

Towards the analysis of the
redshift-space
bispectrum

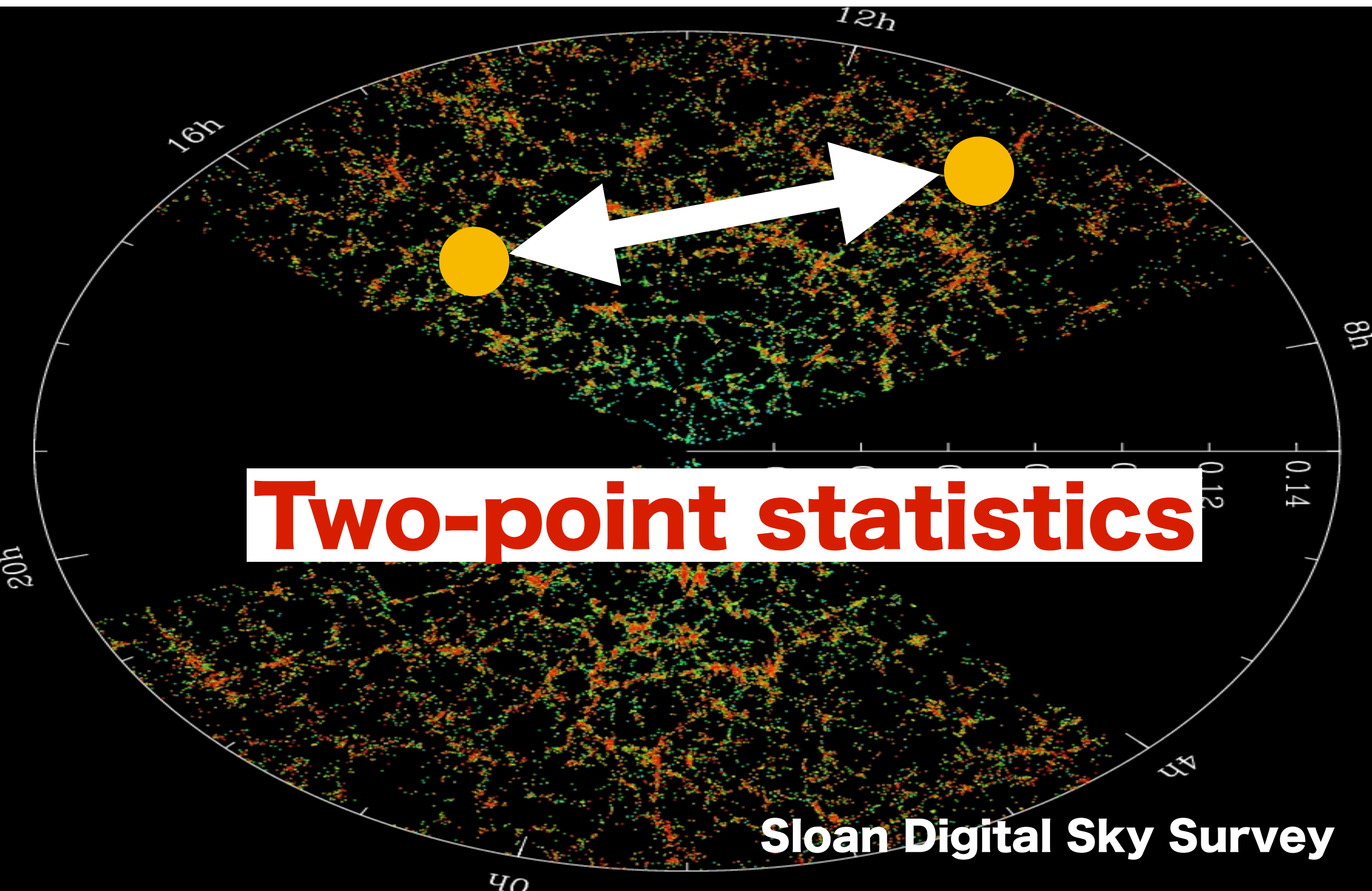
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Collaborators: Shun Saito, Florian Beutler and Hee-Jong Seo



PTchat @ Yukawa Institute (April 8-12, 2019)

3D galaxy map



SDSS DR12 BOSS (2016)

Dard energy: $w = -1.01 \pm 0.06$

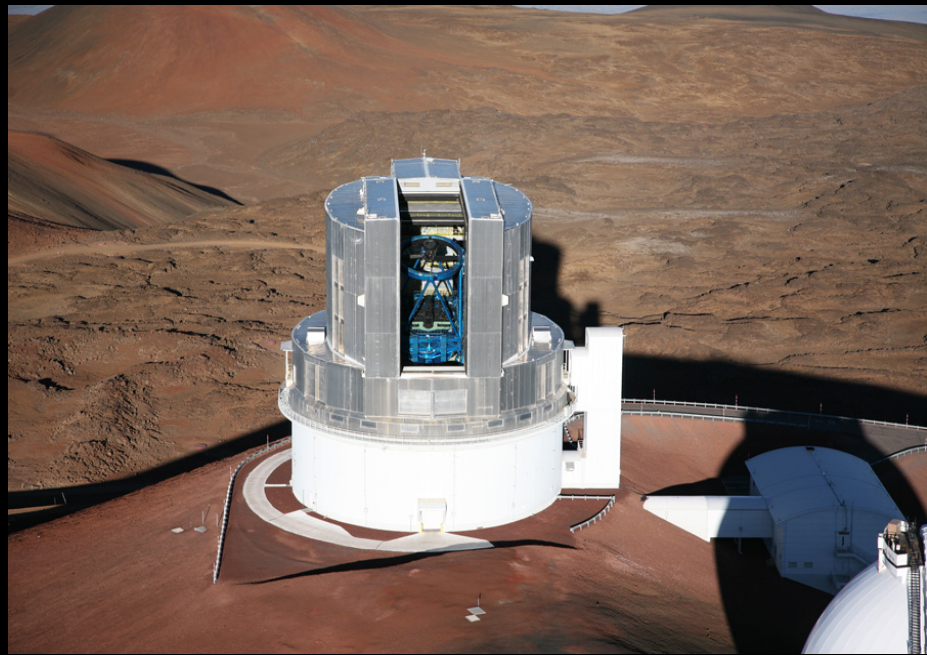
Growth rate(z=0.5): 0.452 ± 0.058

Total neutrino mass: $< 0.16 \text{ eV}/c^2$

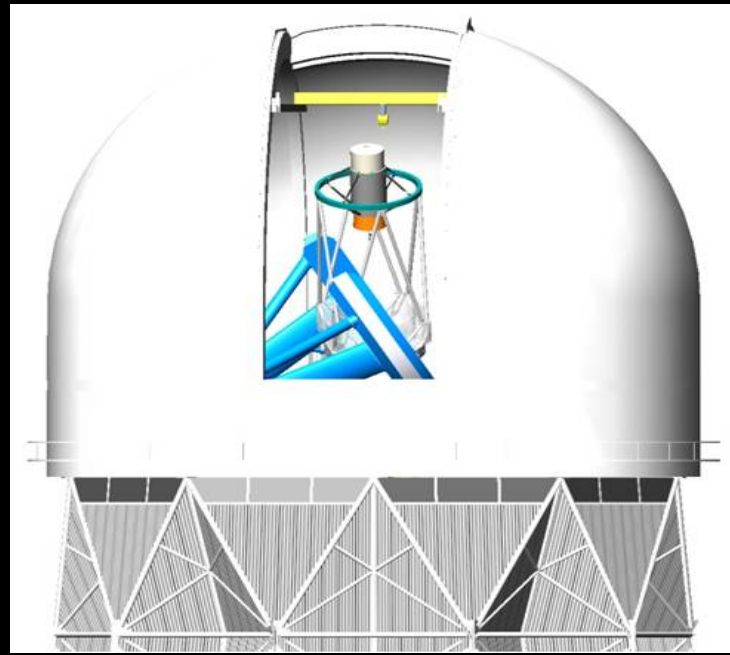
Consistent with LCDM.

Strong upper limit on total neutrino mass.

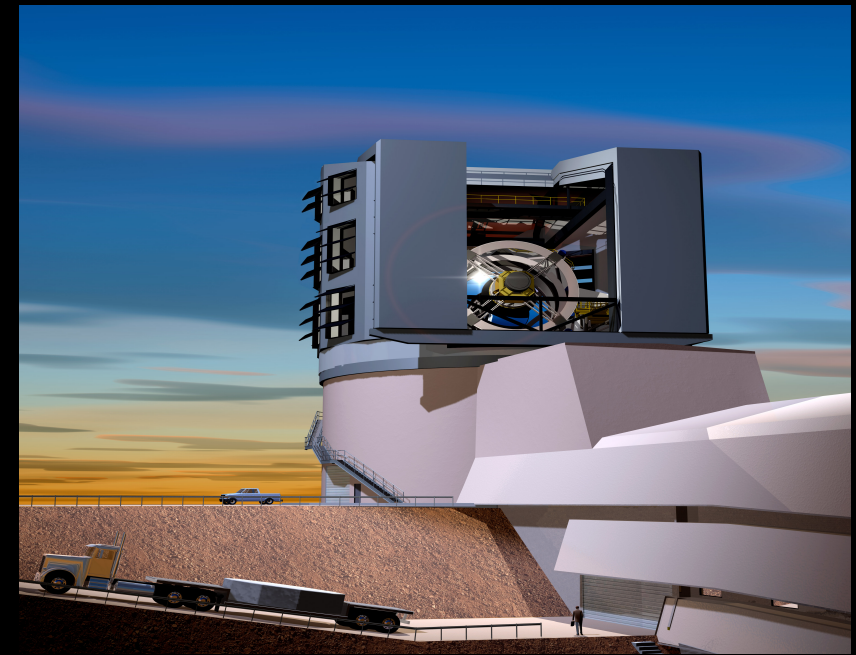
Next Generation Galaxy Surveys



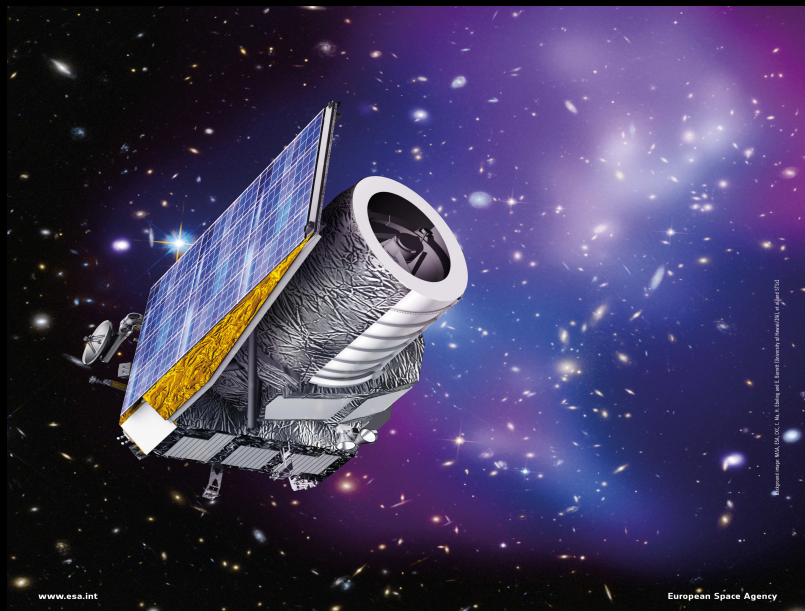
**SuMIRe HSC/PFS
(2015-25)**



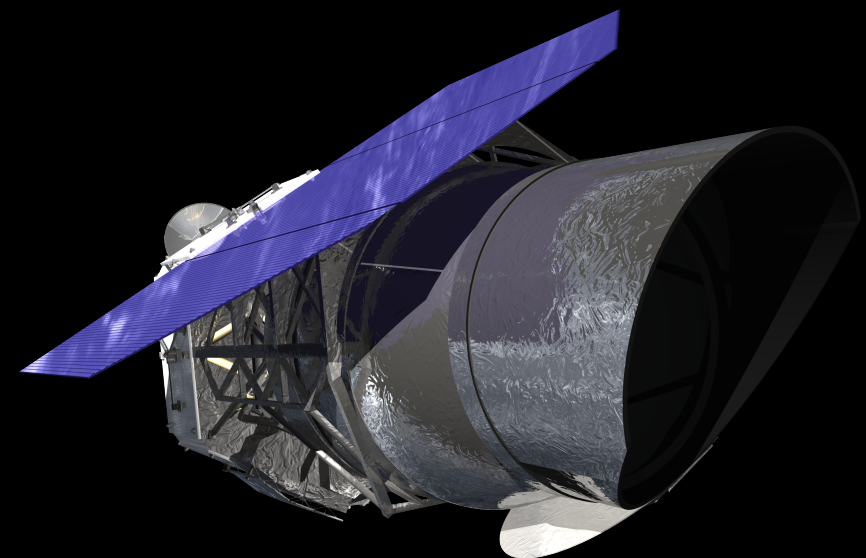
DESI (4m, LBL, 2020-)



LSST (6.5m, SLAC, 2022-)

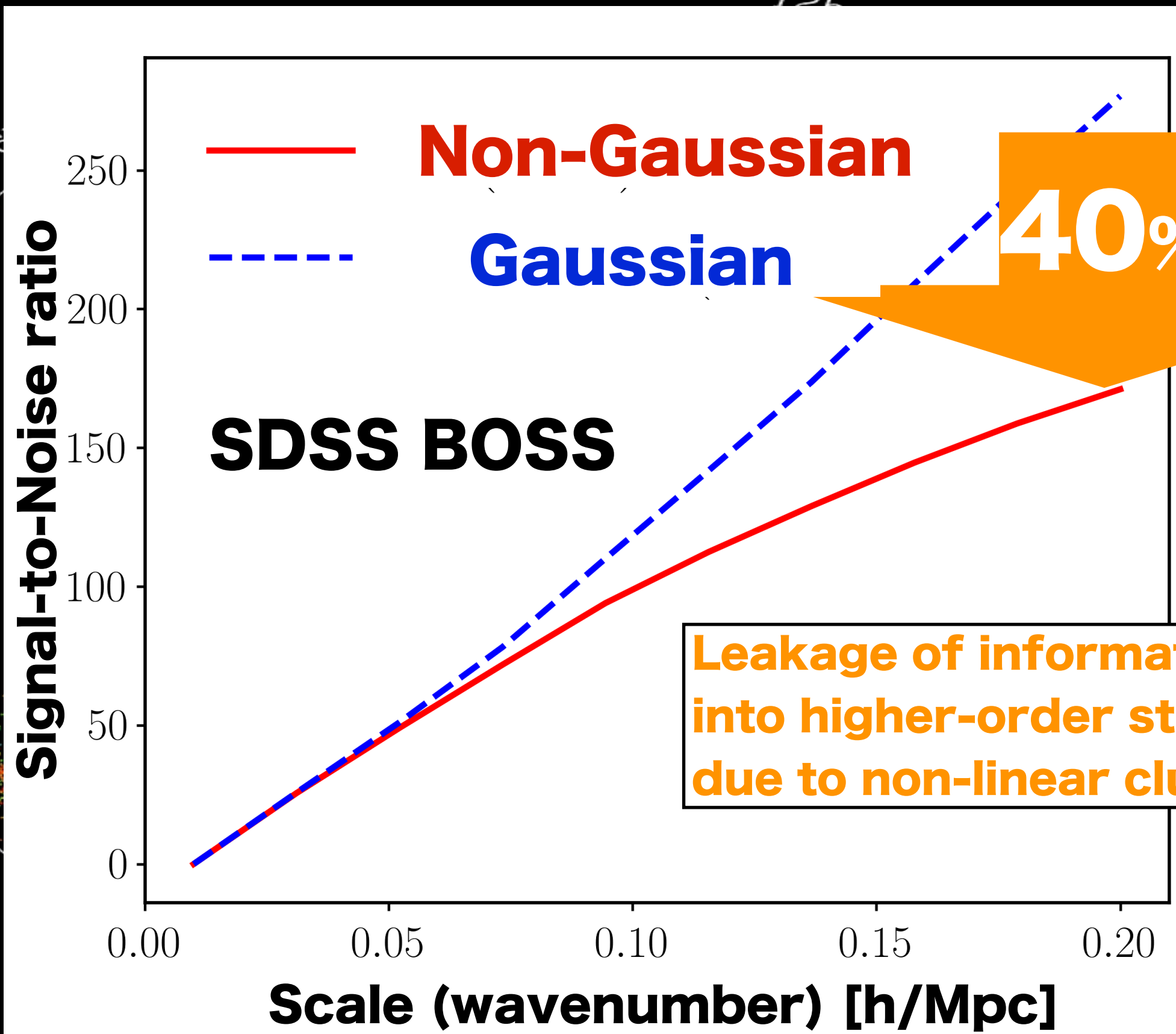


Euclid (ESA, 2022-)

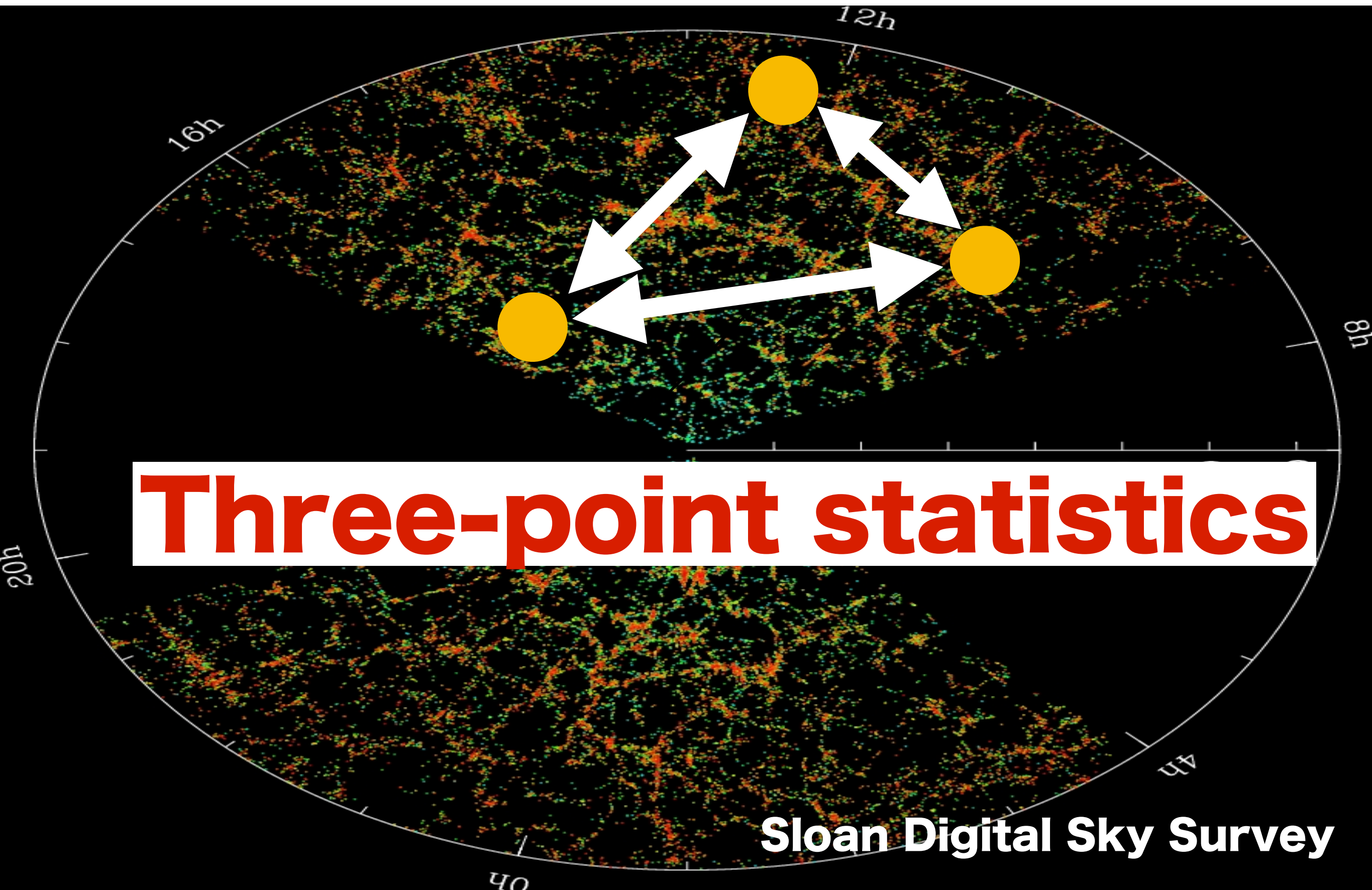


WFIRST (NASA, 2025-)

Limitation of two-point statistics



Towards full information extraction



Three-point statistics

Sloan Digital Sky Survey

History

Angular catalogues:

Peebles & Groth (1975)

Groth & Peebles (1977);
Fry & Slender (1982)

Spectroscopic surveys in configuration-space

Kayo et al. (2004);
Jing & Boerner (2004);
Wang et al. (2004);
Gaztanaga et al. (2005);
Nichol et al. (2006);
Kulkarni et al. (2007);
Gaztanaga et al. (2009);
McBride et al. (2011a, b);
Marin (2011);
Marin et al. (2013);
Guo et al. (2013);
Slepian et al. (2017a,b);

Spectroscopic surveys in Fourier-space

Scoccimarro et al. 2001;
Feldman et al. 2001;
Verde et al. 2002;
Gil-Marin et al. 2015a,b;

Gil-Marin et al. 2017 (SDSS BOSS analysis)

Pearson & Samushia 2017;

**Joint analysis of P + B will become
the standard method for future galaxy surveys.**

**Is there anything else
we should do?**

**Anisotropic bispectrum
analysis has not been
done yet.**

Bispectrum Project

[1] NS, Saito, Beutler and Seo 2018

- A new decomposition formalism
- Detection of the quadrupole bispectrum (14σ)

[2] In progress

- Modeling the bispectrum
- Modeling the bispectrum covariance
- Fisher analysis

[3] Future works

- Analysis using BOSS data

Final goal

- Application to future galaxy survey,
PFS, DESI and Euclid.

Decomposition formalism

$$B(k_1, k_2, \hat{n})$$

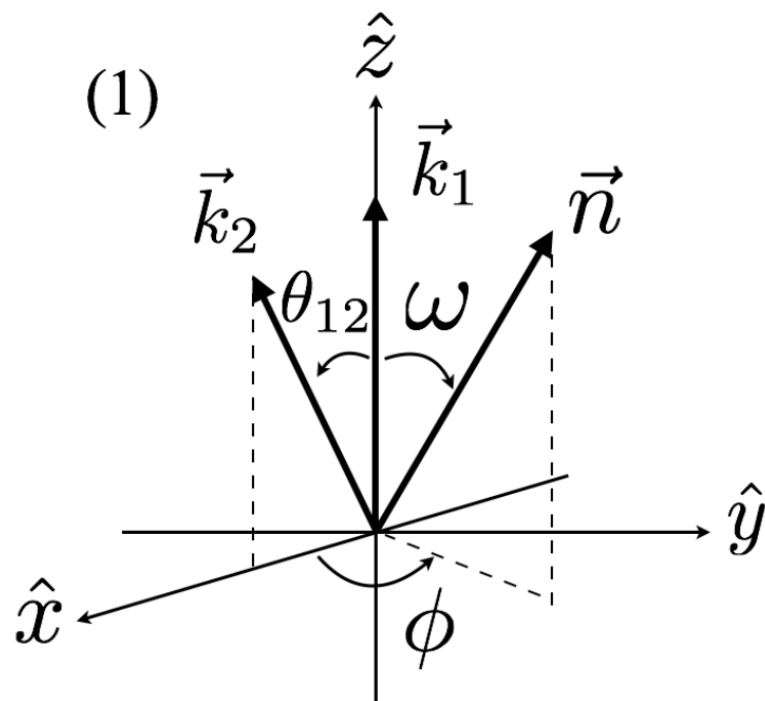
Wavevectors **Line-of-sight**

k_1 is the z-axis

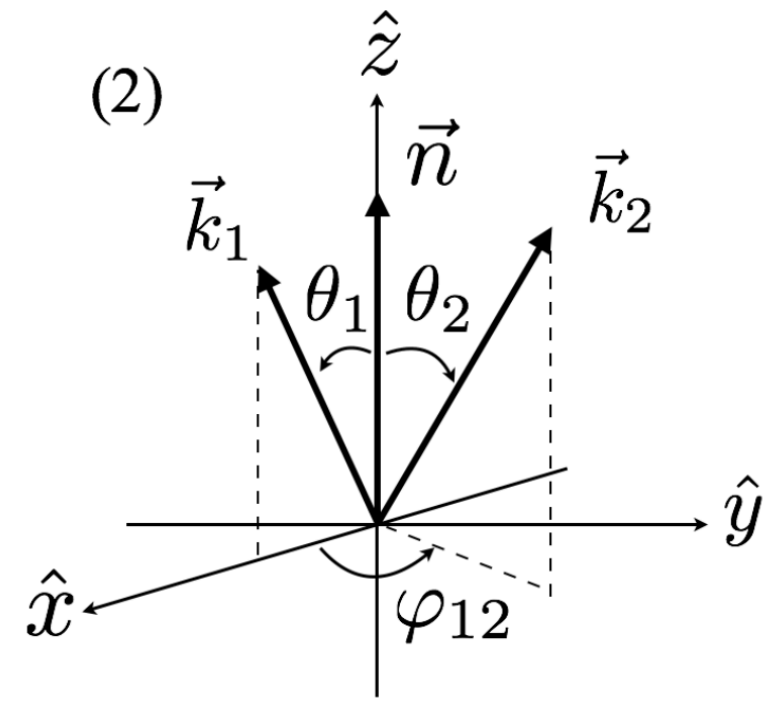
$$B = B_{LM} Y_{LM}(\hat{n})$$

LOS is the z-axis

$$B = B_{l_1 l_2}^m Y_{l_1}^m(\hat{k}_1) Y_{l_2}^{m*}(\hat{k}_2)$$



Scoccimarro et al. (1999)



Slepian et al. (2017)

New decomposition formalism not depending on coordinate systems

1) Expand the bispectrum in three spherical harmonics

$$B_{\ell_1 \ell_2 L}^{m_1 m_2 M}(k_1, k_2) = N_{\ell_1 \ell_2 L} \int \frac{d^2 \hat{k}_1}{4\pi} \int \frac{d^2 \hat{k}_2}{4\pi} \int \frac{d^2 \hat{n}}{4\pi} \\ \times y_{\ell_1}^{m_1*}(\hat{k}_1) y_{\ell_2}^{m_2*}(\hat{k}_2) y_L^{M*}(\hat{n}) B(k_1, k_2, \hat{n}),$$

2) Sum up over all m-modes with wigner 3j symbol.

$$B_{\ell_1 \ell_2 L}(k_1, k_2) \propto \sum_{m_1 m_2 m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_{\ell_1 \ell_2 L}^{m_1 m_2 M}(k_1, k_2)$$

3) Restrict the allowed multipoles to $l_1 + l_2 + L = \text{even}$

New Bispectrum Multipoles

Three multipole indexes

Two wavenumber

$$B_{\ell_1 \ell_2 L}(k_1, k_2)$$

k_1, k_2

LOS

- 1) Translational symmetry
(triangle condition)
- 2) Rotational symmetry
- 3) Parity symmetry

New Bispectrum Multipoles

Three multipole indexes

Two wavenumber

$$B_{\ell_1 \ell_2 L}(k_1, k_2)$$

k_1, k_2

LOS

L = 0: monopole

L = 2: quadrupole

Advantages (1)

For example,

$$\underline{B_{\ell_1 \ell_2 L}(k_1, k_2)} \propto \int \frac{d \cos \theta}{2} \sum_M \begin{pmatrix} \ell_1 & \ell_2 & L \\ 0 & -M & M \end{pmatrix}$$

Ours $\times \mathcal{L}_{\ell_2}^{-M}(\cos \theta) \underline{B_{LM}(k_1, k_2, \theta)}$

Scoccimarro et al. (1999)

Independent of the choice of the coordinate axis.

Advantages (2)

Following Scoccimarro 2015:

$$\hat{B}_{\ell_1 \ell_2 L}(k_1, k_2) = H_{\ell_1 \ell_2 L} \sum_{m_1 m_2 M} \begin{pmatrix} \ell_1 & \ell_2 & L \\ m_1 & m_2 & M \end{pmatrix} \\ \times \frac{N_{\ell_1 \ell_2 L}}{I} \int d^3 x F_{\ell_1}^{m_1}(\mathbf{x}; k_1) F_{\ell_2}^{m_2}(\mathbf{x}; k_2) G_L^M(\mathbf{x}),$$

$$F_{\ell}^m(\mathbf{x}; k) = \int \frac{d^2 \hat{k}}{4\pi} e^{i\mathbf{k} \cdot \mathbf{x}} y_{\ell}^{m*}(\hat{k}) \frac{\delta n|_{\text{FFT}}(\mathbf{k})}{W_{\text{mass}}(\mathbf{k})}$$

$$G_L^M(\mathbf{x}) = \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{x}} \frac{\delta n_L^M|_{\text{FFT}}(\mathbf{k})}{W_{\text{mass}}(\mathbf{k})}.$$

Requires only FFT processes.

Advantages (3)

Double Hankel transform

$$\underline{B_{\ell_1 \ell_2 L}(k_1, k_2)} \propto \int dr_1 r_1^2 \int dr_2 r_2^2 j_{\ell_1}(k_1 r_1) j_{\ell_1}(k_2 r_2) \underline{\zeta_{\ell_1 \ell_2 L}(r_1, r_2)}$$

Bispec.

3PCF

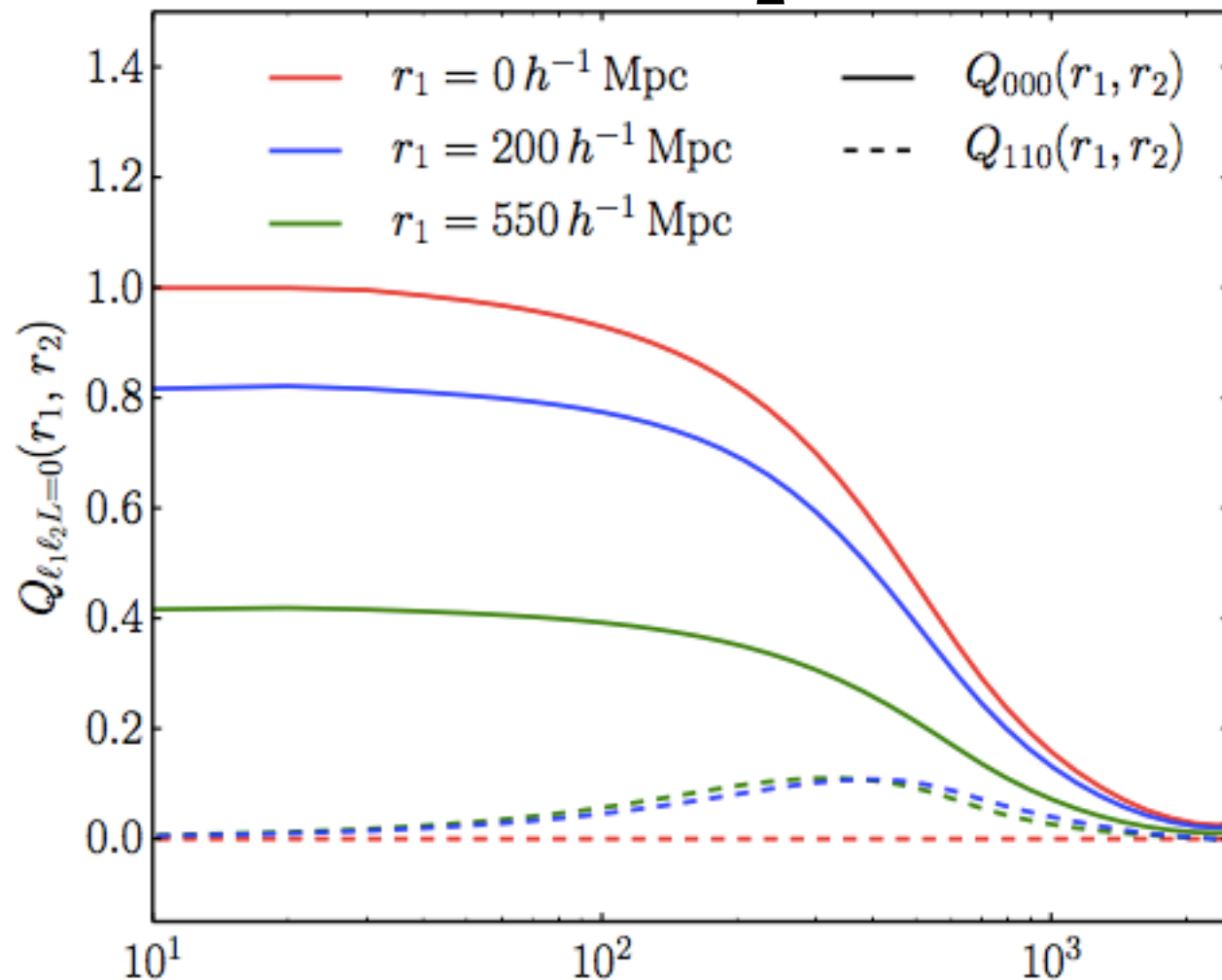
$$\underline{\zeta_{000}^{\text{masked}}(r_1, r_2)} = Q_{000}(r_1, r_2) \zeta_{000}^{\text{theory}}(r_1, r_2) + \underline{Q_{110}(r_1, r_2) \zeta_{110}^{\text{theory}}(r_1, r_2)} + \text{Window function}$$

Gives a simple expression to correct for survey geometry

Three-point window function multipoles

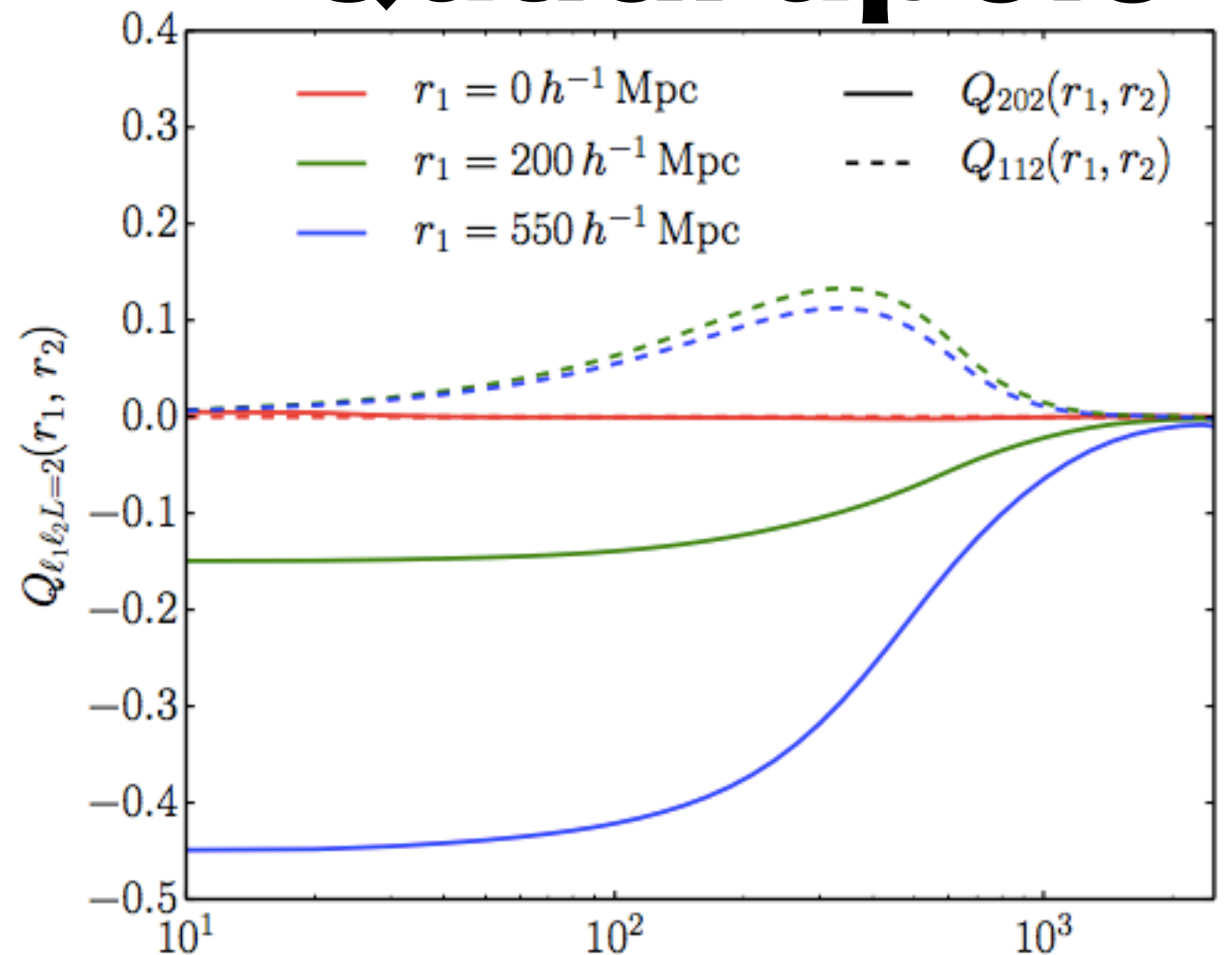
$$Q_{\ell_1 \ell_2 L}(r_1, r_2)$$

Monopole



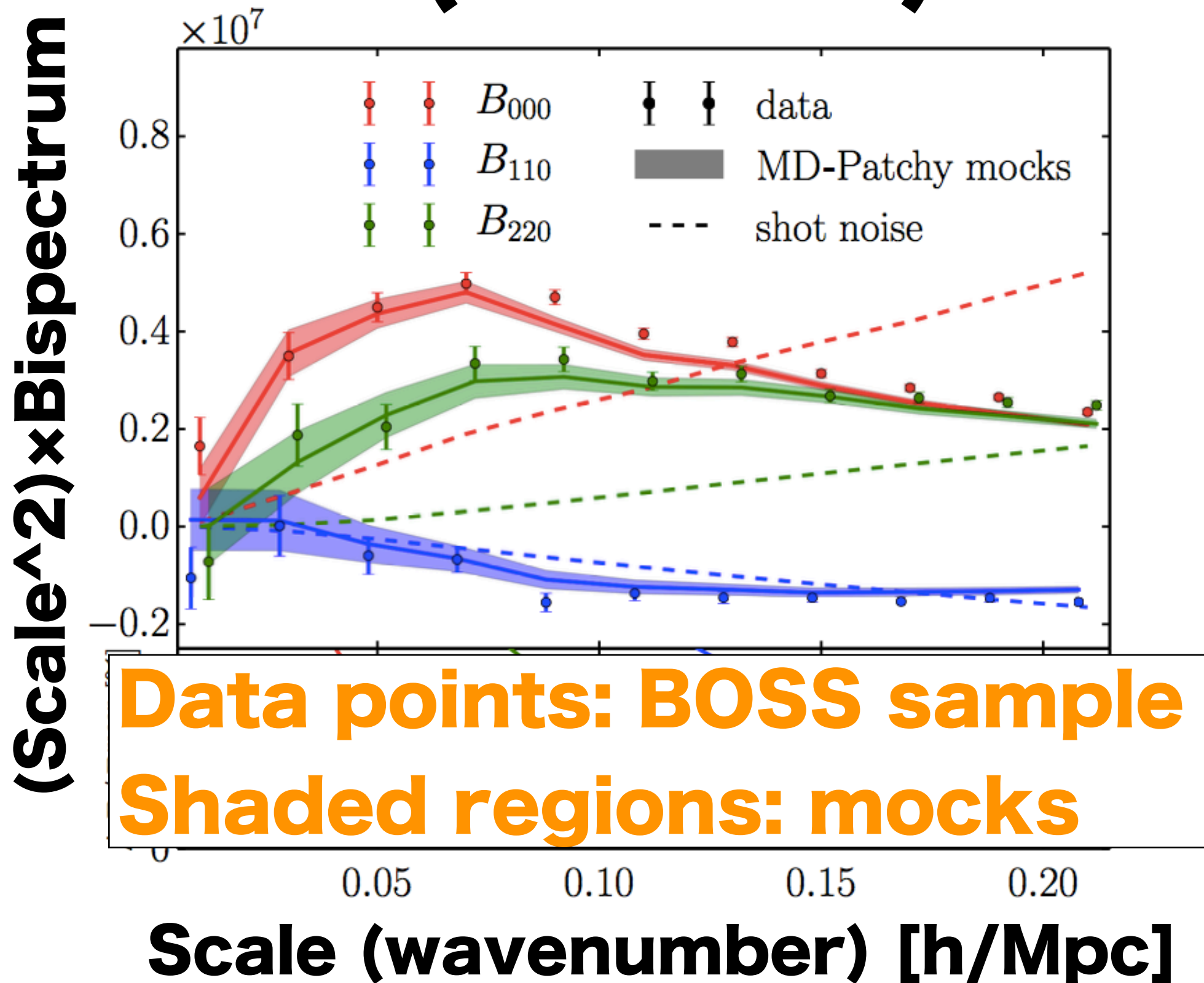
Scale $[r_2]$

Quadrupole



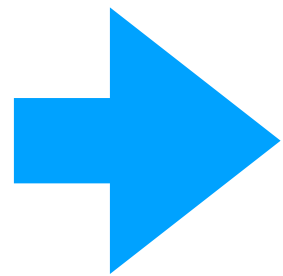
Scale $[r_2]$

Measurements of Monopole (L=0) ($k_1 = k_2$)

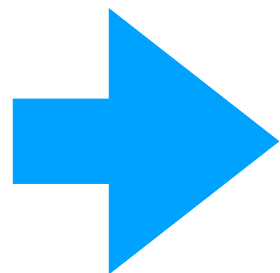


Strategy

1) Focus only on $k_1=k_2$ elements of the bispectrum multipoles.



- **Decreases the number of bins**



- **Smaller number of mocks**

- **Fast measurements**

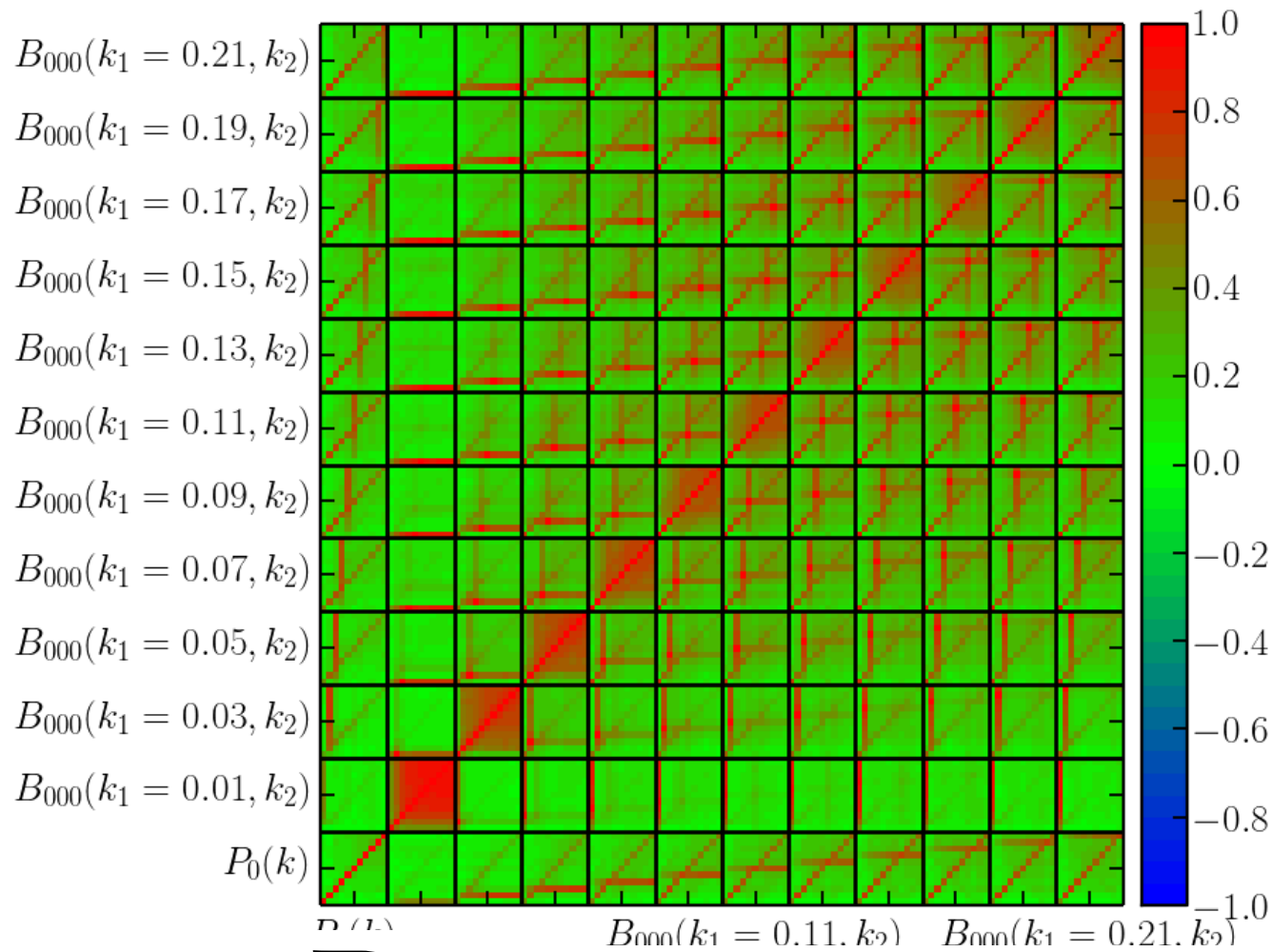
- **Fast analysis**

2) Full bispectrum analysis.

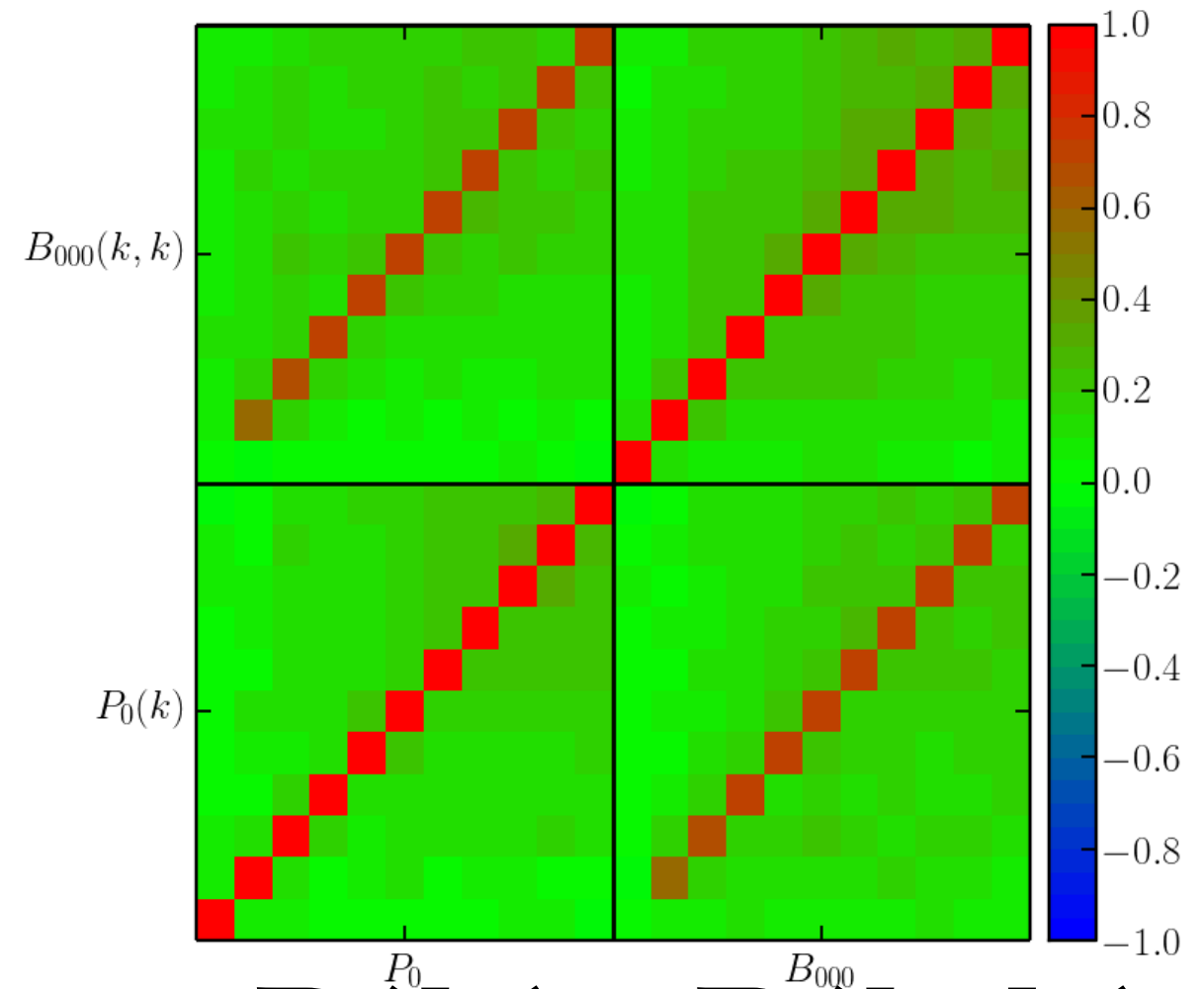
B000

Full(k_1, k_2)

$k_1 = k_2$

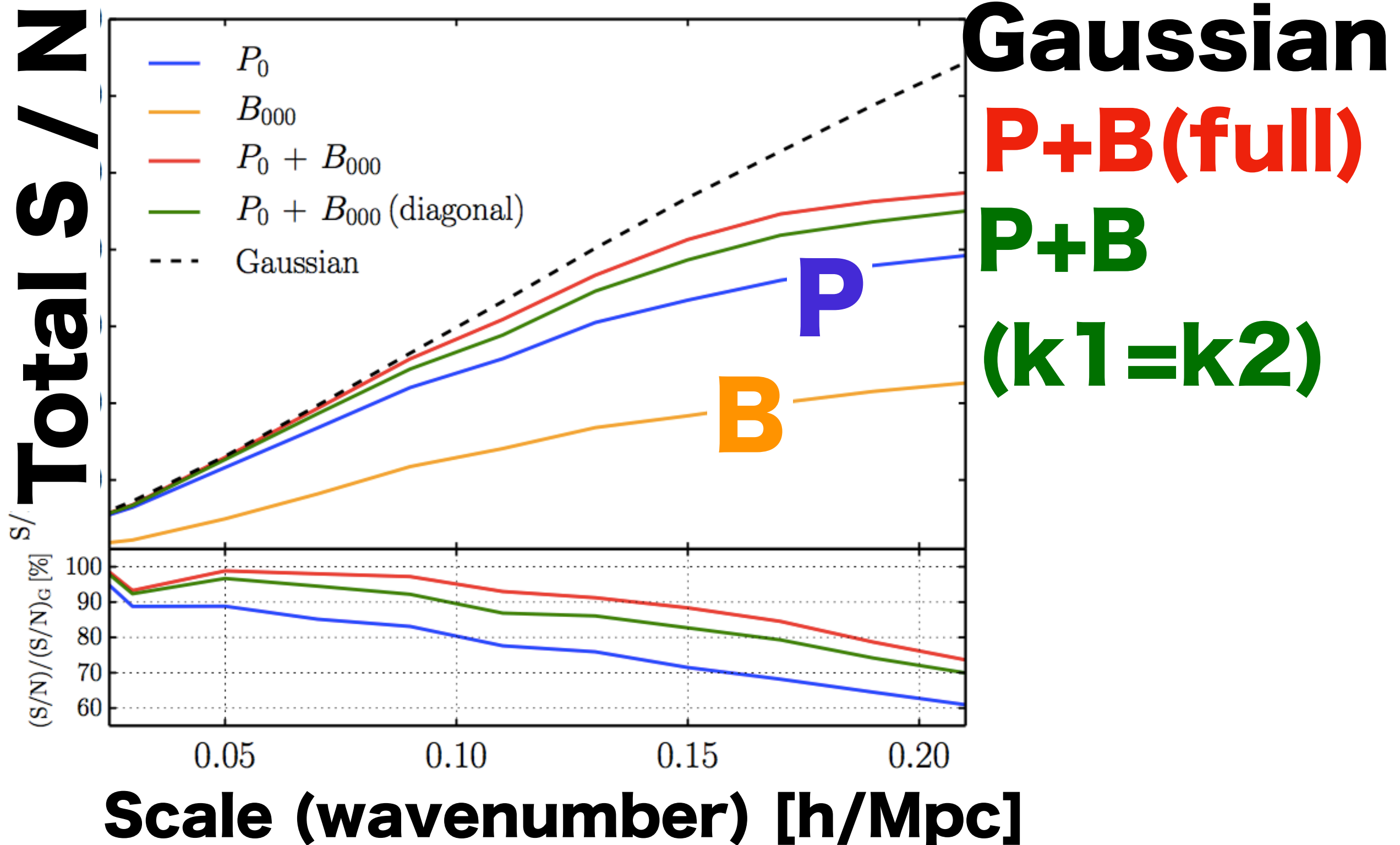


P B(k_1 =fixed, k_2)



P(k) B(k,k)

Cumulative S/N: CMASS North



**How do anisotropic
bispectrum measurements
improve constraints on
cosmological parameters?**

(Preliminary results)

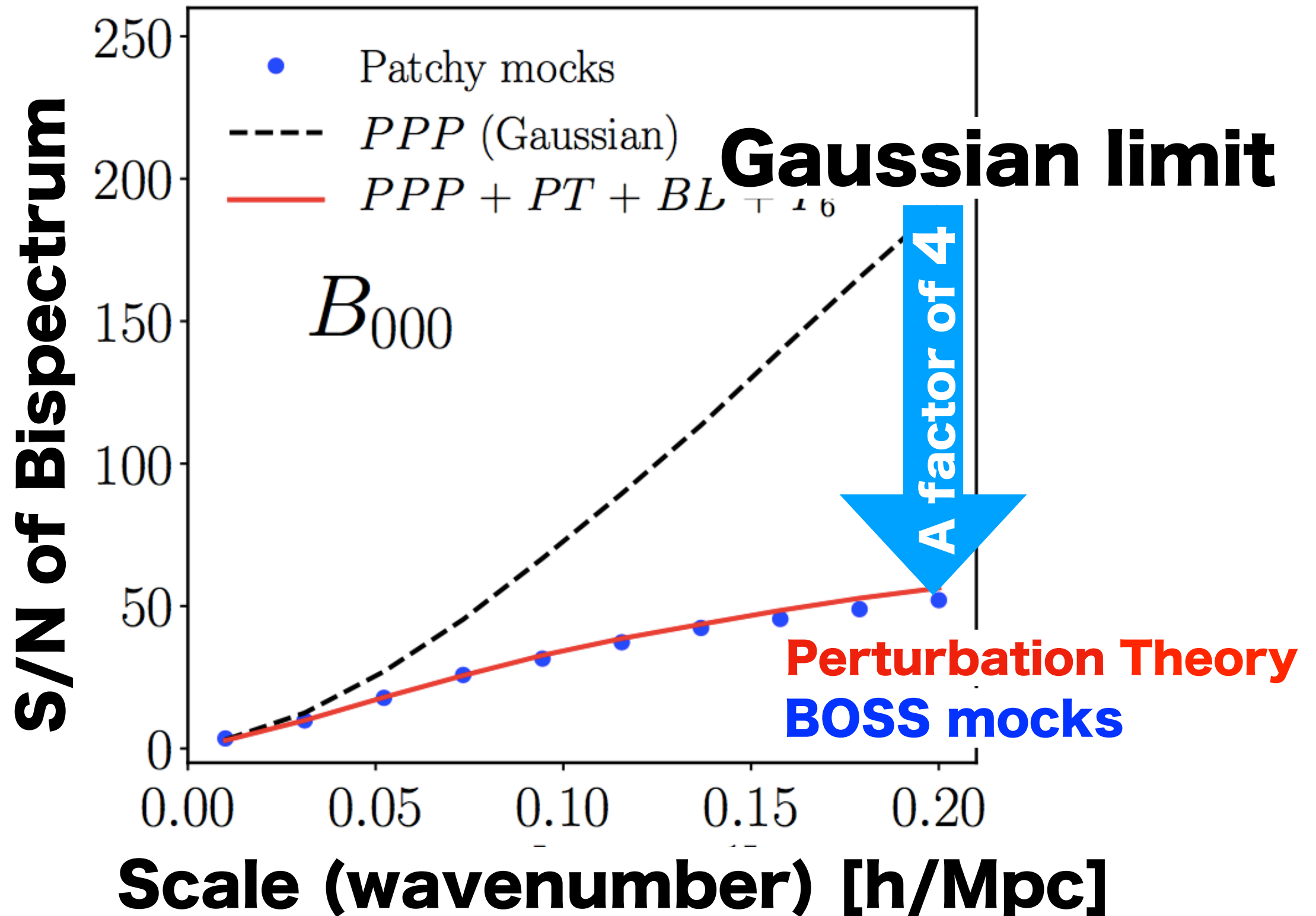
Fisher analysis

Correct error estimates

[non-Gaussian covariance]

Correct theoretical model

Importance of Non-Gaussian errors



Non-Gaussian covariance

P-P covariance -> tri-spectrum

P-B covariance -> 5-point spectrum

B-B covariance -> 6-point spectrum

RSD + linear bias (b1) + shot noise

Shot-noise subtraction effect

$$\text{Cov} (|\delta(\mathbf{k})|^2, |\delta(\mathbf{k})|^2)$$

\neq

$$\text{Cov} (P(\mathbf{k}), P(\mathbf{k}))$$

$$P = |\delta(\mathbf{k})|^2 - \frac{1}{\bar{n}}$$

$$N_{\text{obs}} = N_{\text{true}} (1 + \delta_b)$$

Before shot-noise subtraction

Cov [P, P]

$$\begin{aligned} &= \frac{1}{V} \left\{ T(\mathbf{k}, -\mathbf{k}, \mathbf{k}', -\mathbf{k}') \right. \\ &+ (1/\bar{n}) [B(\mathbf{k}, -\mathbf{k}, \mathbf{0}) + B(\mathbf{k}', -\mathbf{k}', \mathbf{0}) \\ &\quad + B(\mathbf{k}, \mathbf{k}', -\mathbf{k} - \mathbf{k}') + B(\mathbf{k}, -\mathbf{k}, -\mathbf{k} + \mathbf{k}') + B(-\mathbf{k}, \mathbf{k}', \mathbf{k} - \mathbf{k}') + B(-\mathbf{k}, -\mathbf{k}', \mathbf{k} + \mathbf{k}')] \\ &+ (1/\bar{n}^2) [2P(\mathbf{k}) + 2P(\mathbf{k}') + P(\mathbf{0}) + P(\mathbf{k} + \mathbf{k}') + P(\mathbf{k} - \mathbf{k}')] \\ &\left. + (1/\bar{n}^3) \right\}. \end{aligned}$$

After shot-noise subtraction

Cov [P, P]

$$\begin{aligned} &= \frac{1}{V} \left\{ T(\mathbf{k}, -\mathbf{k}, \mathbf{k}', -\mathbf{k}') \right. \\ &+ (1/\bar{n}) [B(\mathbf{k}, \mathbf{0}) + B(\mathbf{k}', \mathbf{0}) \\ &\quad + B(\mathbf{k}, \mathbf{k}', -\mathbf{k} - \mathbf{k}') + B(\mathbf{k}, -\mathbf{k}, -\mathbf{k} + \mathbf{k}') + B(-\mathbf{k}, \mathbf{k}', \mathbf{k} - \mathbf{k}') + B(-\mathbf{k}, -\mathbf{k}', \mathbf{k} + \mathbf{k}')] \\ &+ (1/\bar{n}^2) [2P(\mathbf{k}) + 2P(\mathbf{k}') + P(\mathbf{k} + \mathbf{k}') + P(\mathbf{k} - \mathbf{k}')] \\ &+ (1/\bar{n}^2) \left. \right\}. \end{aligned}$$

Shot-noise subtraction effect

$$\text{Cov} (B, B)$$

\neq

$$\text{Cov} (\delta^3, \delta^3)$$

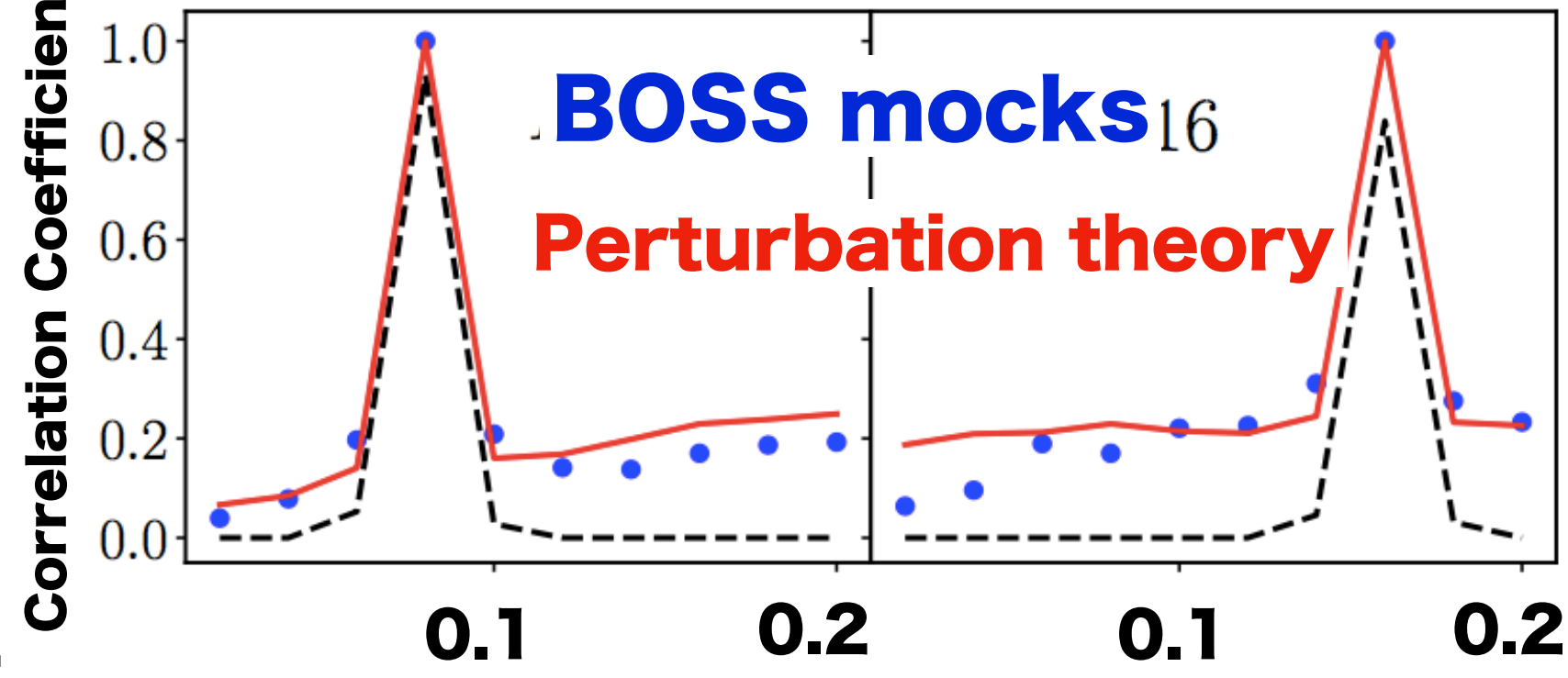
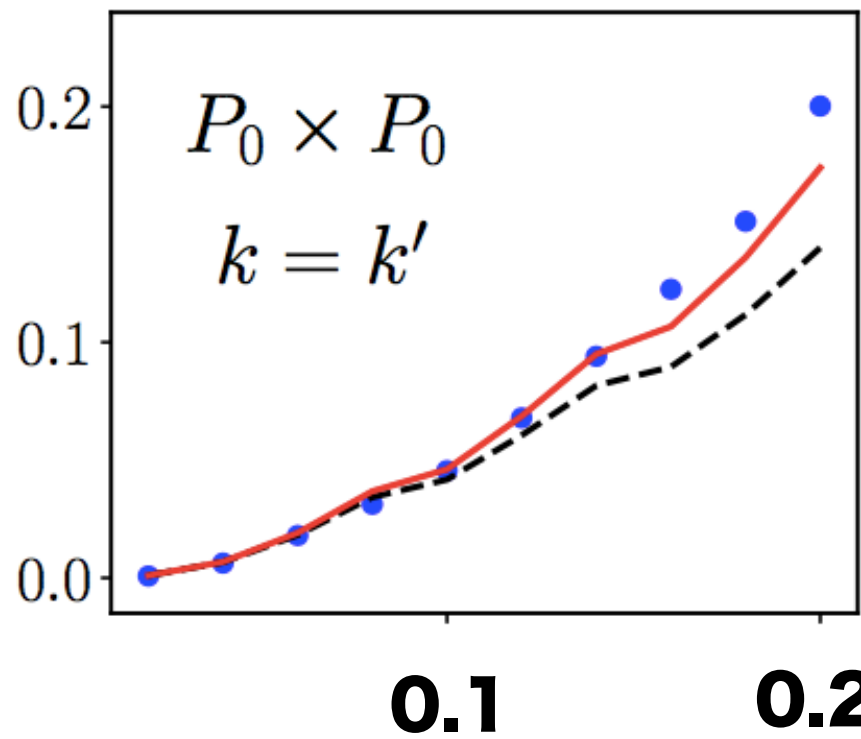
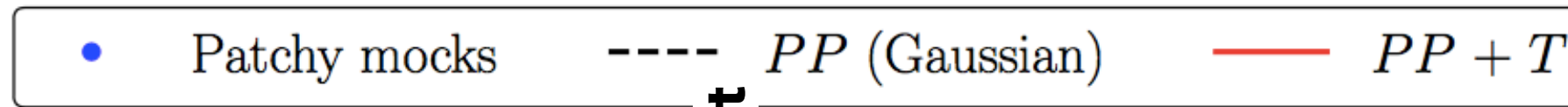
$$B = \delta^3 - \frac{1}{\bar{n}} P - \frac{1}{\bar{n}^2}$$

Cov [P0, P0]

Diagonal

Off-diagonal (**Trispectrum**)

Covariance $\times k^6$



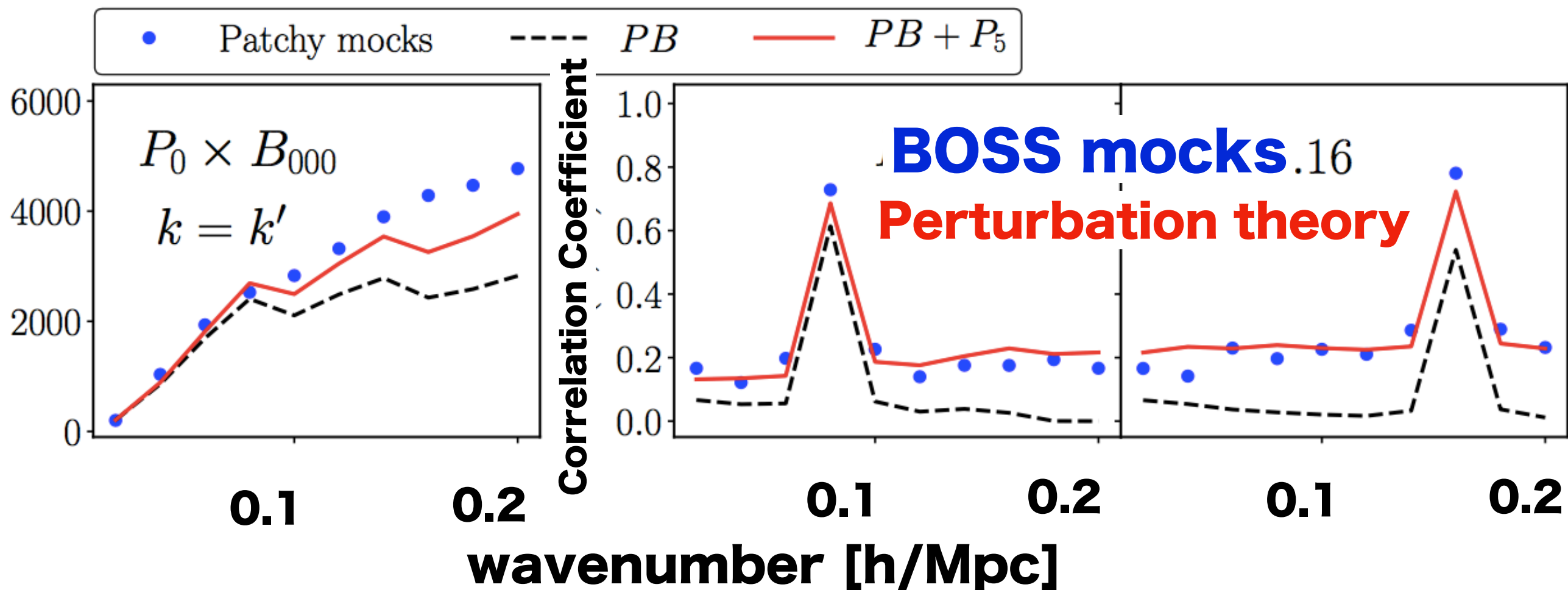
$$\text{Cov}[P, P] = PP + T$$

Cov [PO, BOOO]

Diagonal

Off-diagonal (5point spectrum)

Covariance $\times k^6$

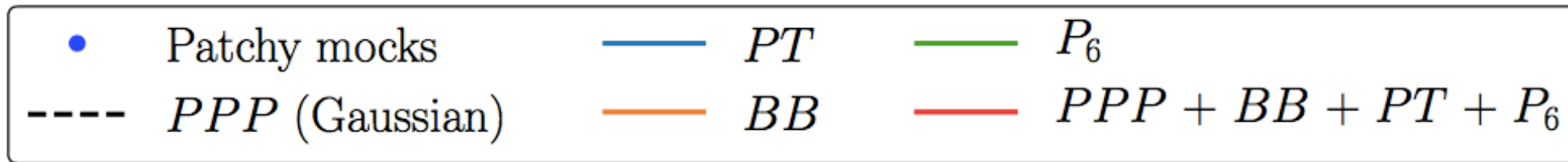


$$\text{Cov}[P,B] = PB + P5$$

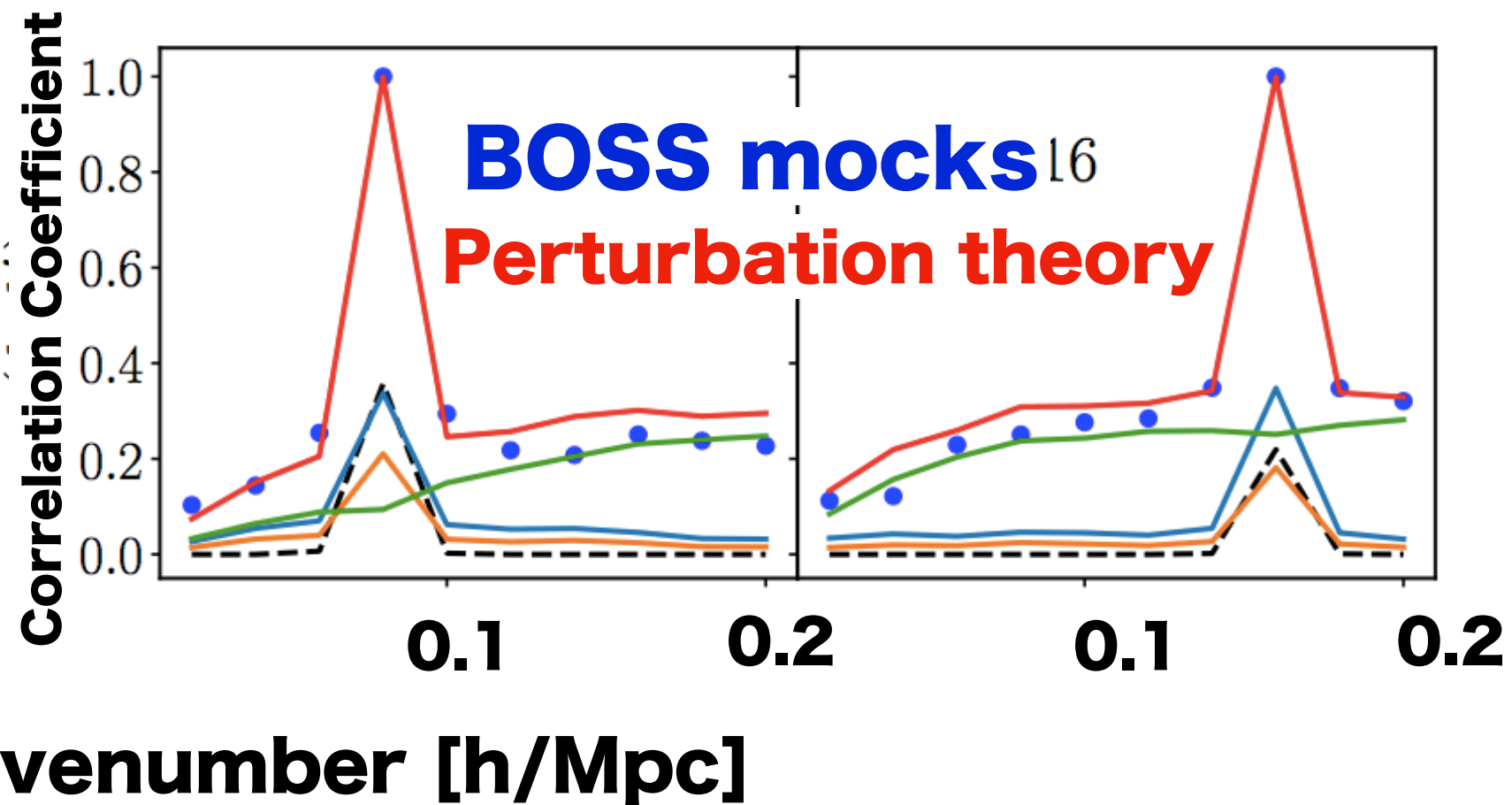
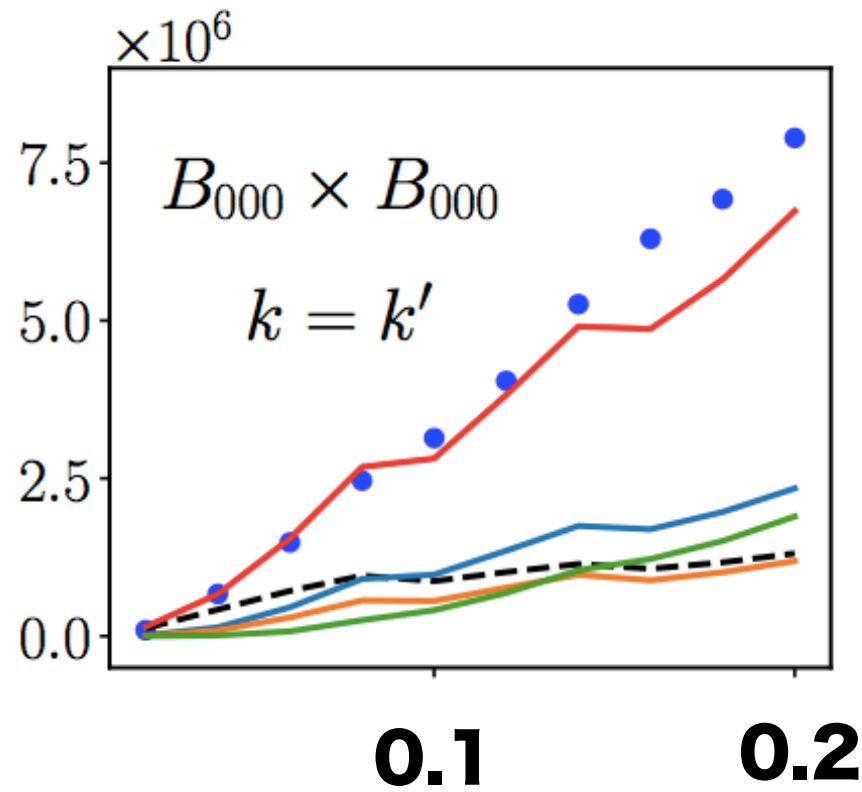
Cov [B000, B000]

Diagonal

Off-diagonal (6point spectrum)



Covariance x k^6

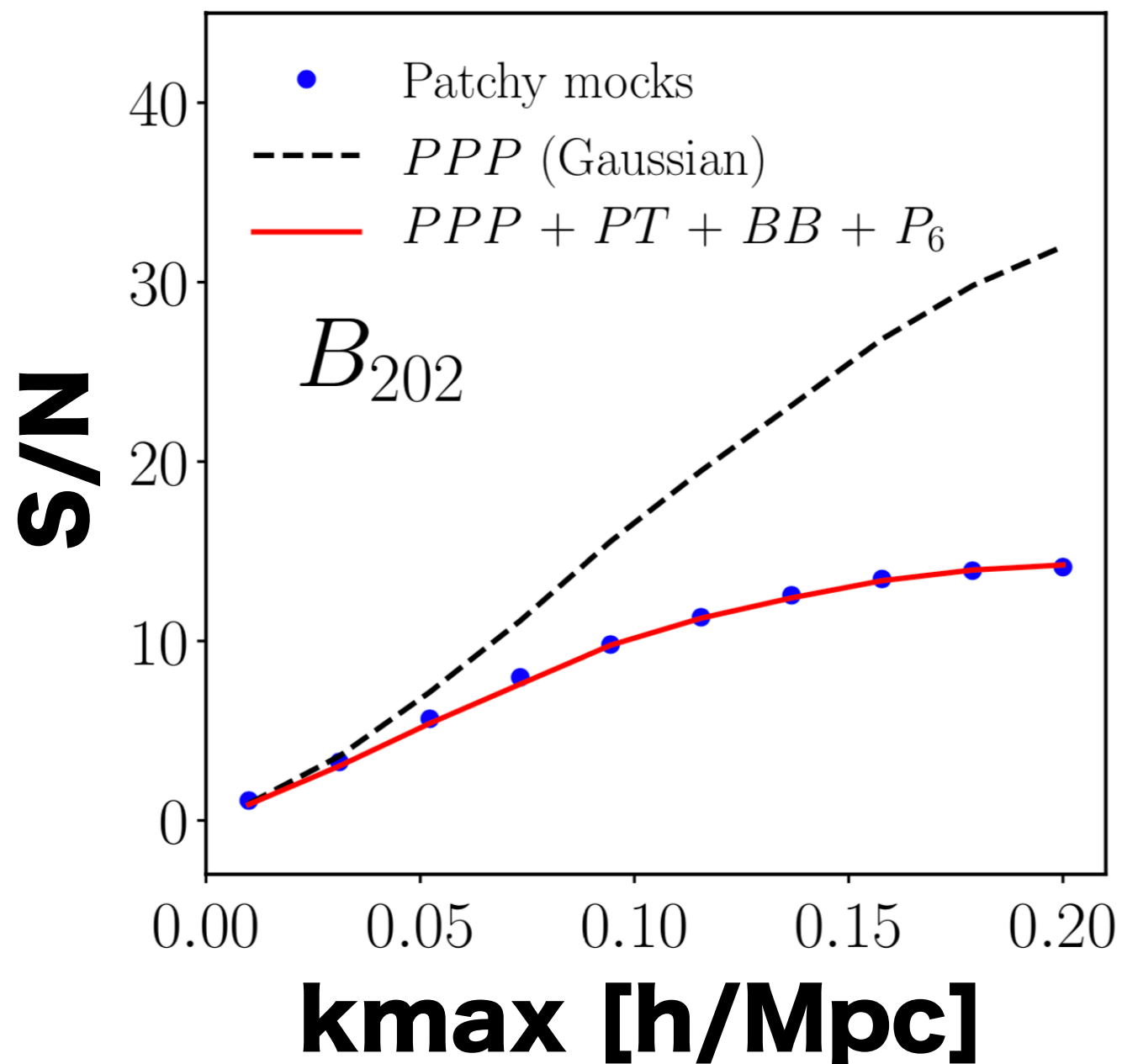
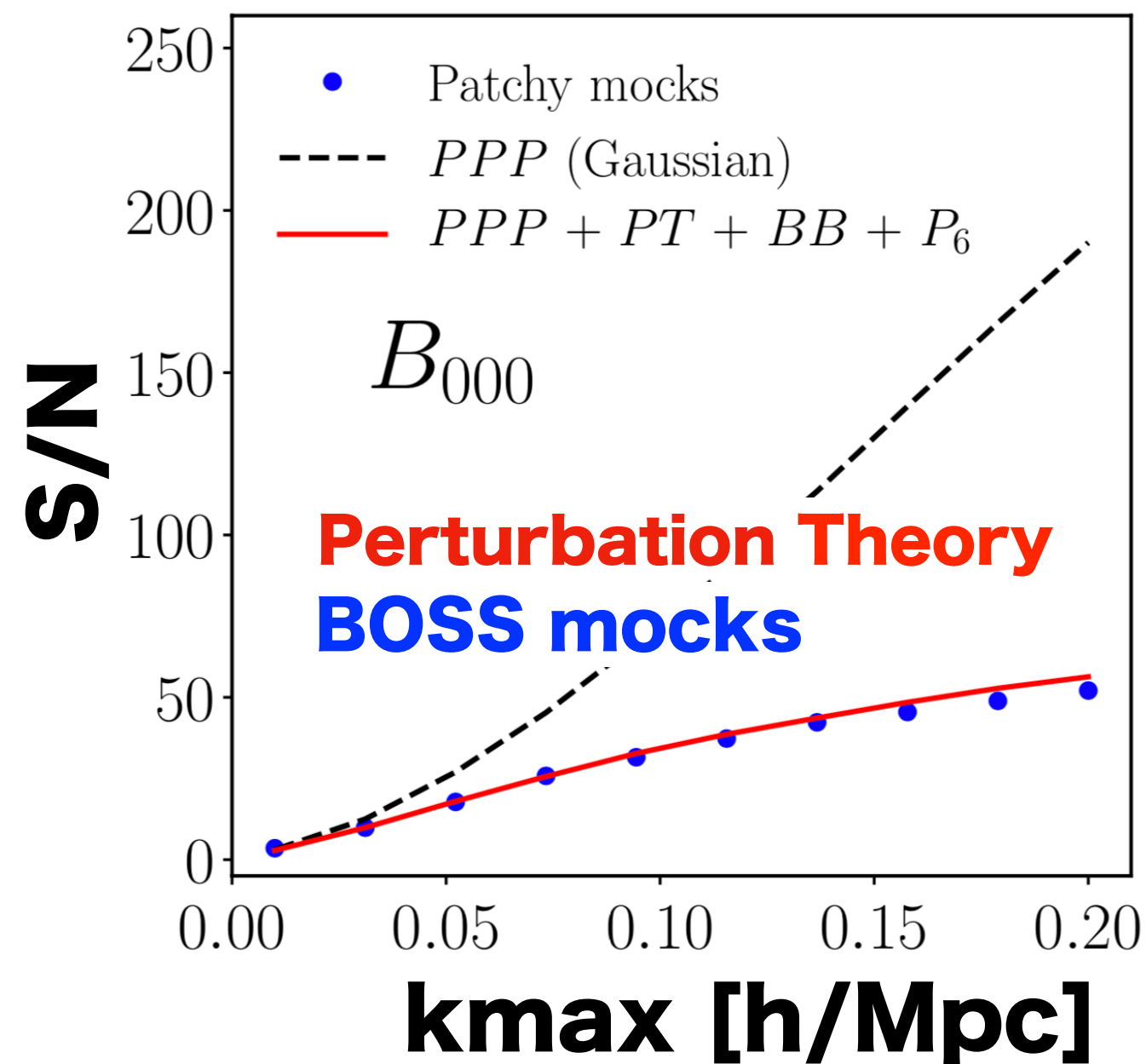


$$\mathbf{Cov}[B, B] = \mathbf{PPP} + \mathbf{PT} + \mathbf{BB} + \mathbf{P6}$$

S / N

Monopole (B000)

Quadrupole (B202)



The same signal is used for both the mock and PT.

Bispectrum model

We need a model that explains the anisotropic galaxy power spectrum and bispectrum simultaneously.

- 1) Non-linear clustering**
- 2) Non-linear RSDs**
- 3) Non-linear bias**

Bispectrum model

We need a model that explains the anisotropic galaxy power spectrum and bispectrum simultaneously.

- 
- 1) Non-linear clustering**
 - 2) Non-linear RSDs**
 - 3) Non-linear bias**

Bispectrum model describing the BAO damping

In the power spectrum case
(Eisenstein+2007):

$$P(\mathbf{k}, \hat{n}) = [Z^{[1]}(\mathbf{k}, \hat{n})]^2 \left[\underbrace{D^2(\mathbf{k}, \hat{n}) P_{\text{BAO}}(k)}_{\text{BAO}} + \underbrace{P_{\text{nw}}(k)}_{\text{Power spec. without BAO}} \right]$$

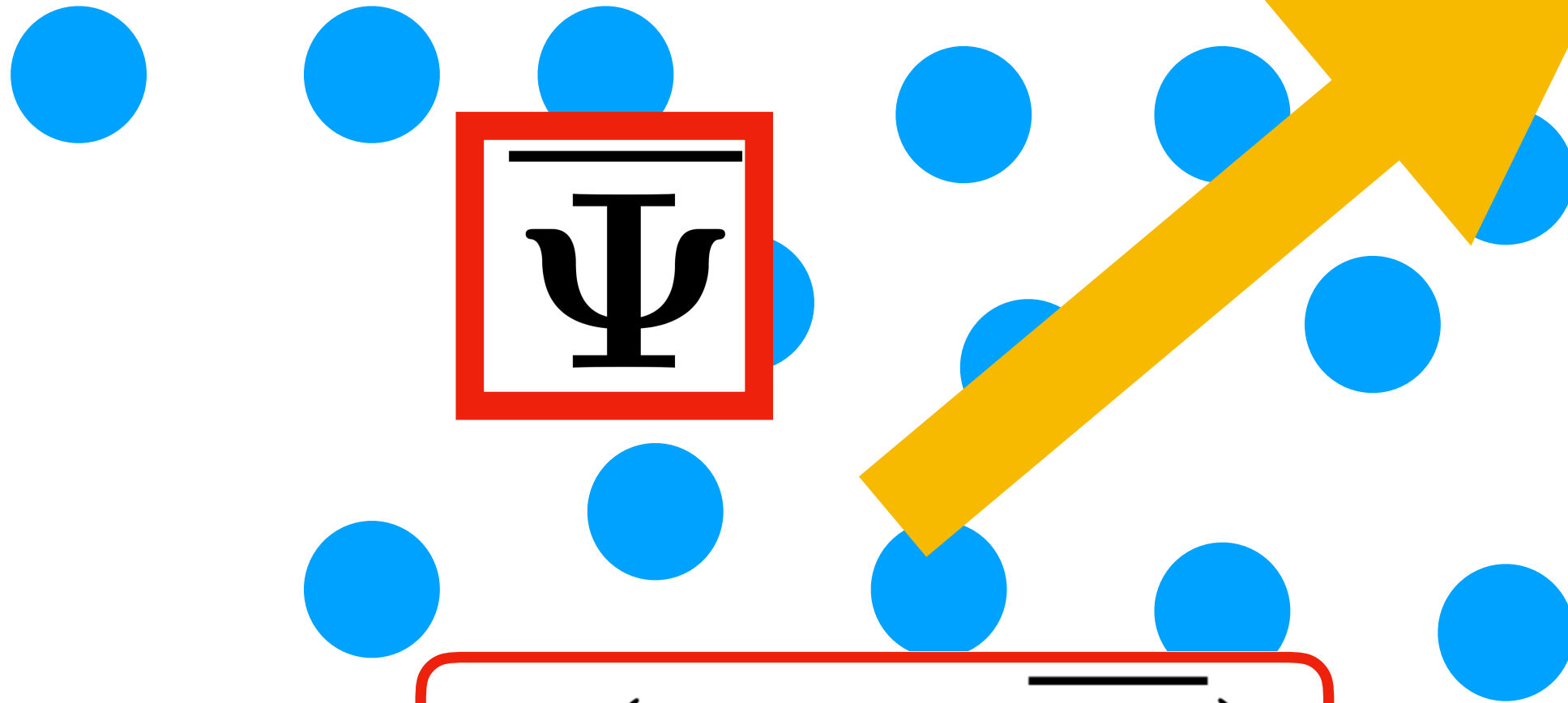
BAO

**Power spec.
without BAO**

$$D(\mathbf{k}, \hat{n}) = \exp \left(- \frac{k^2 \mu^2 \Sigma_{\parallel}^2 + k^2 (1 - \mu^2) \Sigma_{\perp}^2}{4} \right)$$

$$P_{\text{BAO}}(k) = P_{\text{lin}}(k) - P_{\text{nw}}(k)$$

Large scale (infrared, IR) flow



$$\delta(x - \bar{\Psi})$$

$$e^{-ik \cdot \bar{\Psi}} \tilde{\delta}(k)$$

IR cancellation

Assuming that the IR flow is **NOT**
correlated with the density field:

$$\begin{aligned}\langle \delta(x - \bar{\Psi}) \delta(y - \bar{\Psi}) \rangle &= \langle \delta(x) \delta(y) \rangle \\ &= \xi(x - y)\end{aligned}$$

**The effect from the IR flow
completely cancels out
because of translational symmetry
(Galilean invariance)**

IR cancellation

Assuming that the IR flow is **NOT**
correlated with the density field:

$$\begin{aligned} & \langle e^{-ik_1 \cdot \bar{\Psi}} \tilde{\delta}(\mathbf{k}_1) e^{-ik_2 \cdot \bar{\Psi}} \tilde{\delta}(\mathbf{k}_2) \rangle \\ = & \langle e^{-ik_1 \cdot \bar{\Psi}} e^{-ik_2 \cdot \bar{\Psi}} \rangle \langle \tilde{\delta}(\mathbf{k}_1) \tilde{\delta}(\mathbf{k}_2) \rangle \\ = & (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2) P(\mathbf{k}_1) \end{aligned}$$

**The effect from the IR flow
completely cancels out
because of translational symmetry
(Galilean invariance)**

IR (high-k limit) cancellation

$$e^{-ik \cdot \bar{\Psi}_{\text{lin}}} Z^{[1]}(\mathbf{k}) \tilde{\delta}_{\text{lin}}(\mathbf{k})$$

$$P_{22} = \int \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p_2}{(2\pi)^3} \times (2\pi)^3 \delta_{\text{D}}(\mathbf{k} - \mathbf{p}_1 - \mathbf{p}_2) \underbrace{F^2(\mathbf{p}_1, \mathbf{p}_2)}_{\text{Shift}} P_{\text{lin}}(p_1) P_{\text{lin}}(p_2)$$

$$p_1 \gg p_2$$

Shift

$$\rightarrow k^2 \sigma^2 P_{\text{lin}}(k)$$

$$P_{13} + P_{22} \rightarrow 0$$

IR (high- k limit) cancellation Gamma-expansion (all orders)

$$P(k) = \underbrace{G^2(k) P_{\text{lin}}(k)}_{\downarrow} + \underbrace{P_{\text{MC}}(k)}_{\downarrow}$$
$$\underbrace{\mathcal{D}^2(\mathbf{k}) [Z^{[1]}(\mathbf{k})]^2 P_{\text{lin}}(k)}_{\downarrow}$$
$$\underbrace{[1 - \mathcal{D}^2(k)] [Z^{[1]}(k)]^2 P_{\text{lin}}(k)}_{\downarrow}$$
$$\rightarrow [Z^{[1]}(k)]^2 P_{\text{lin}}(k)$$

**The IR flow never
affects the power
spectrum??**

**To extract physical effects from
the IR flow, the breaking of the IR
cancellation should be considered.**

Breaking of the IR (high-k limit) cancellation

$$P_{22} = \int \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p_2}{(2\pi)^3} \times (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{p}_1 - \mathbf{p}_2) F^2(\mathbf{p}_1, \mathbf{p}_2) P_{\text{lin}}(p_1) P_{\text{lin}}(p_2)$$

$$p_1 \gg p_2$$

Too strong

$$\rightarrow k^2 \sigma^2 P_{\text{lin}}(k)$$

Breaking of the IR (high-k limit) cancellation

$$P_{22} = \int \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p_2}{(2\pi)^3} \times (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{p}_1 - \mathbf{p}_2) F^2(\mathbf{p}_1, \mathbf{p}_2) P_{\text{lin}}(p_1) P_{\text{lin}}(p_2)$$

$$p_1 \gg p_2$$

Too strong

$$\rightarrow k^2 \sigma^2 P_{\text{nw}}(k)$$

Eisenstein's template

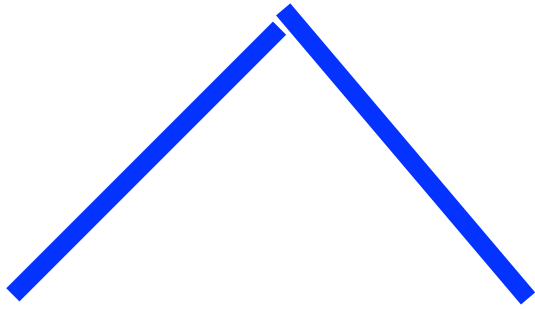
$$P(\mathbf{k}) = \underbrace{G^2(\mathbf{k}) P_{\text{lin}}(\mathbf{k})}_{\text{blue}} + \underbrace{P_{\text{MC}}(\mathbf{k})}_{\text{red}}$$

$$\underbrace{\mathcal{D}^2(\mathbf{k}) [Z^{[1]}(\mathbf{k})]^2 P_{\text{lin}}(\mathbf{k})}_{\text{blue}}$$

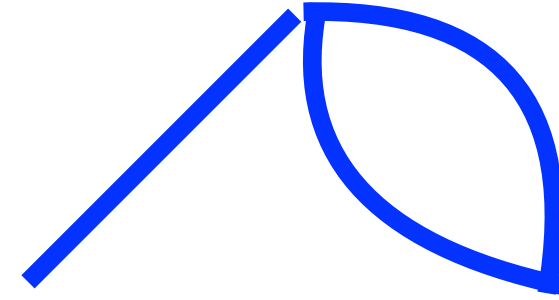
$$\underbrace{[1 - \mathcal{D}^2(\mathbf{k})] [Z^{[1]}(\mathbf{k})]^2}_{\text{red}} P_{\text{nw}}(\mathbf{k})$$

$$P(\mathbf{k}, \hat{n}) = [Z^{[1]}(\mathbf{k}, \hat{n})]^2 [\mathcal{D}^2(\mathbf{k}, \hat{n}) P_{\text{BAO}}(\mathbf{k}) + P_{\text{nw}}(\mathbf{k})]$$

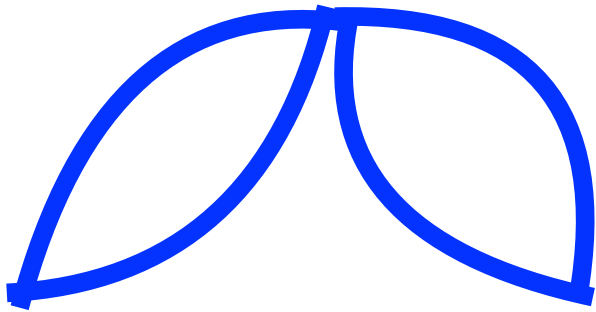
Bispectrum case



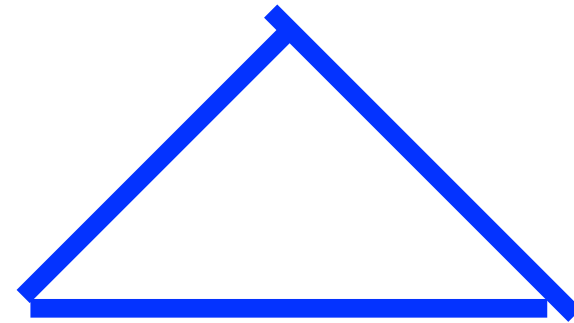
$$\mathcal{D}(\mathbf{k}_1)\mathcal{D}(\mathbf{k}_2)\mathcal{D}(\mathbf{k}_{12})B_{\text{tree}}(\mathbf{k}_1, \mathbf{k}_2)$$



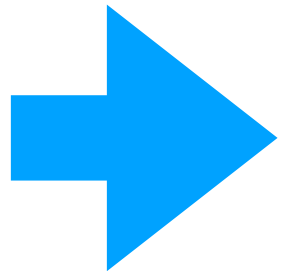
$$\left(\mathcal{D}^2(\mathbf{k}_1) - \mathcal{D}(\mathbf{k}_1)\mathcal{D}(\mathbf{k}_2)\mathcal{D}(\mathbf{k}_{12})\right) B_{\text{tree}}(\mathbf{k}_1, \mathbf{k}_2)$$



$$\left(\mathcal{D}(\mathbf{k}_1)\mathcal{D}(\mathbf{k}_2)\mathcal{D}^{-1}(\mathbf{k}_{12}) - \mathcal{D}^2(\mathbf{k}_1) - \mathcal{D}^2(\mathbf{k}_2) + \mathcal{D}(\mathbf{k}_1)\mathcal{D}(\mathbf{k}_2)\mathcal{D}(\mathbf{k}_{12})\right) B_{\text{tree}}(\mathbf{k}_1, \mathbf{k}_2)$$

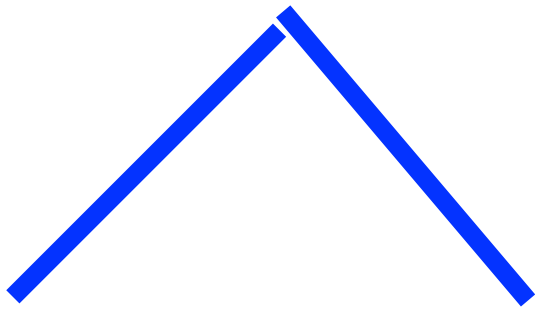


$$\left(1 - \mathcal{D}(\mathbf{k}_1)\mathcal{D}(\mathbf{k}_2)\mathcal{D}^{-1}(\mathbf{k}_{12})\right) B_{\text{tree}}(\mathbf{k}_1, \mathbf{k}_2)$$

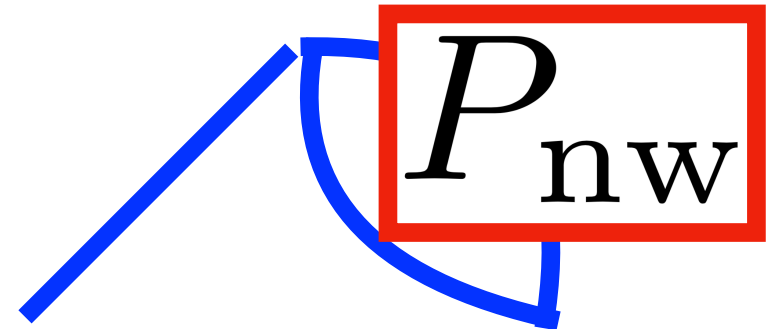


Tree-level solution

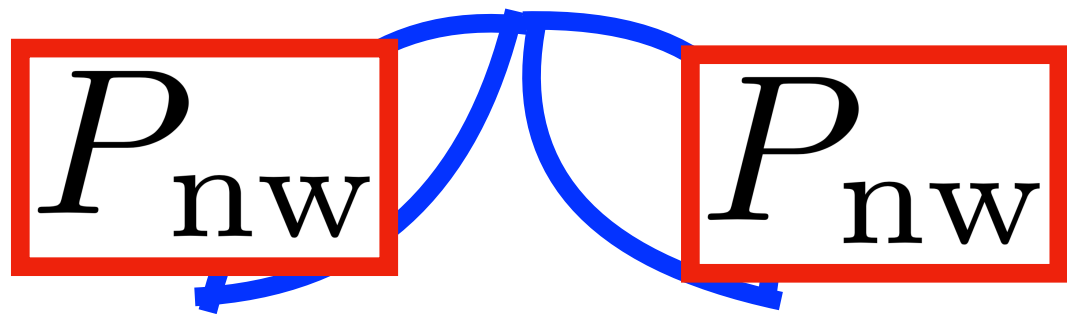
Bispectrum case



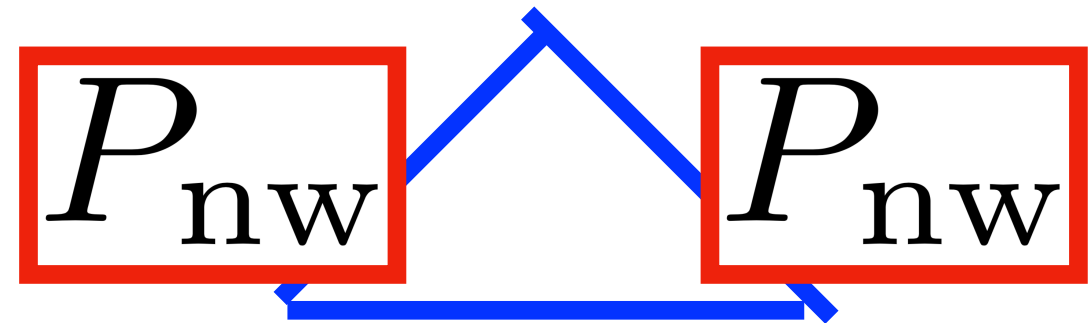
$$\mathcal{D}(\mathbf{k}_1)\mathcal{D}(\mathbf{k}_2)\mathcal{D}(\mathbf{k}_{12})B_{\text{tree}}(\mathbf{k}_1, \mathbf{k}_2)$$



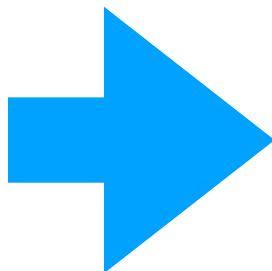
$$(\mathcal{D}^2(\mathbf{k}_1) - \mathcal{D}(\mathbf{k}_1)\mathcal{D}(\mathbf{k}_2)\mathcal{D}(\mathbf{k}_{12})) B_{\text{tree}}(\mathbf{k}_1, \mathbf{k}_2)$$



$$\left(\mathcal{D}(\mathbf{k}_1)\mathcal{D}(\mathbf{k}_2)\mathcal{D}^{-1}(\mathbf{k}_{12}) - \mathcal{D}^2(\mathbf{k}_1) - \mathcal{D}^2(\mathbf{k}_2) + \mathcal{D}(\mathbf{k}_1)\mathcal{D}(\mathbf{k}_2)\mathcal{D}(\mathbf{k}_{12}) \right) B_{\text{tree}}(\mathbf{k}_1, \mathbf{k}_2)$$



$$\left(1 - \mathcal{D}(\mathbf{k}_1)\mathcal{D}(\mathbf{k}_2)\mathcal{D}^{-1}(\mathbf{k}_{12}) \right) B_{\text{tree}}(\mathbf{k}_1, \mathbf{k}_2)$$



Model

$$\begin{aligned} & B(\mathbf{k}_1, \mathbf{k}_2) \\ &= 2 Z^{[1]}(\mathbf{k}_1) Z^{[1]}(\mathbf{k}_2) Z^{[2]}(\mathbf{k}_1, \mathbf{k}_2) \\ &\times \left\{ \mathcal{D}(\mathbf{k}_1) \mathcal{D}(\mathbf{k}_2) \mathcal{D}(\mathbf{k}_{12}) \underline{P_{\text{BAO}}(k_1) P_{\text{BAO}}(k_2)} \right. \\ &\quad + \mathcal{D}^2(\mathbf{k}_1) P_{\text{BAO}}(k_1) P_{\text{nw}}(k_2) + \mathcal{D}^2(\mathbf{k}_2) \underline{P_{\text{nw}}(k_1) P_{\text{BAO}}(k_2)} \\ &\quad \left. + \underline{P_{\text{nw}}(k_1) P_{\text{nw}}(k_2)} \right\} + 2 \text{ cyc..} \end{aligned}$$

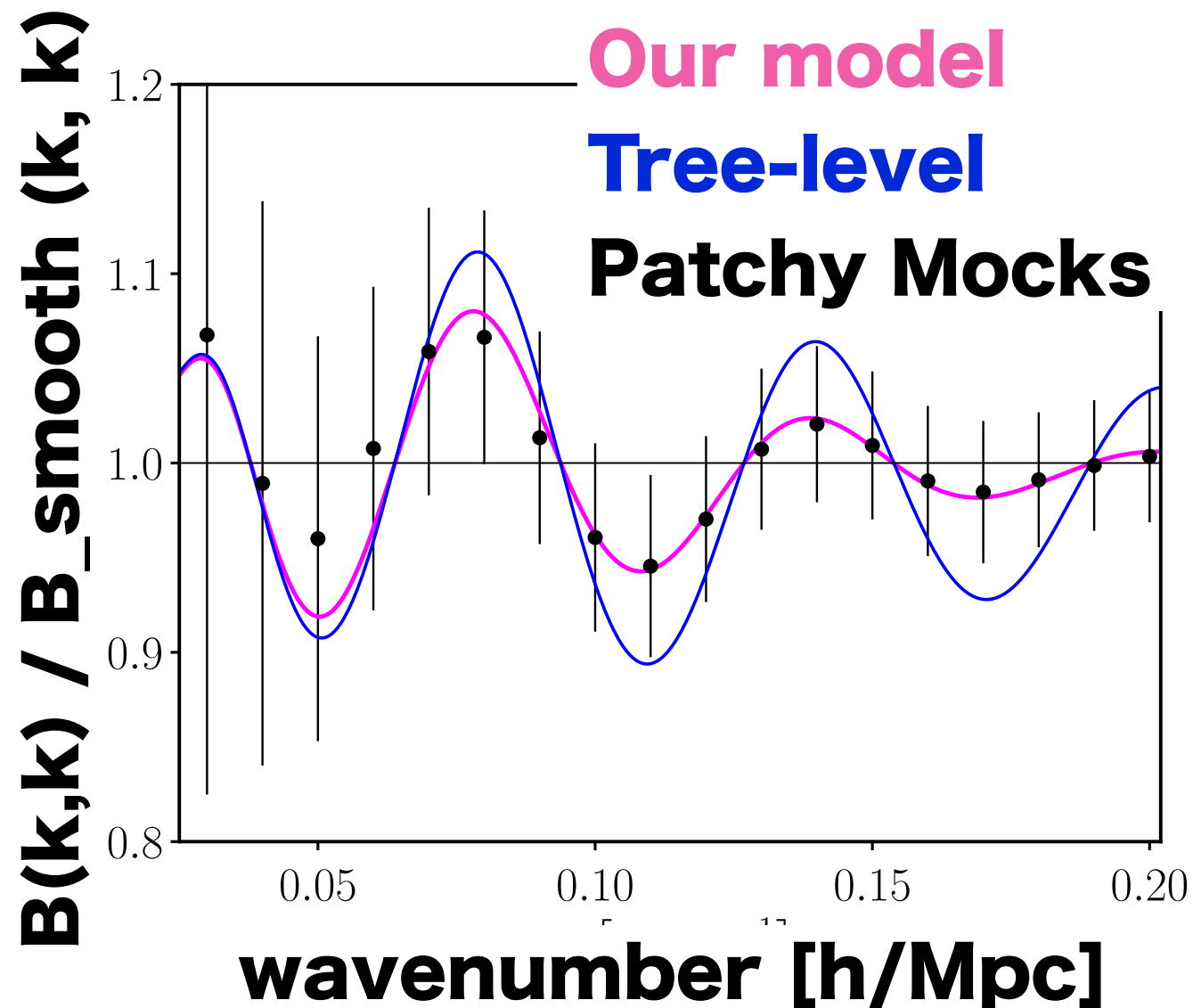
$$\mathcal{D}(\mathbf{k}, \hat{n}) = \exp \left(- \frac{k^2 \mu^2 \Sigma_{\parallel}^2 + k^2 (1 - \mu^2) \Sigma_{\perp}^2}{4} \right)$$

$$P_{\text{BAO}}(k) = P_{\text{lin}}(k) - P_{\text{nw}}(k)$$

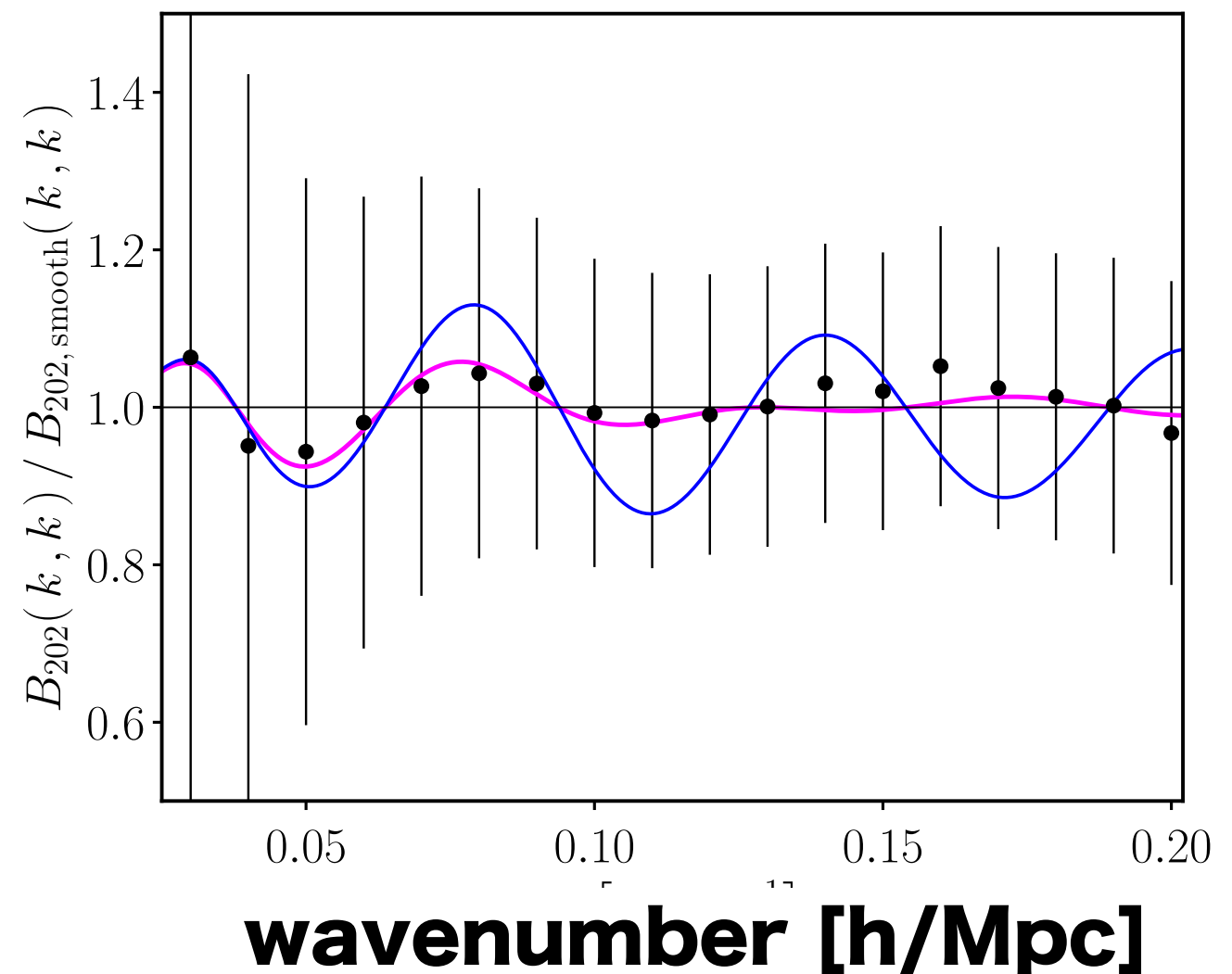
BAO damping

$$B_{000}(k, k) = B_{\text{BAO}, 000}(k, k) + B_{\text{nw}, 000}(k, k) [ak + bk^2 + ck^3]$$

Monopole (B000)



Quadrupole (B202)



Bispectrum model for the fisher analysis

Power spectrum and bispectrum:

Tree-level

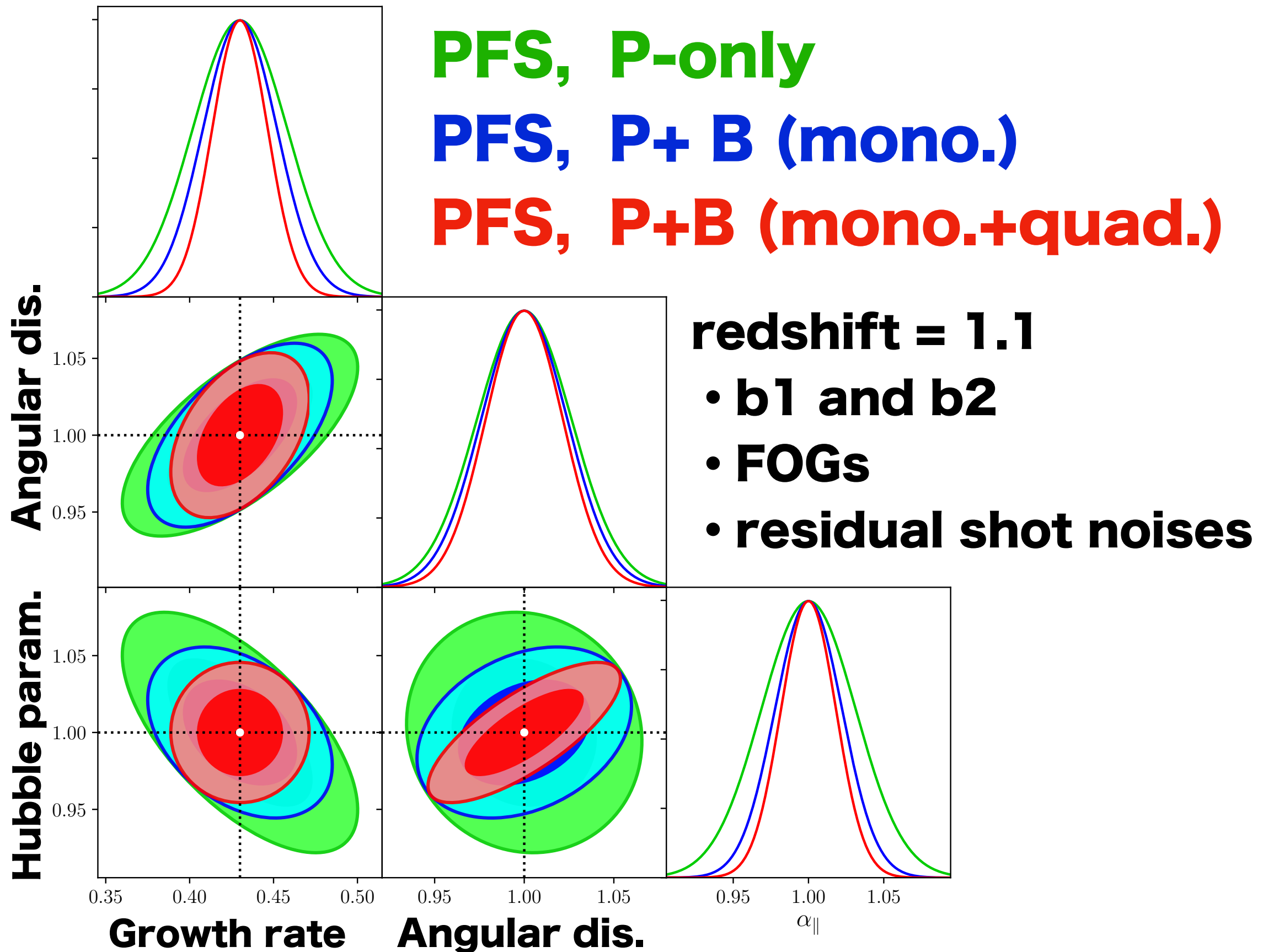
+ non-linear BAO damping

+ Gaussian FOG  **non-linear RSD**

+ local bias (b_1, b_2)  **non-linear bias**

+ residual shot-noise terms

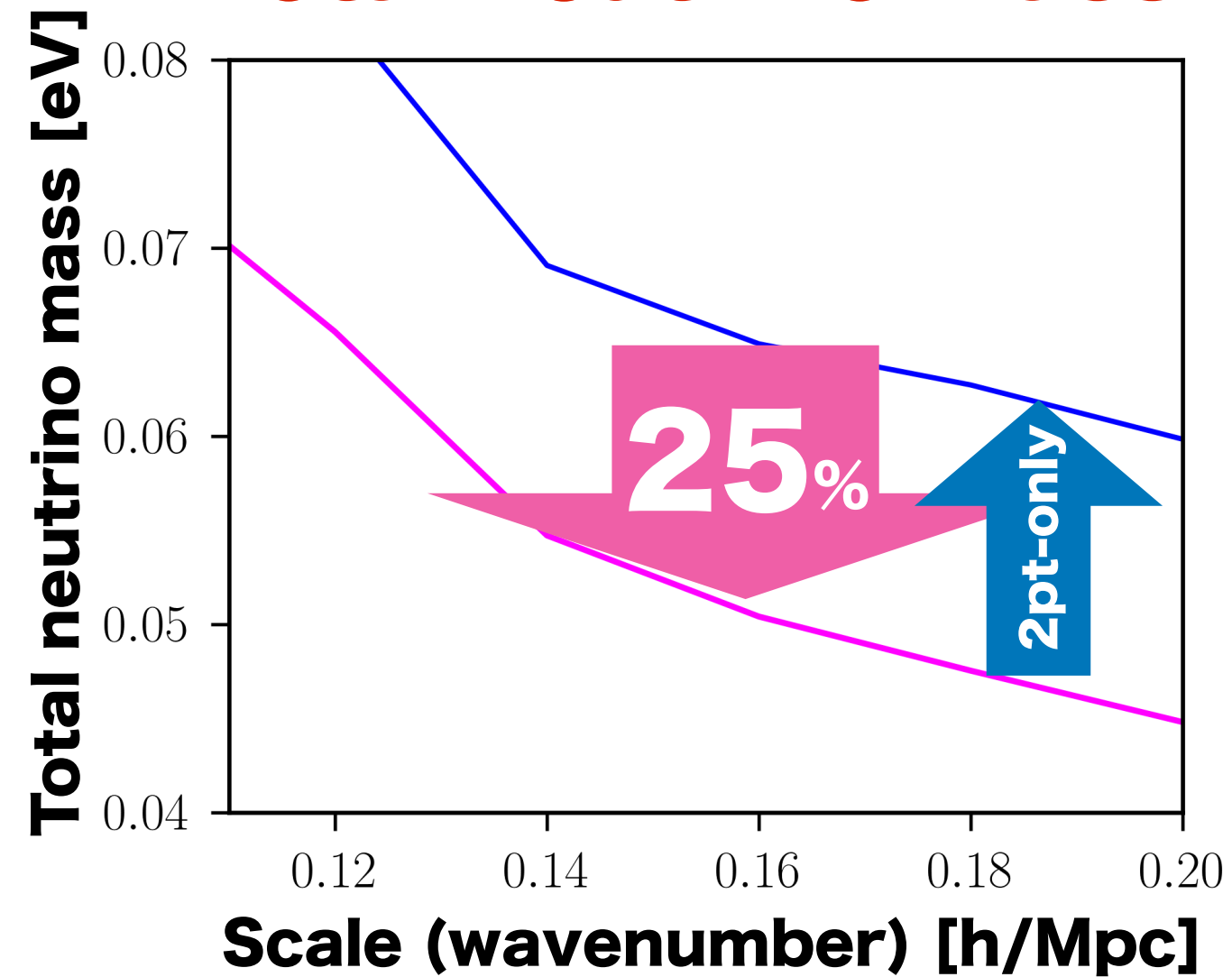
Fisher analysis for PFS



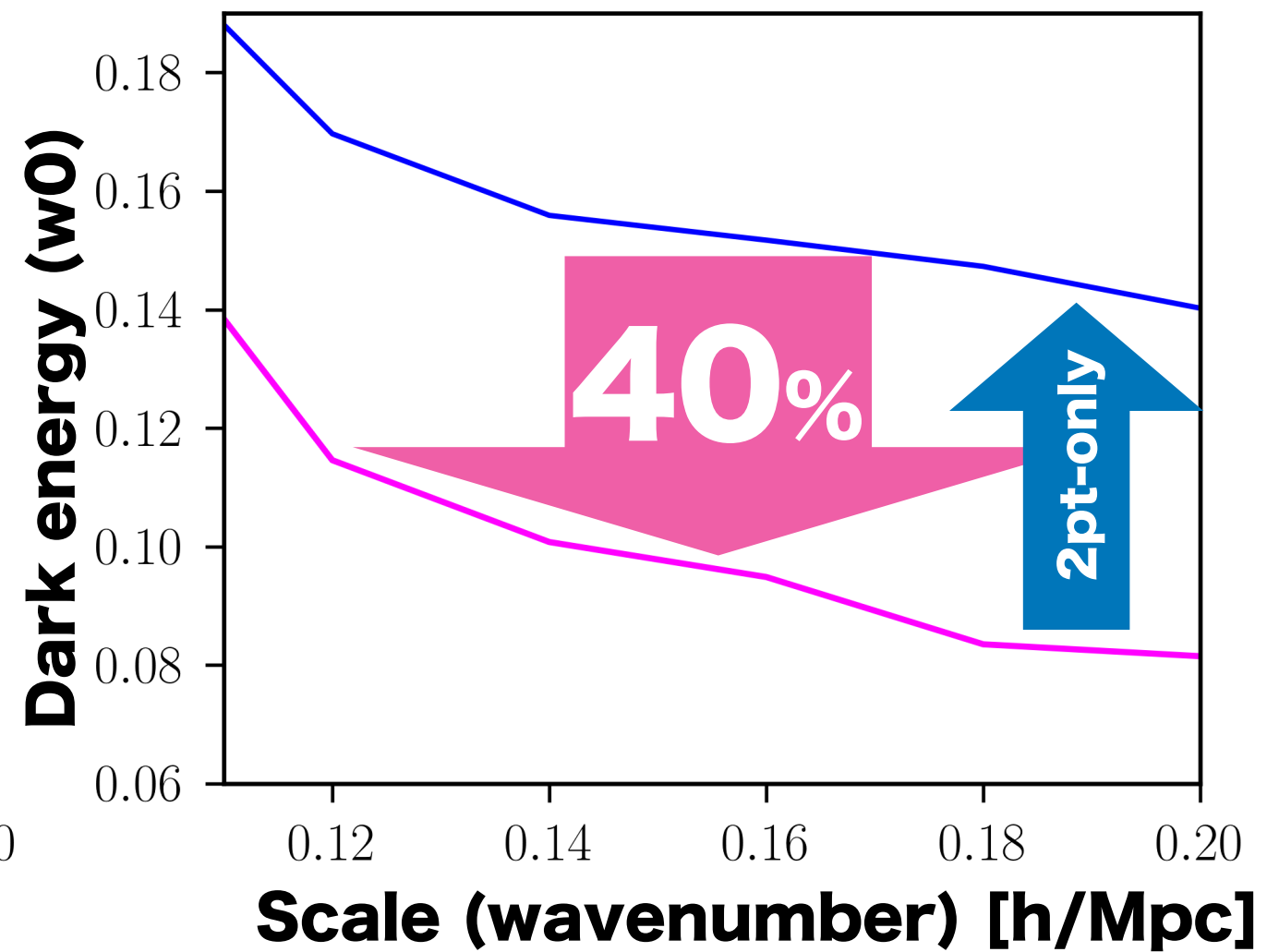
PFS Project

($0.6 < z < 2.2$)

Total neutrino mass



Dark energy



Bispectrum Project

[1] NS, Saito, Beutler and Seo 2018

- A new decomposition formalism
- **Detection of the quadrupole bispectrum (14σ)**

[2] In progress

- Modeling the bispectrum covariance
- Fisher analysis
- Modeling the anisotropic bispectrum

This talk



[3] Future works

- Analysis using BOSS data

Final goal

- Application to future galaxy survey,
PFS, DESI and Euclid.