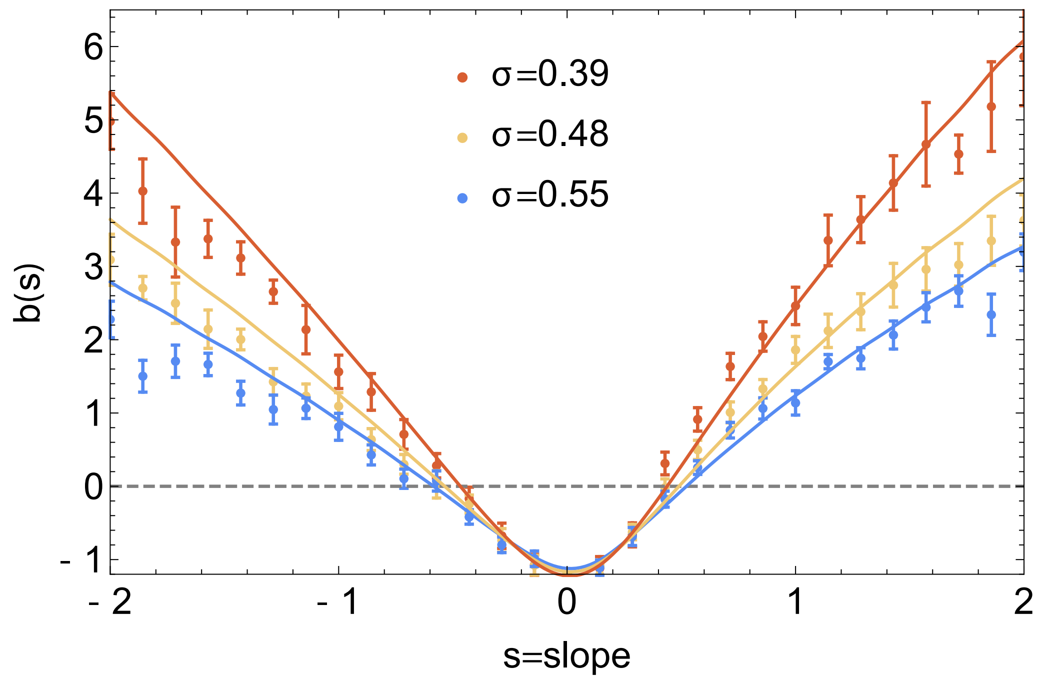
Consistency relations

$$\int_0^\infty \mathrm{d}\rho\ b(\rho)\ P(\rho) = 0$$

$$\int_0^\infty \mathrm{d}\rho\ \rho\, b(\rho)\ P(\rho) = 1$$



For the slope



A regime of large-deviation functions can be identified in LSS cosmology.

- *Observables can be related to joint PDFs of the density in concentric cells but also to the cumulant generating function.*

Perspectives - what are the domains of application ?:

- *These calculations can be applied to 3D and projected mass maps, and to joint density of multiple tracers;*
- *biasing of over-dense/under-dense regions can also be computed = statistical properties of clipped regions;*
- *it can be applied to some non-linear transforms of the density field;*
- *other configuration/geometries ?*