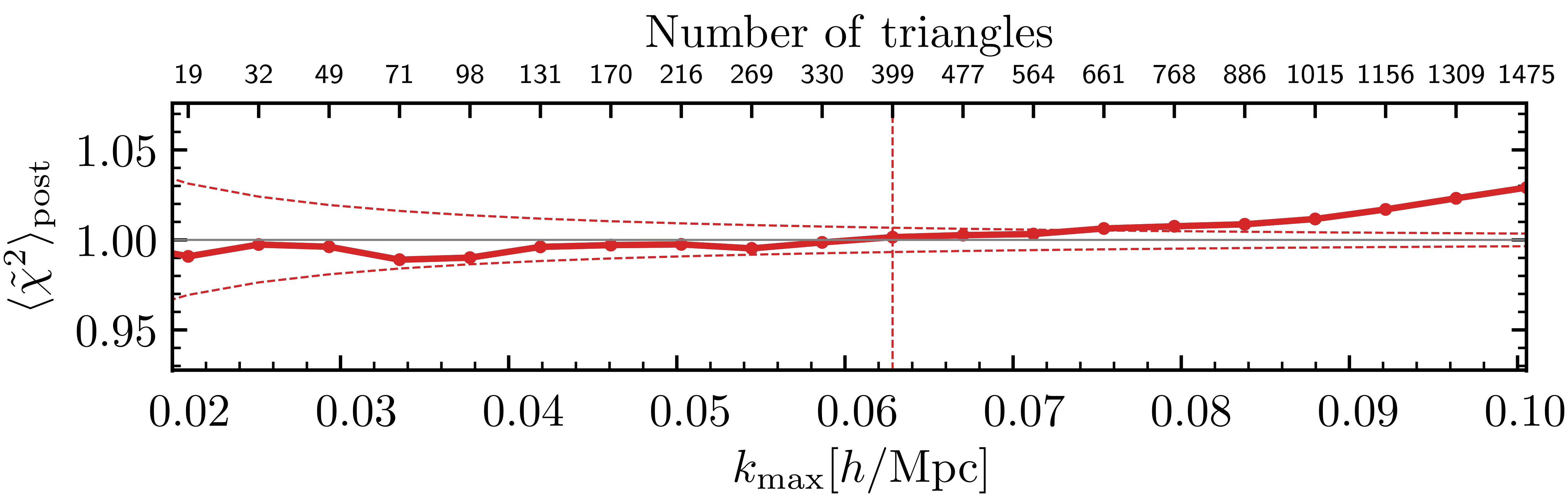
Understanding *something* of the halo bispectrum *at least* at very large scales

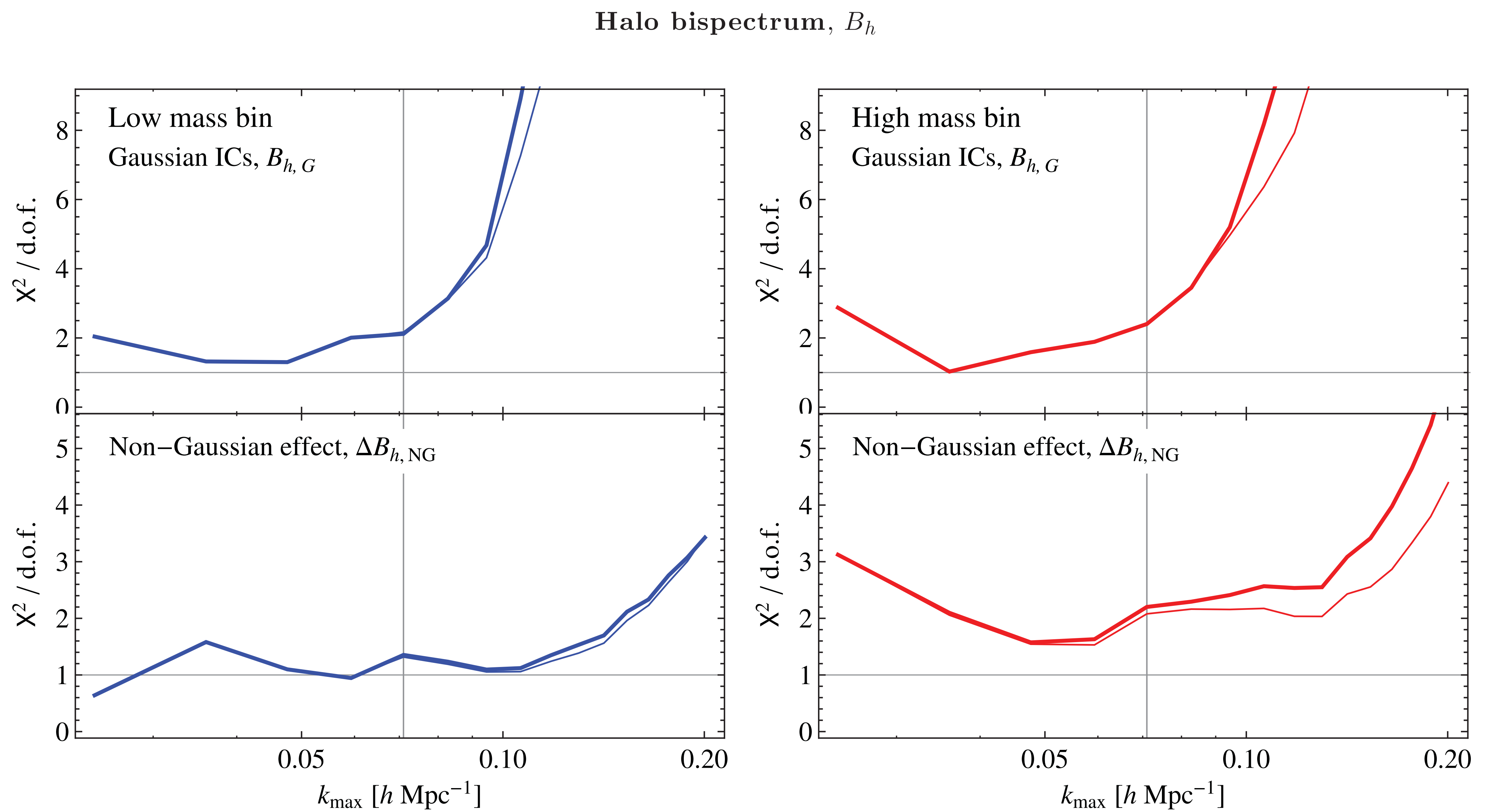


Emiliano Sefusatti
w/ **Andrea Oddo**, Cristiano Porciani
for the *Euclid Higher Order WP*

PTchat@Kyoto, April 10th 2019

Prelude

Over the years, comparing bispectrum models and predictions, I got used to χ^2 of this sort:



ES, Crocce & Desjacques (2012)

Why is that?

Many reasons

- no covariance …
- … or a poorly estimated covariance?
- … or a wrong covariance?
- non-Gaussian likelihood?
- binning effects?
- surprises?

  ….
- wrong model?

Why is that?

Many reasons

- **no covariance …**
- **… or a poorly estimated covariance?**
- **… or a wrong covariance?**
- non-Gaussian likelihood?
- **binning effects?**
- **surprises?**

  ….

- wrong model?

For fun with the bispectrum wait for the talks of Naonori, Alex & Paco …!

The model

Standard, **tree-level** model

$$B_h(k_1,k_2,k_3) = b_1^3 B_m(k_1,k_2,k_3) + b_1^2 b_2 \left[P(k_1)P(k_2) + \text{perm.}\right]$$
$$+2b_1^2\gamma_2 \left[(\cos^2\mu_{12} - 1)P(k_1)P(k_2) + \text{perm.}\right]$$
$$+\frac{1}{(2\pi)^3\bar{n}}(1+\alpha_1)\left[P(k_1)+P(k_2)+P(k_3)\right] + \frac{1}{(2\pi)^6\bar{n}^2}(1+\alpha_2)$$

5 parameters: b_1, b_2, γ_2 plus α_1 and α_2

We assume the cosmology

We have:

- very fast chains
- exact binning of theoretical predictions

Simulations and halo catalogs

Minerva simulations: Grieb *et al.* (2016)

$L = 1500\, h^{-1}\text{Mpc}, \quad N_{\text{part}} = 1000^3$

300 realisations

Halo catalogs:

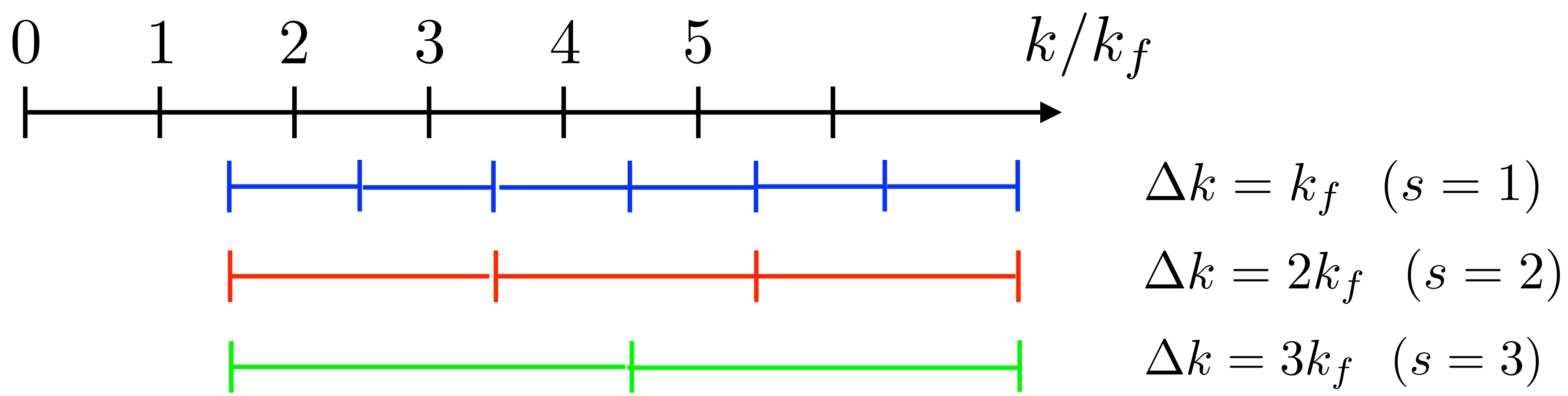
FoF halos of mass $M \geq 1.12 \times 10^{13}\, h^{-1} M_\odot$ at $z = 1$

with density $n \simeq 2 \times 10^{-4}\, h^3\text{Mpc}^{-3}$

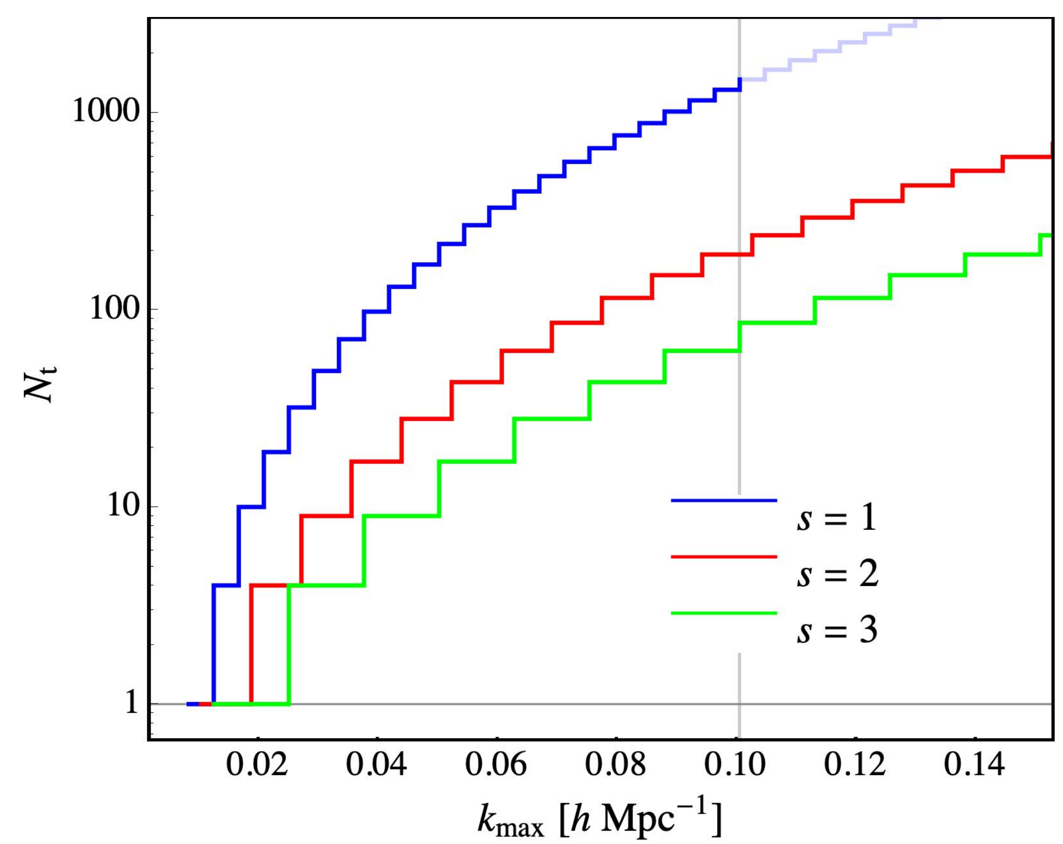
and linear bias $b_1 \simeq 2.8$

The measurements

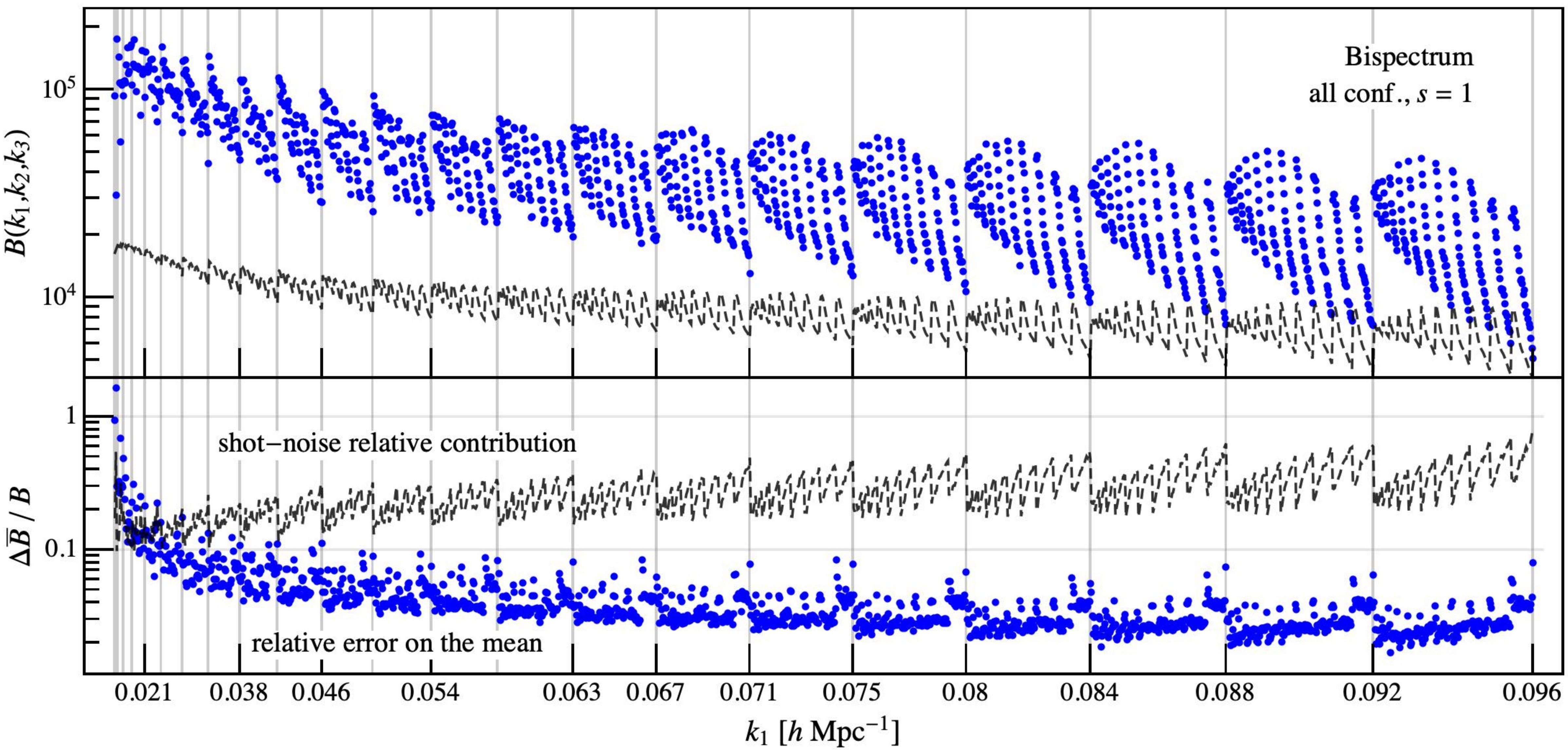
Three linear binning schemes:



Number of triangular configurations



The measurements



The mocks

10,000 `Pinocchio` mocks
with the first 300 matching the N-body ICs

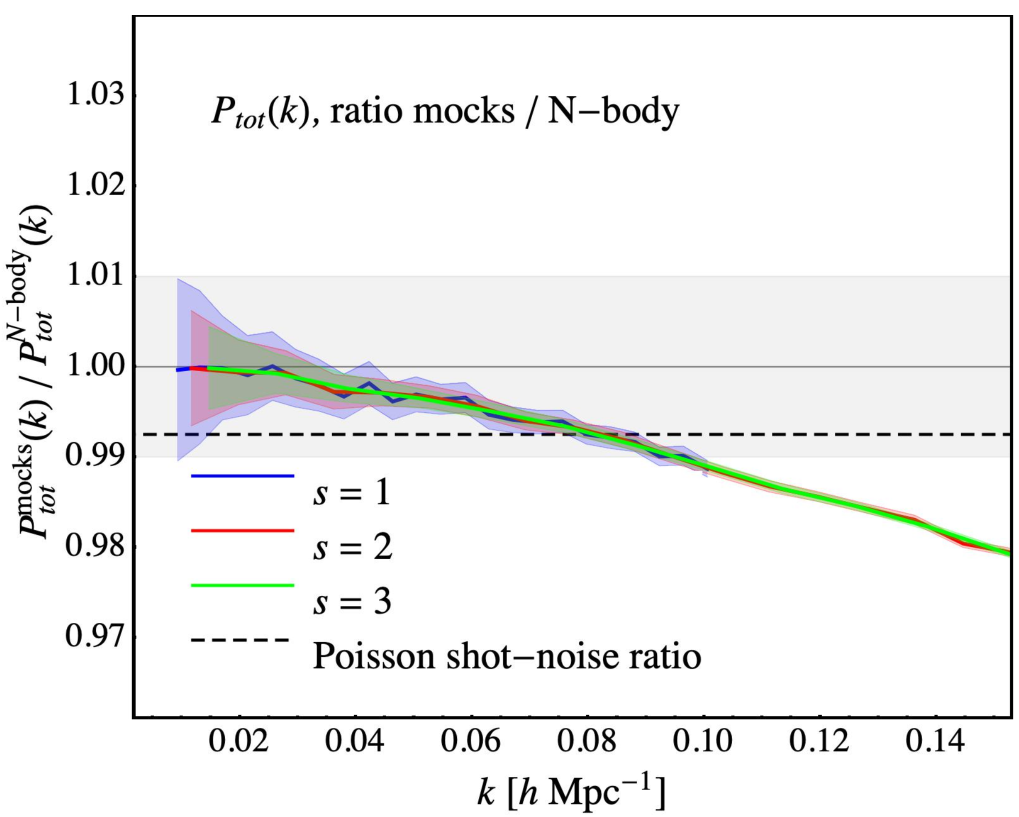
1

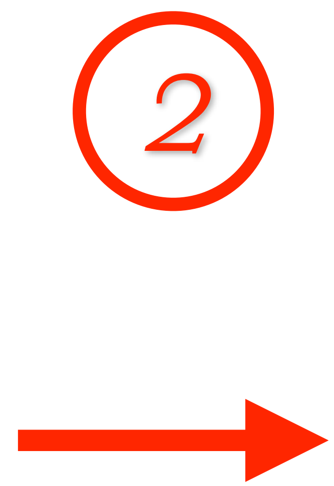
Lippich *et al.* (2018)
Blot *et al.* (2018)
Colavincenzo *et al.* (2018)

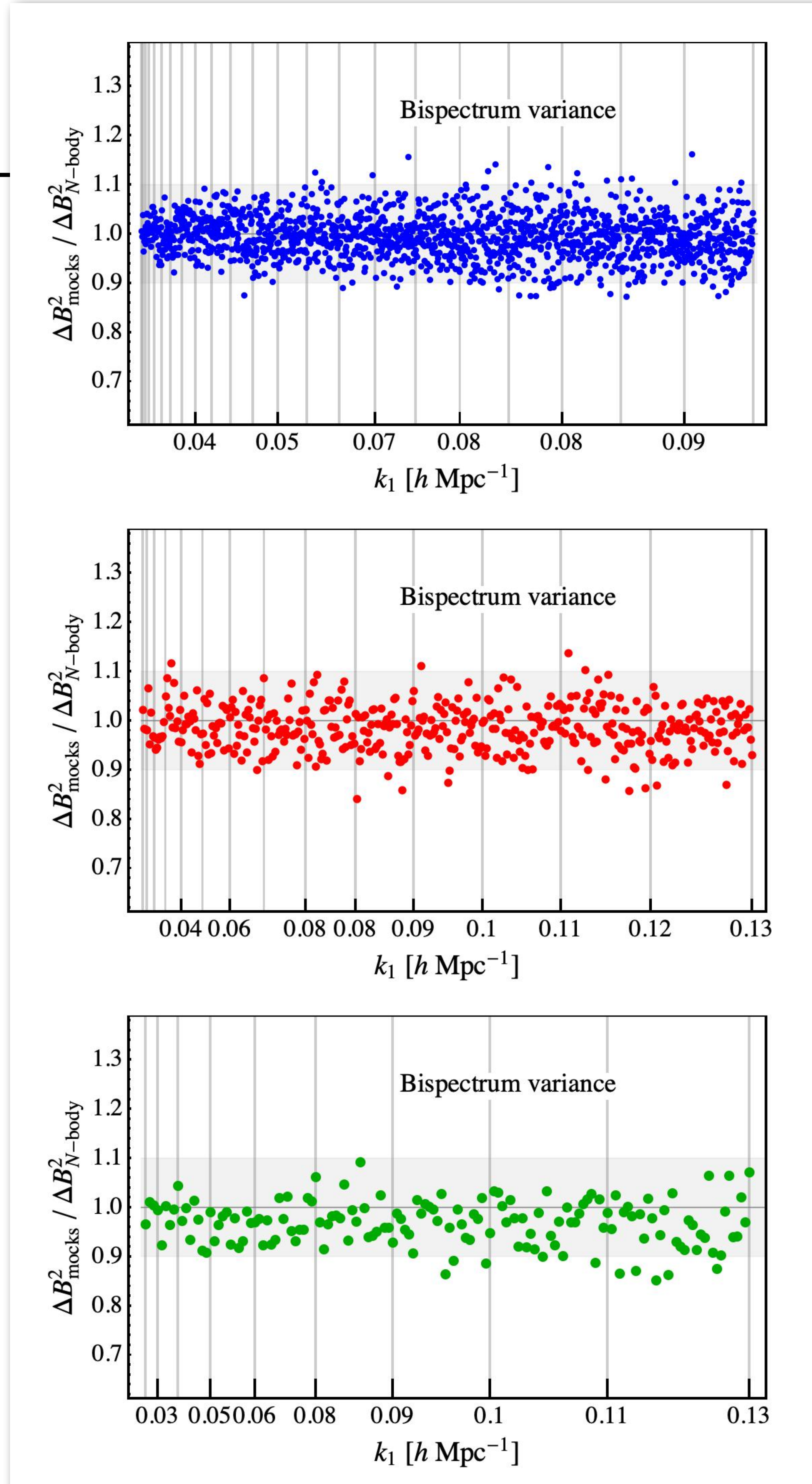
The mocks

10,000 `Pinocchio` mocks
with the first 300 matching the N-body ICs

We choose the mass threshold to ***match the N-body large-scale power spectrum (with shot noise)***





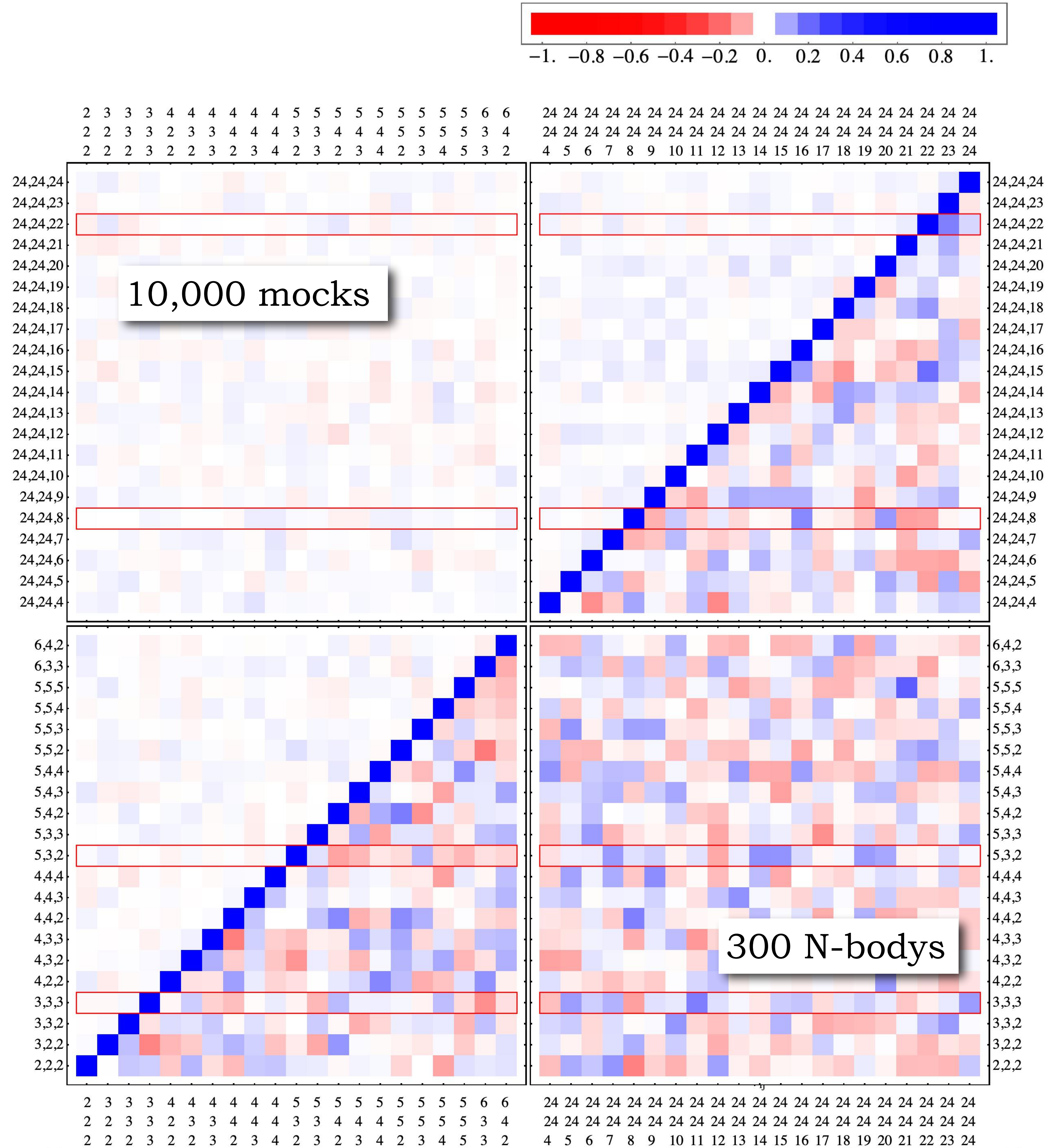


$$\Delta B^2 = s_{123}\frac{k_f^3}{V_B}P_{tot}(k_1)P_{tot}(k_2)P_{tot}(k_3)$$

The covariance

$$r_{i,j} \equiv \frac{C_{ij}}{\sqrt{C_{ii}C_{jj}}}$$

smallest binning



The covariance

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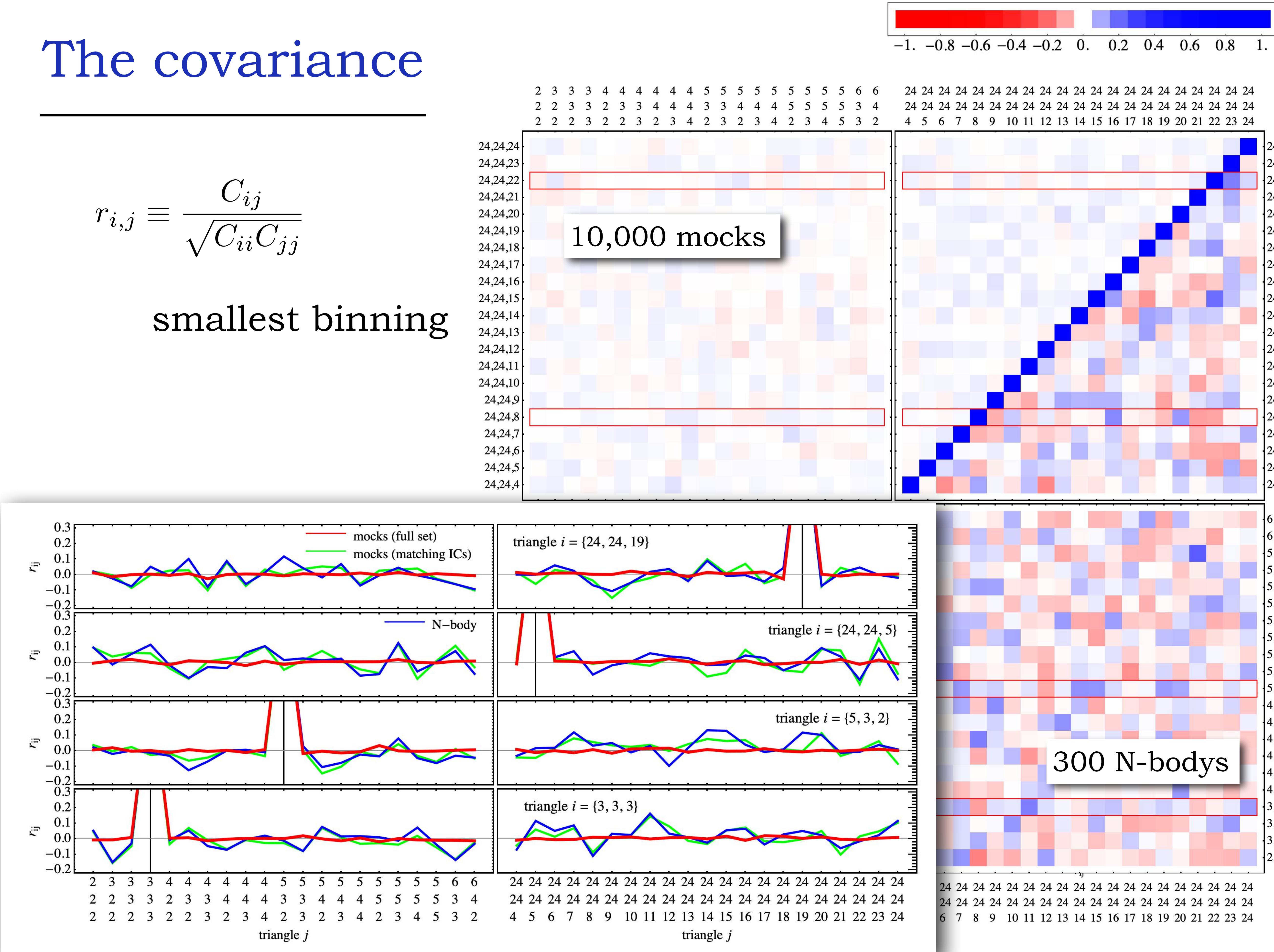
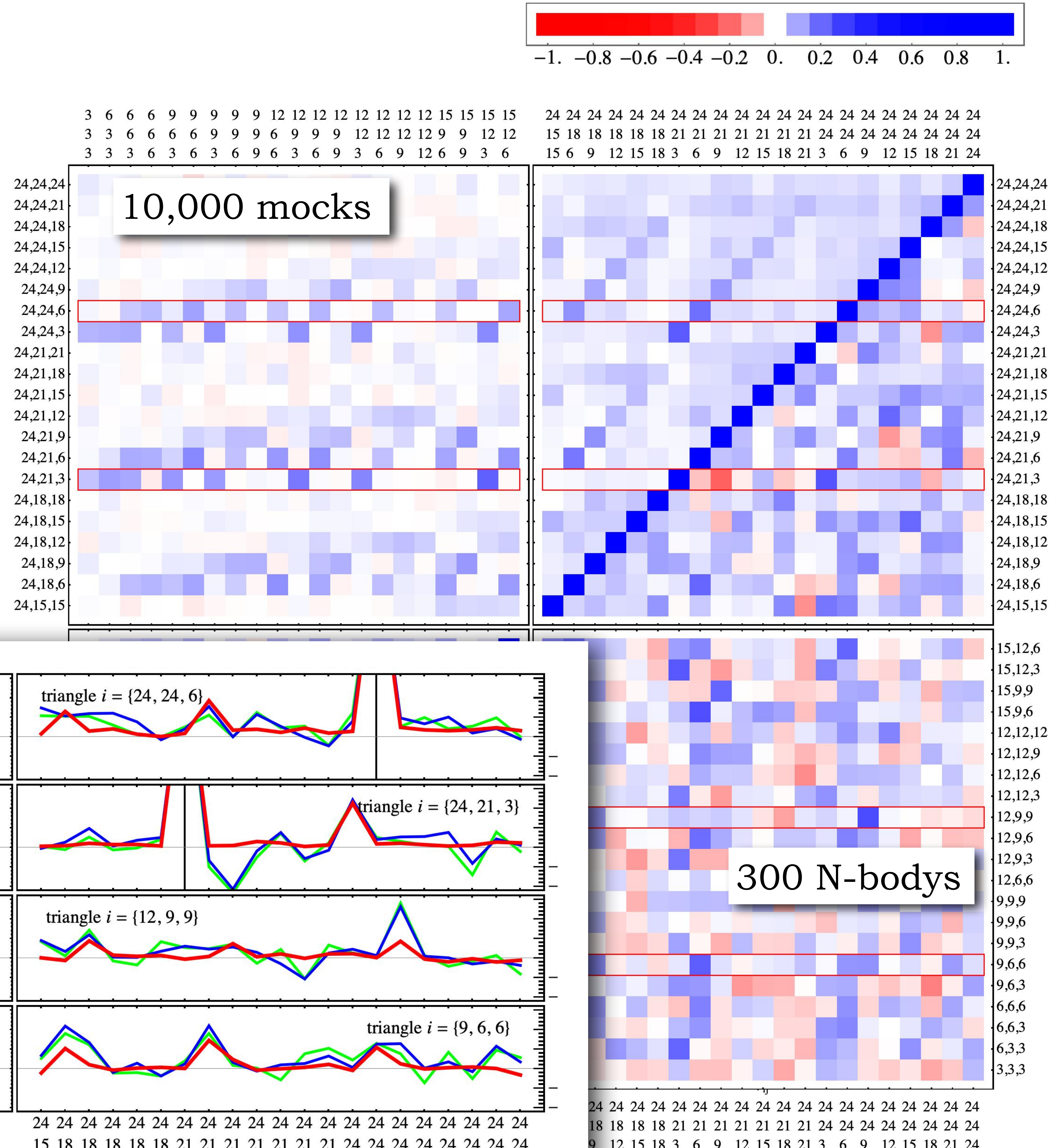
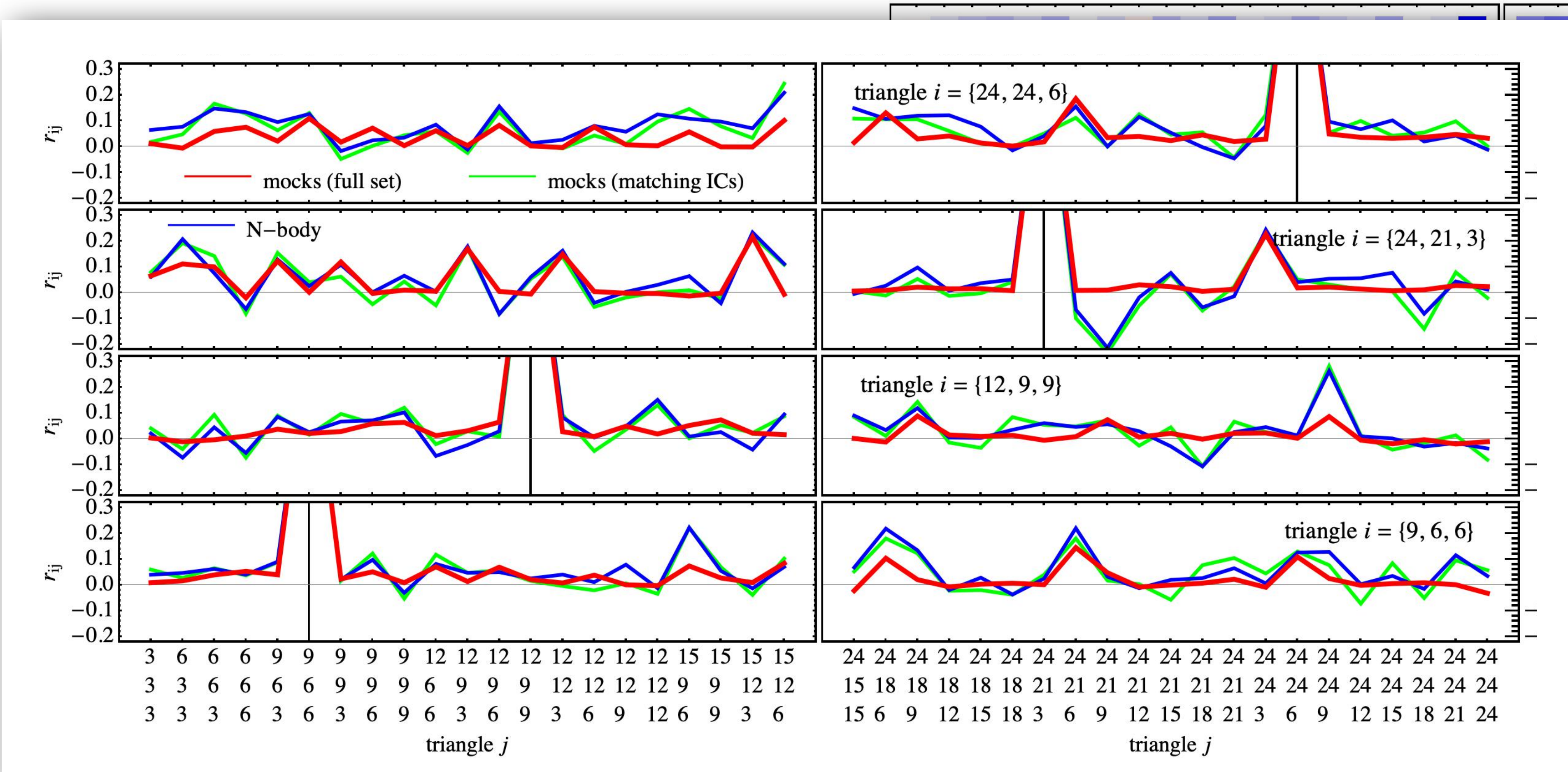


The covariance

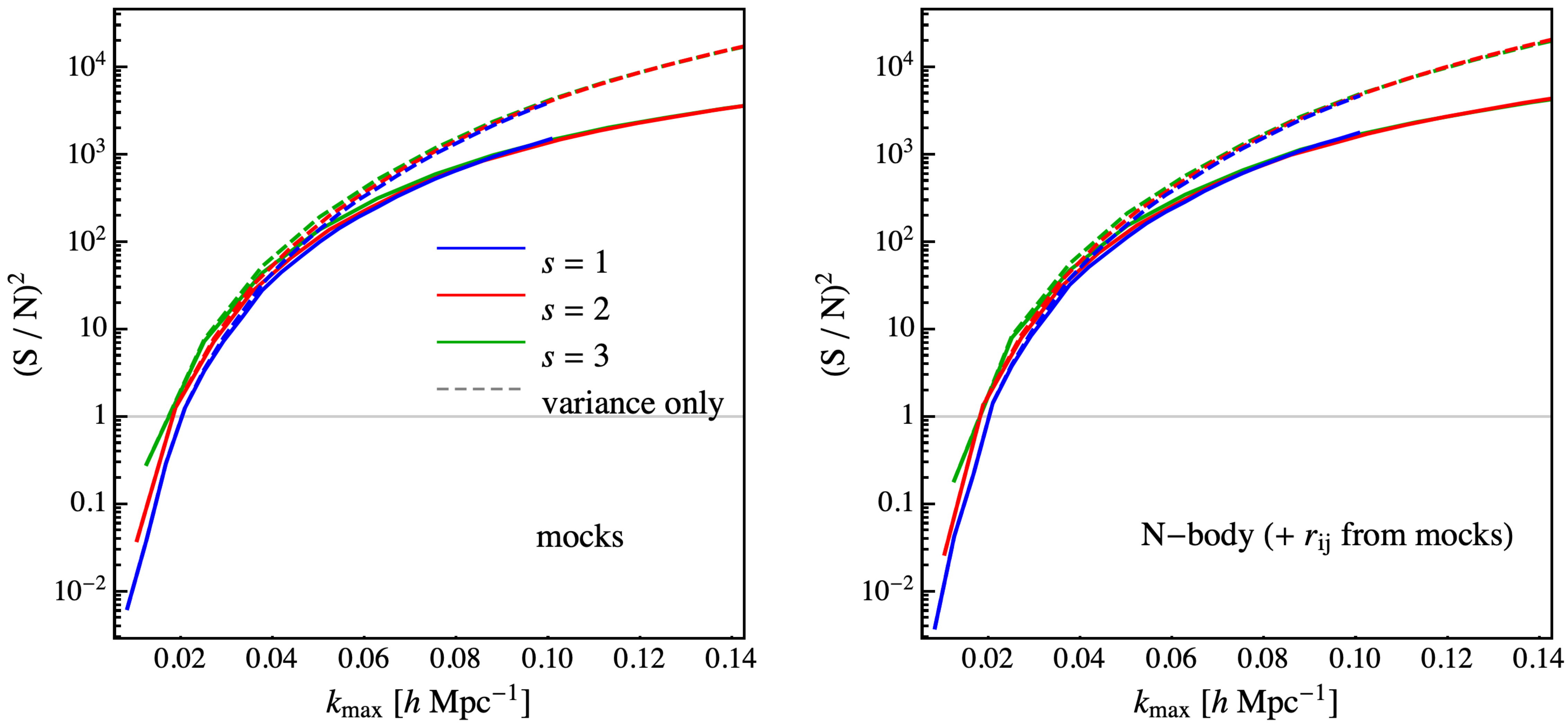
$$r_{i,j} \equiv \frac{C_{ij}}{\sqrt{C_{ii}C_{jj}}}$$

largest binning





Signal-to-noise



$$\left(\frac{S}{N}\right)^2 = \sum_{i,j}^{N_t(k_{\rm max})} B_i \left[C^B_{ij}\right]^{-1} B_j\,, \quad \left(\frac{S}{N}\right)^2 = \sum_{i}^{N_t(k_{\rm max})} \frac{B_i^2}{\Delta B_i^2}$$

The likelihood

We *assume* a **Gaussian likelihood**

but account for a possibly **poorly estimated covariance** in two ways:

1) Anderson/Hartlap correction to the inverse

Anderson (2003)
Hartlap *et al.* (2007)

$$C^{-1} \longrightarrow \frac{n_r - n_t - 2}{n_r - 1} C^{-1}$$

2) Sellentin & Heavens likelihood

Sellentin & Heavens (2008)

$$\ln \mathcal{L} = -\frac{n_r}{2} \ln \left[1 + \sum_{ij}^{n_t} \frac{\delta B_i \, C_{ij}^{-1} \, \delta B_j}{n_r - 1} \right] + \text{const}$$

1

Fit the mean or fit them all?

Suppose you have a model f

$$y = f(\theta) + \epsilon = f_{true} + b + \epsilon$$

to test with R realisations of D measurements $\mathbf{y}$

$$\chi^2_{all} = \sum_{\alpha=1}^{R} (\mathbf{y}_\alpha - \mathbf{f})^{\mathbf{T}} \mathbf{C}^{-1} (\mathbf{y}_\alpha - \mathbf{f}) \quad \rightarrow \quad \langle \chi^2_{all} \rangle = R\, \mathbf{b}^{\mathbf{T}} \mathbf{C}^{-1} \mathbf{b} + R\, D$$

$$\chi^2_{mean} = R(\bar{\mathbf{y}} - \mathbf{f})^{\mathbf{T}} \mathbf{C}^{-1} (\bar{\mathbf{y}} - \mathbf{f}) \quad \rightarrow \quad \langle \chi^2_{mean} \rangle = R\, \mathbf{b}^{\mathbf{T}} \mathbf{C}^{-1} \mathbf{b} + D$$

$$\mathbf{C} \rightarrow \mathbf{C}/R$$

Fit the mean or fit them all?

Suppose you have a model f

$$y = f(\theta) + \epsilon = f_{true} + b + \epsilon$$

3

to test with R realisations of D measurements $\mathbf{y}$

$$\chi^2_{all} = \sum_{\alpha=1}^{R} (\mathbf{y}_\alpha - \mathbf{f})^\mathbf{T} \mathbf{C}^{-1} (\mathbf{y}_\alpha - \mathbf{f}) \rightarrow \langle \chi^2_{all} \rangle = R\, \mathbf{b}^\mathbf{T} \mathbf{C}^{-1} \mathbf{b} + R\, D$$

$$\chi^2_{mean} = R(\bar{\mathbf{y}} - \mathbf{f})^\mathbf{T} \mathbf{C}^{-1} (\bar{\mathbf{y}} - \mathbf{f}) \rightarrow \langle \chi^2_{mean} \rangle = R\, \mathbf{b}^\mathbf{T} \mathbf{C}^{-1} \mathbf{b} + D$$

$$\mathbf{C} \to \mathbf{C}/R$$

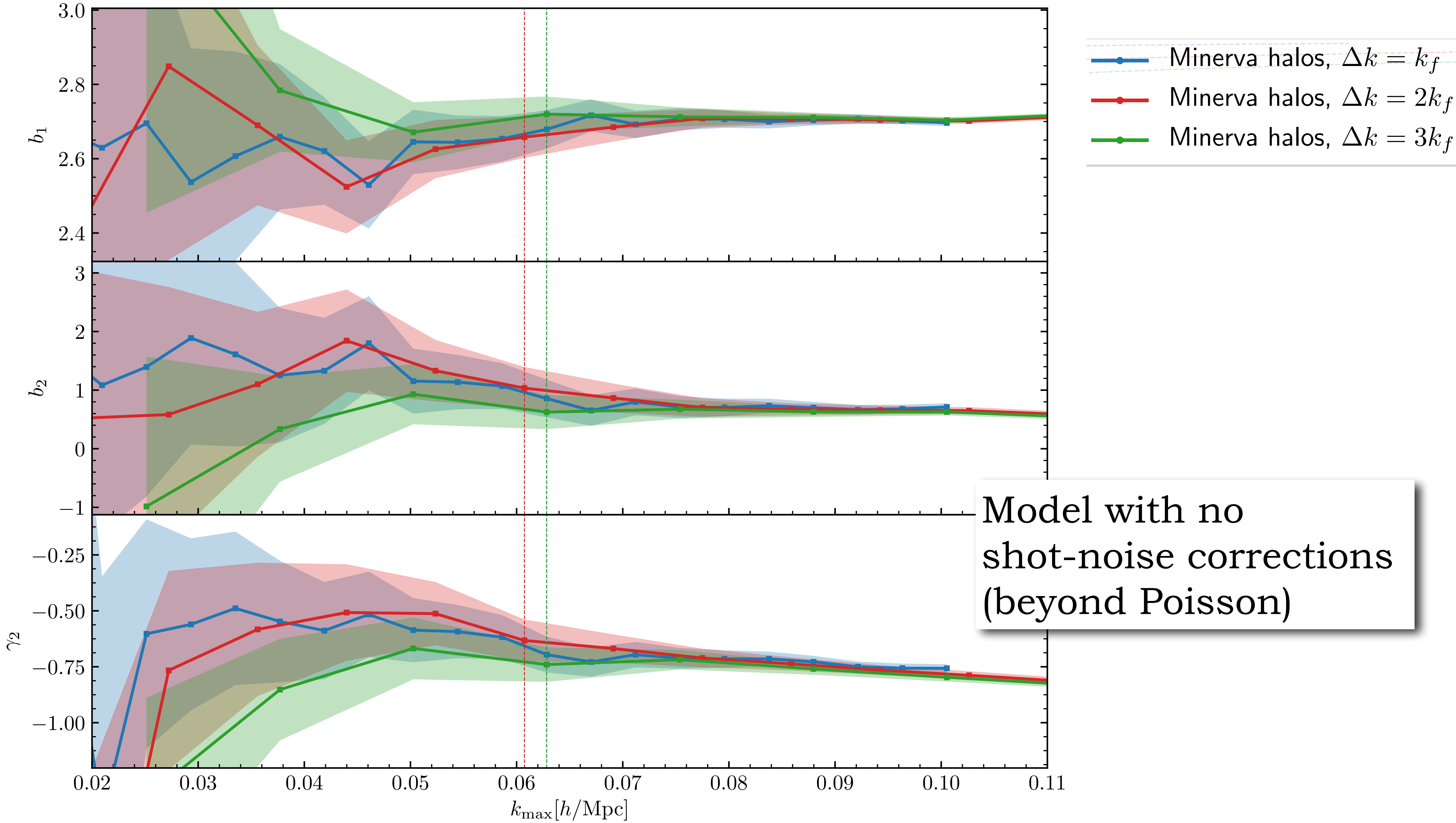
Example:

With D = 5, R = 20 to be within 95%CL we need

$$\chi^2_{all} < 124.34 \rightarrow \mathbf{b}^\mathbf{T} \mathbf{C}^{-1} \mathbf{b} < 1.22$$

but $\chi^2_{mean} < 11.7 \rightarrow \mathbf{b}^\mathbf{T} \mathbf{C}^{-1} \mathbf{b} < 0.3$ a tighter requirement!

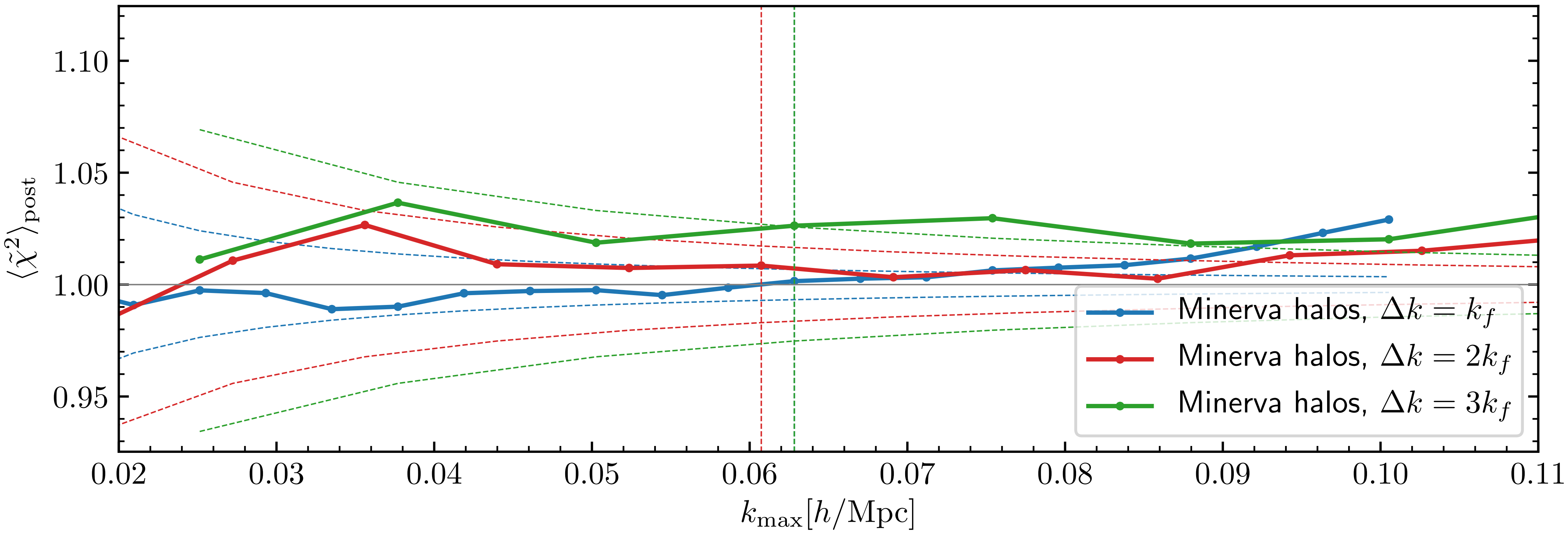
Results: the parameters



Model with no shot-noise corrections (beyond Poisson)

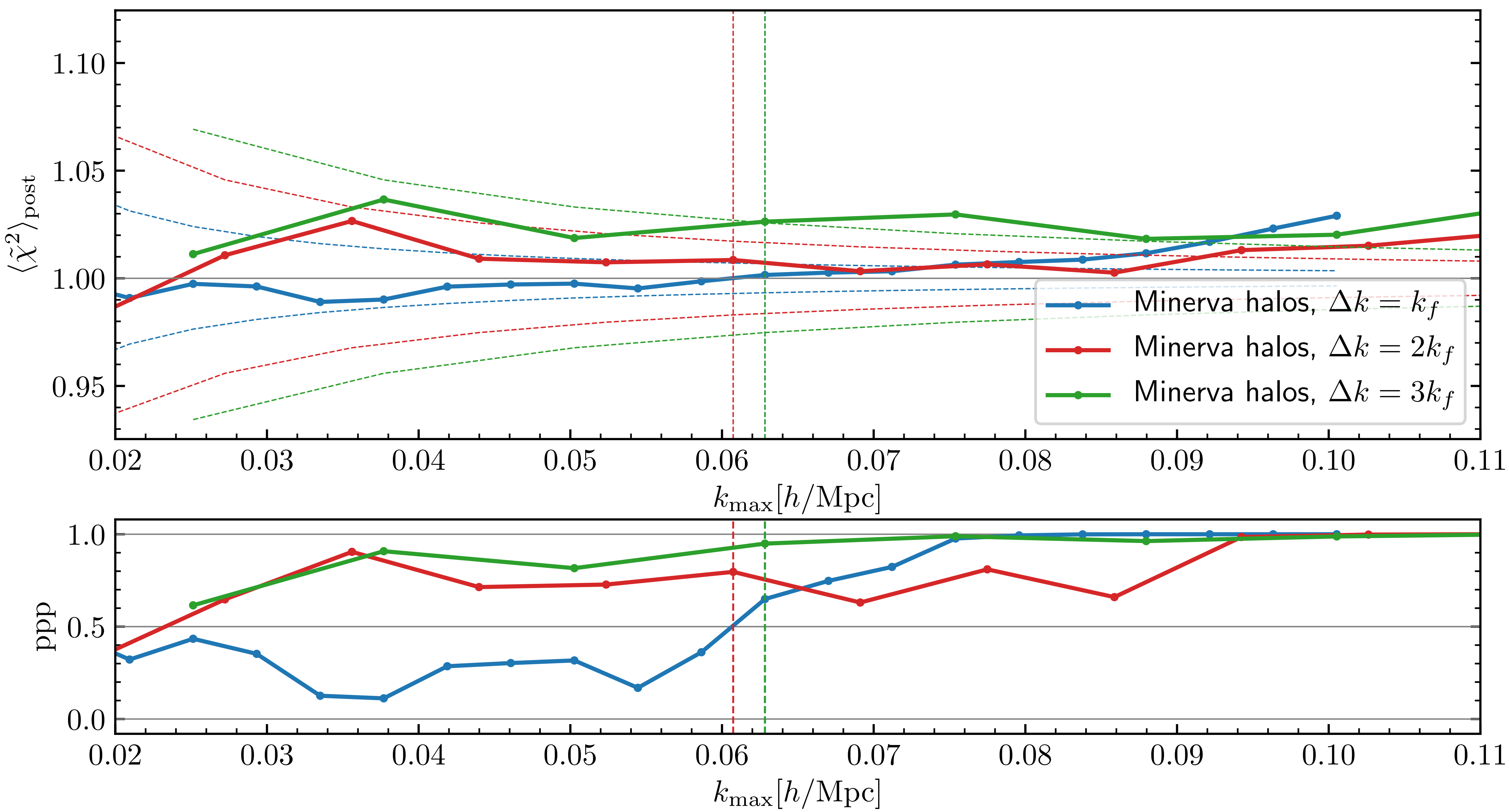
Results: goodness-of-fit

χ^2 per degree of freedom, averaged over the posterior



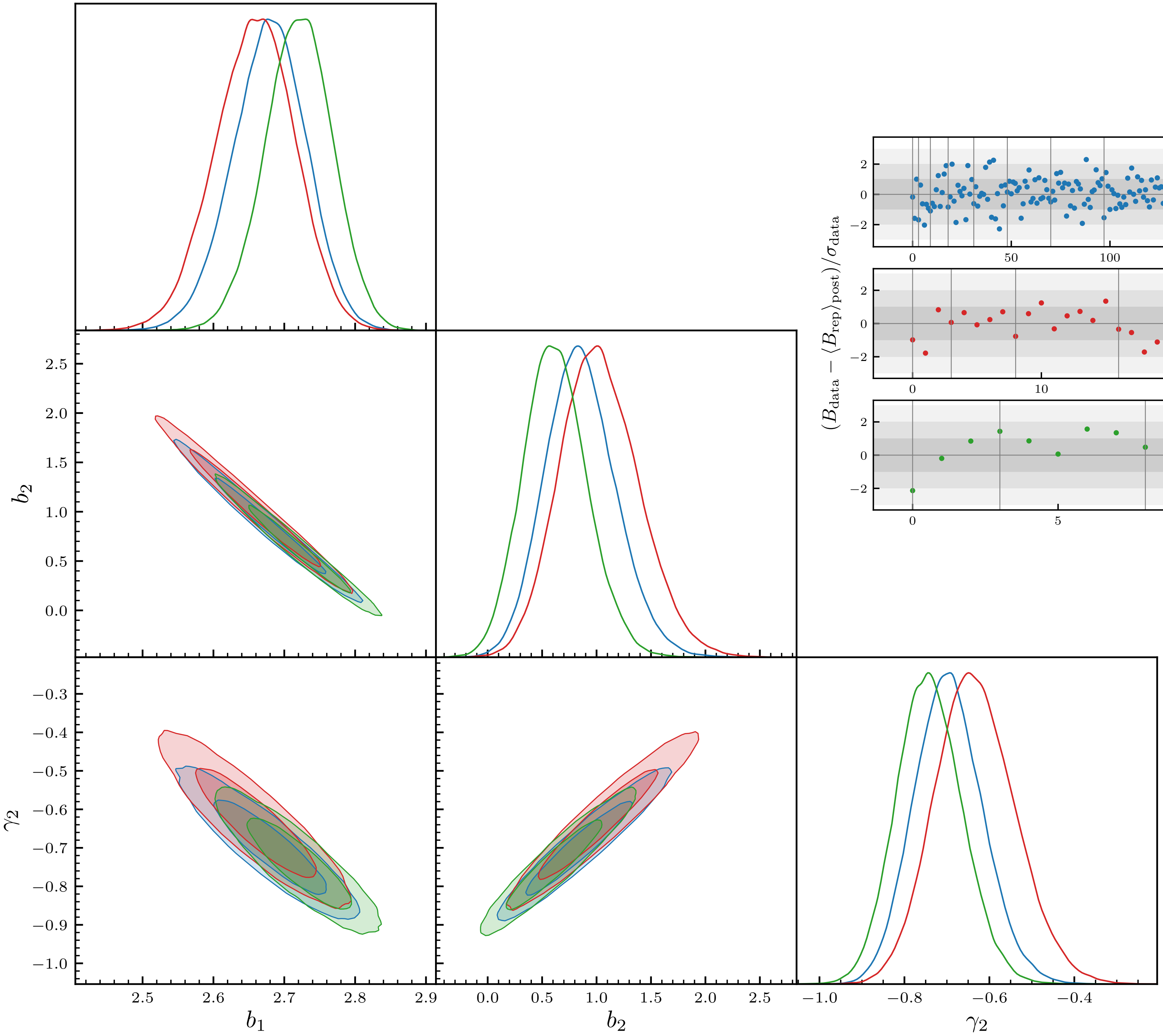
Results: goodness-of-fit

χ^2 per degree of freedom, averaged over the posterior

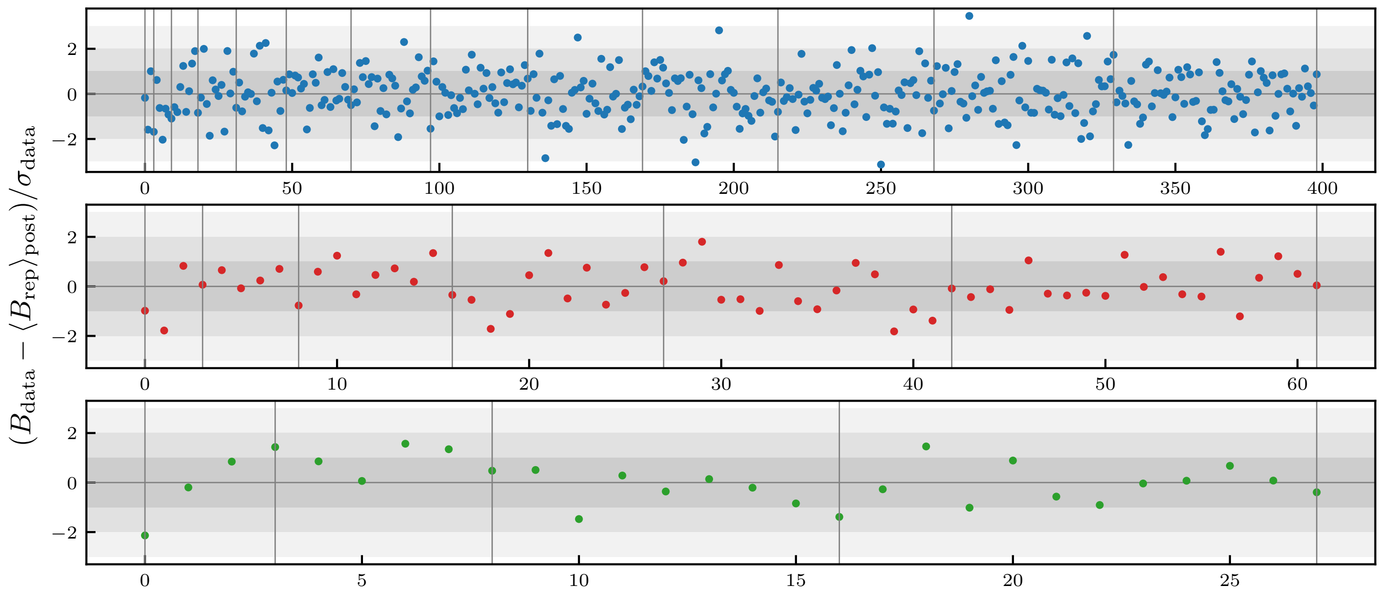


posterior predictive p-value

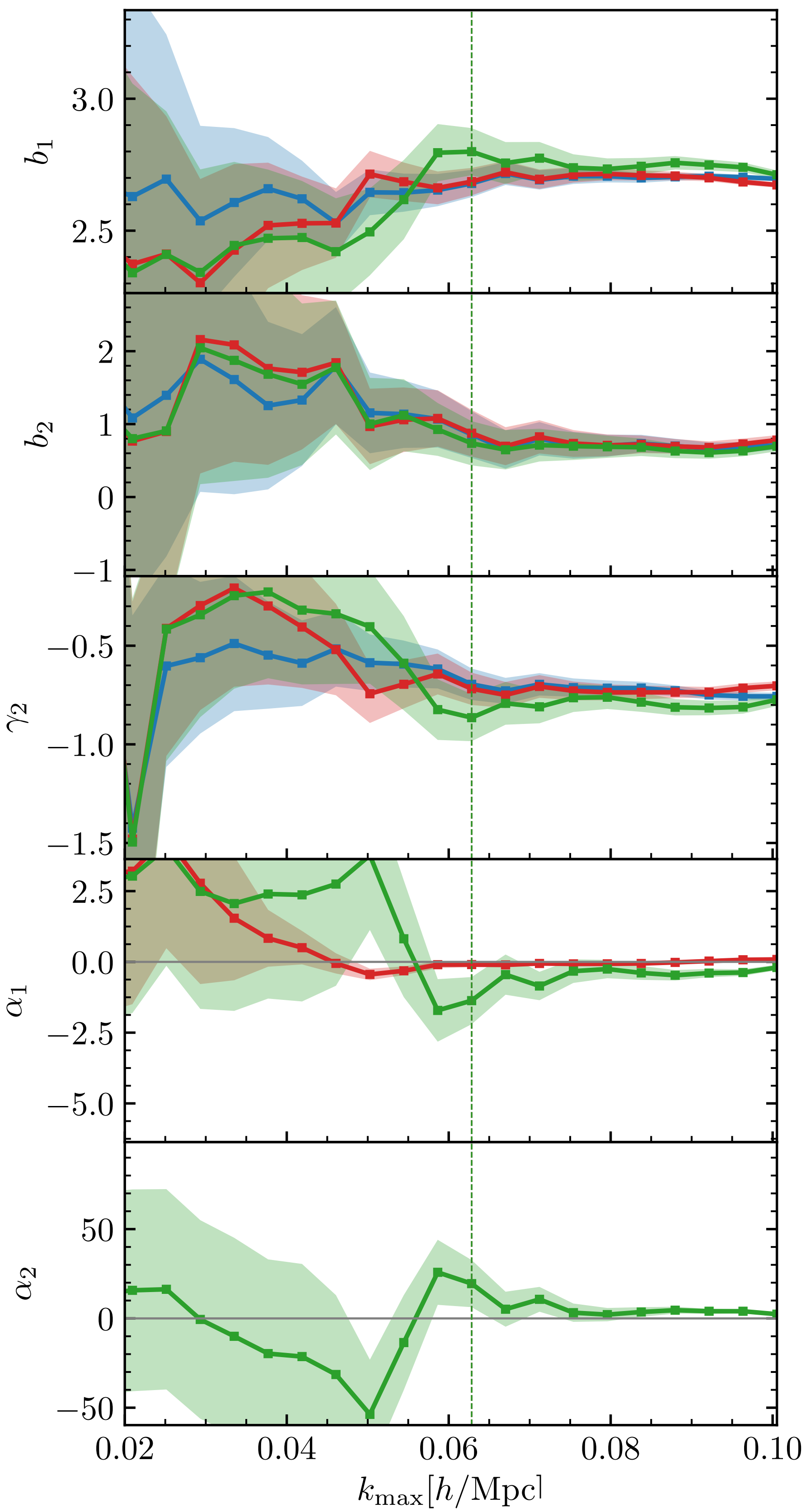
Results (fixed $k_{\rm max}$)

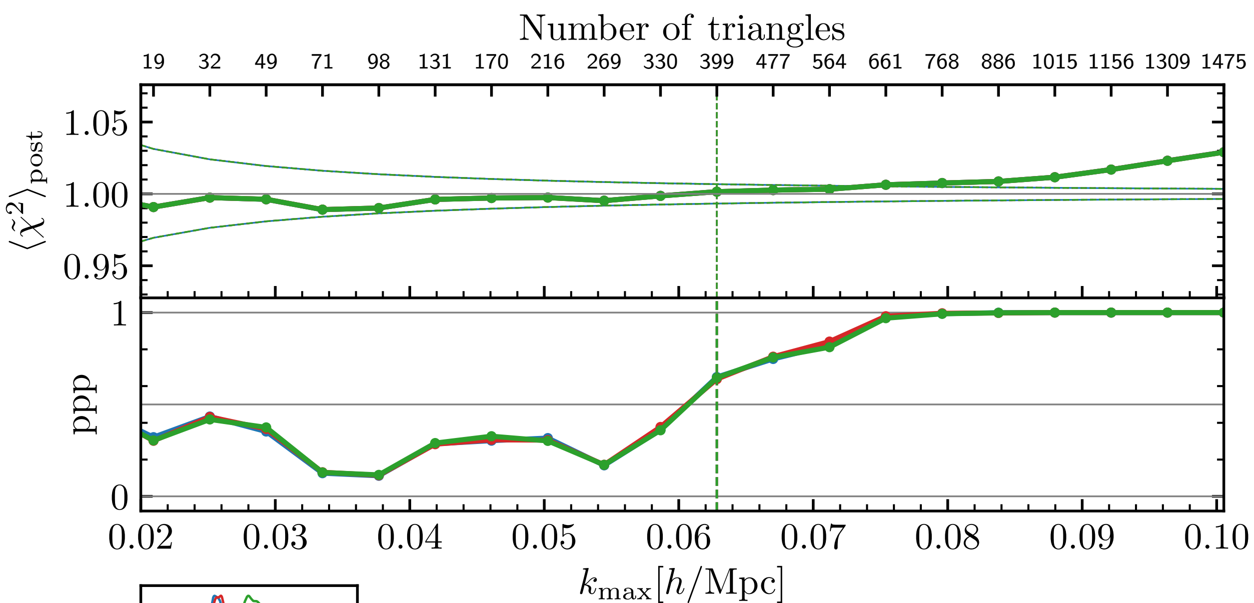


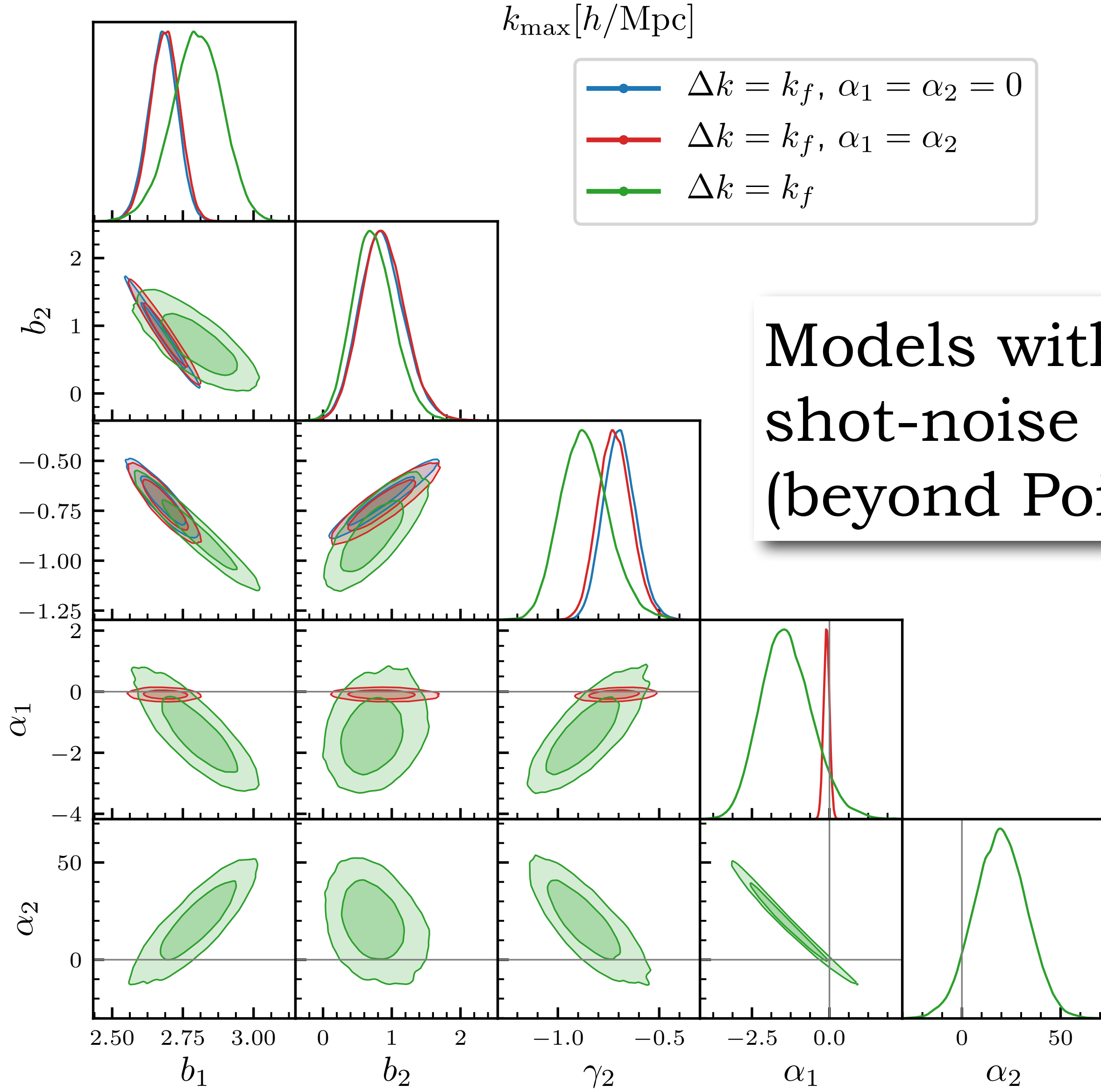
$k_{\rm max} \simeq 0.06\, h^{-1}{\rm Mpc}$



Model comparison





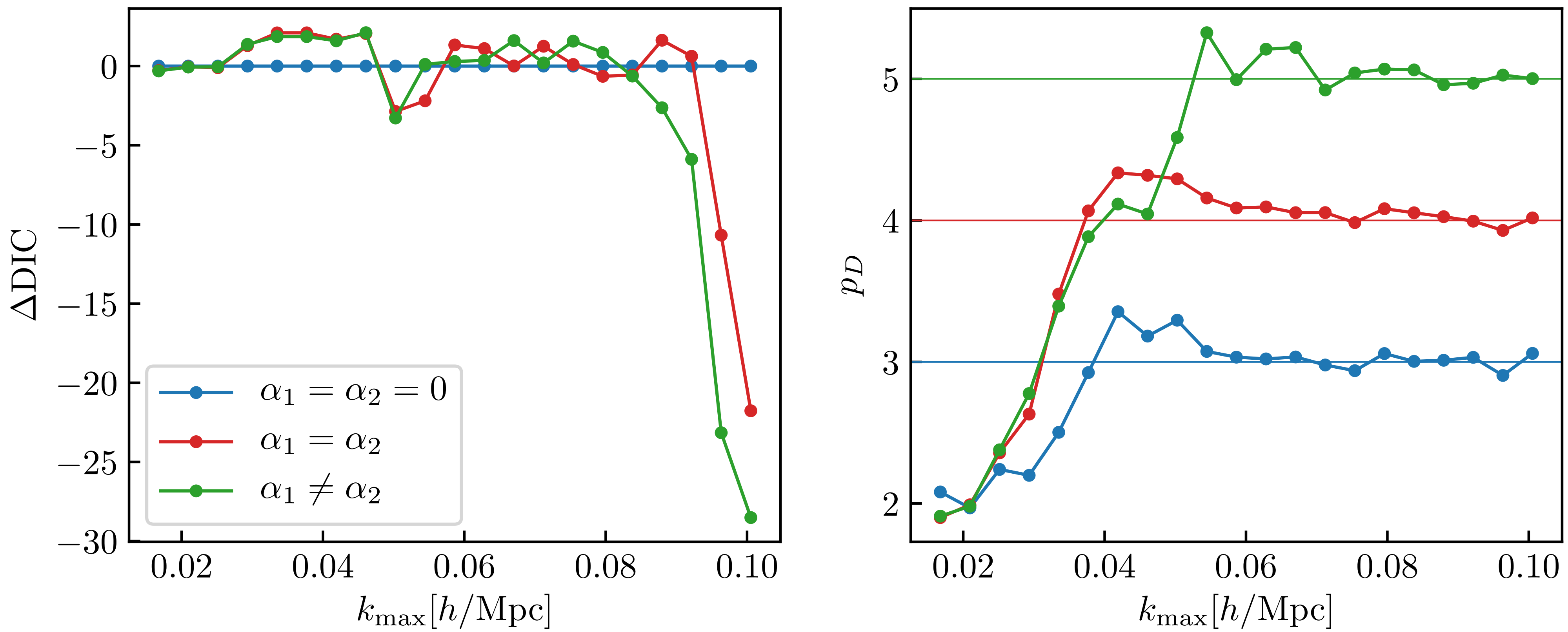


Models with 2, 1 or no shot-noise corrections (beyond Poisson)

Model comparison

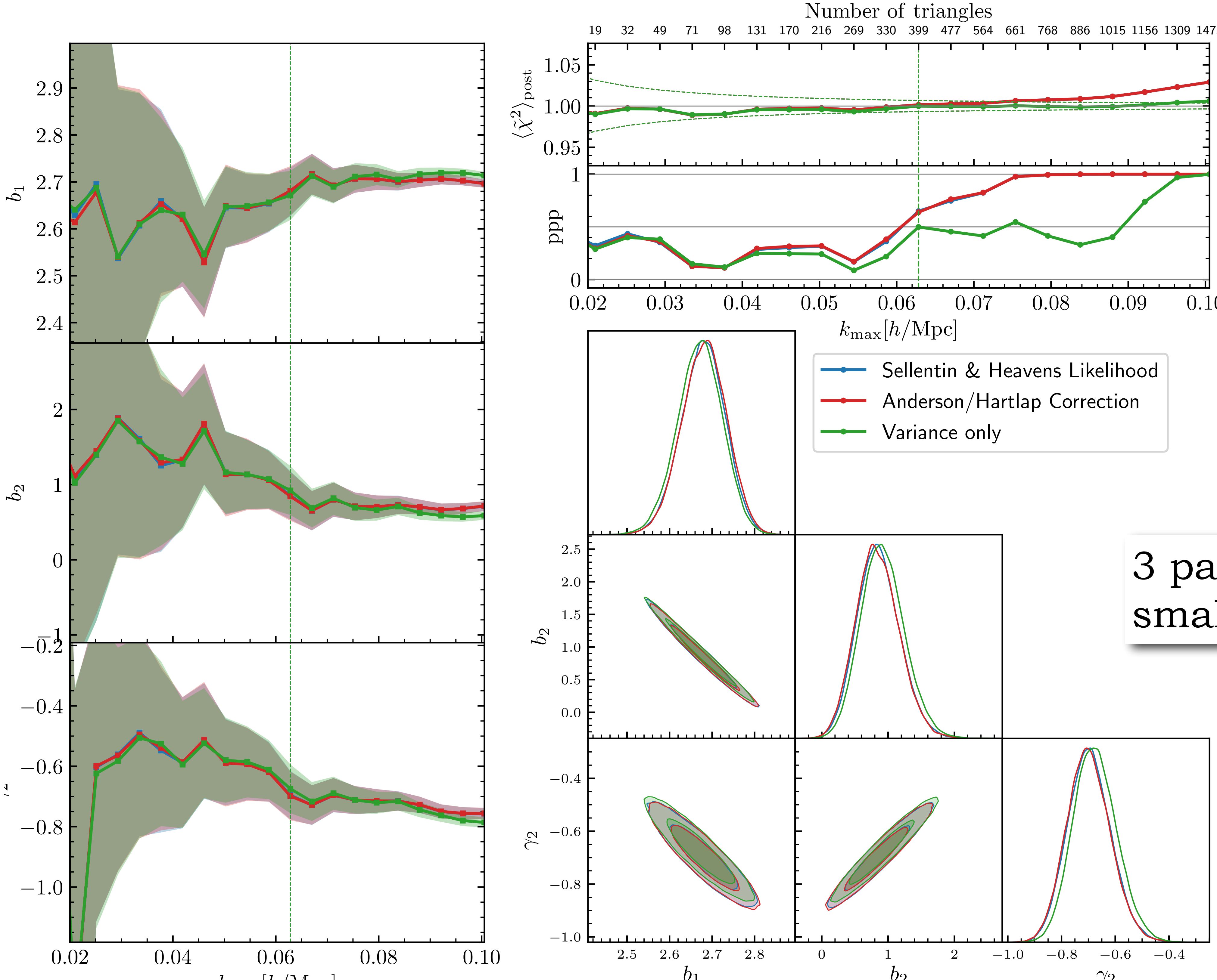
Deviance Information Criterion (DIC)

$$\mathrm{DIC} \equiv \langle D(\theta) \rangle + \frac{1}{2}\,\mathrm{var}[D(\theta)], \quad D(\theta) \equiv -2\,\ln \mathcal{L}(\theta) \quad \text{deviance}$$



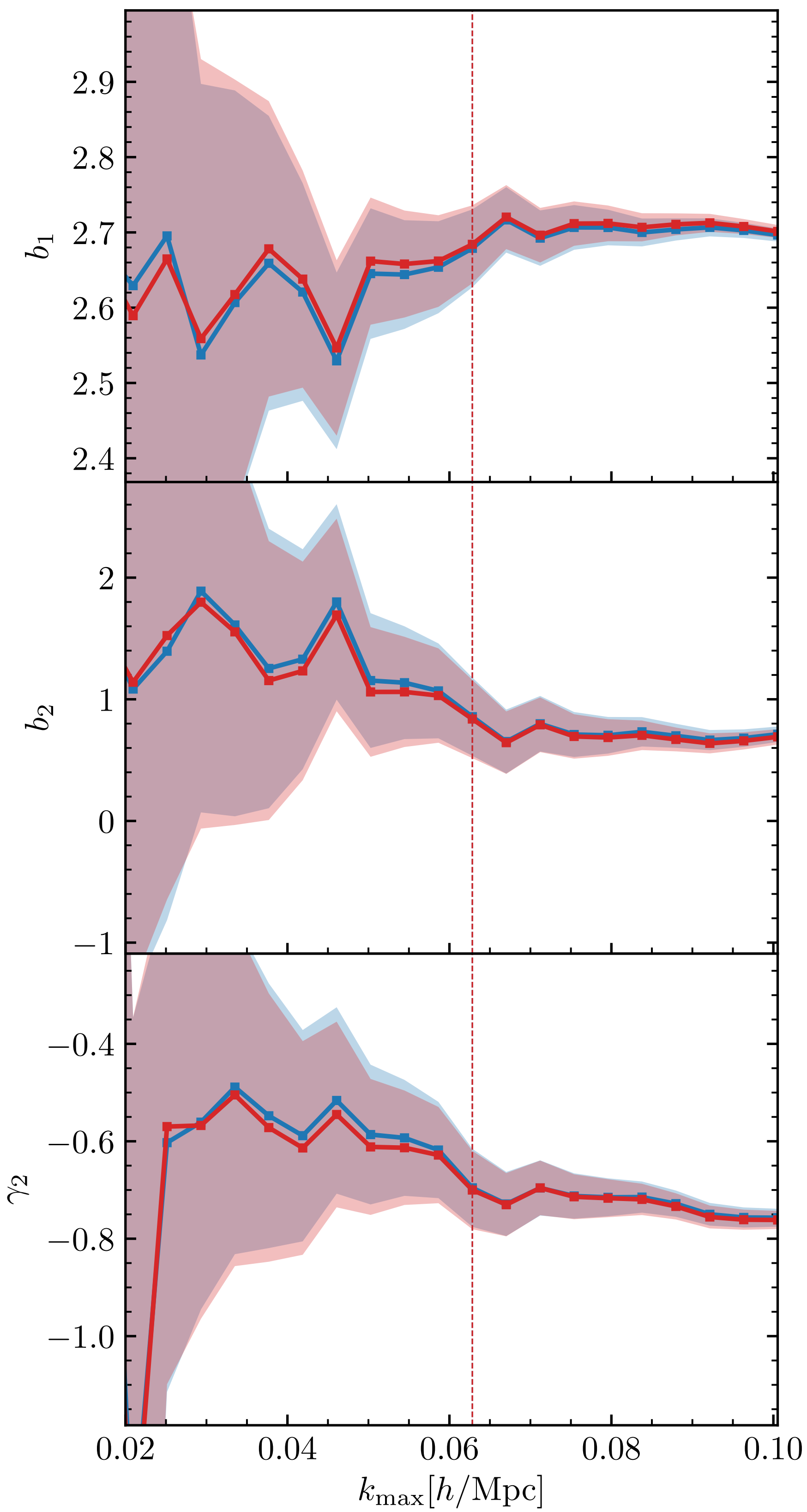
$$p_D \equiv \frac{1}{2}\,\mathrm{var}[D(\theta)] \qquad \text{effective number of parameters}$$

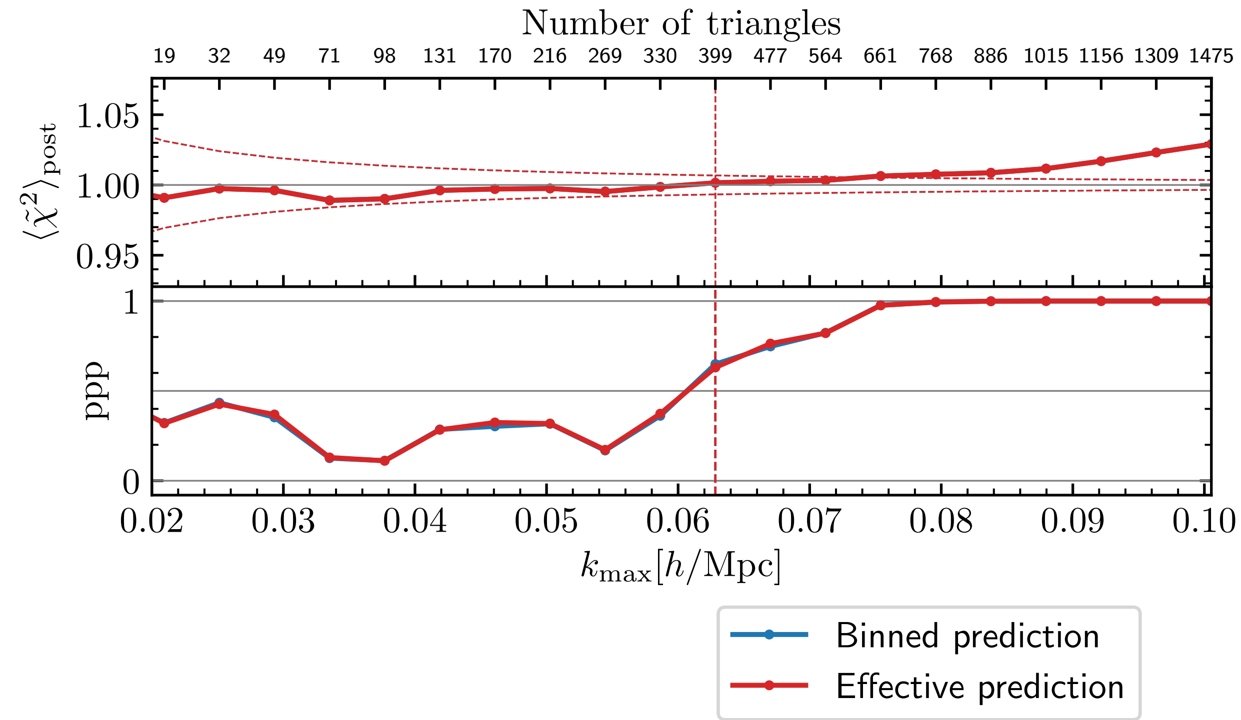
Covariance uncertainty



3 parameters, smallest binning

Binning (we are almost there)





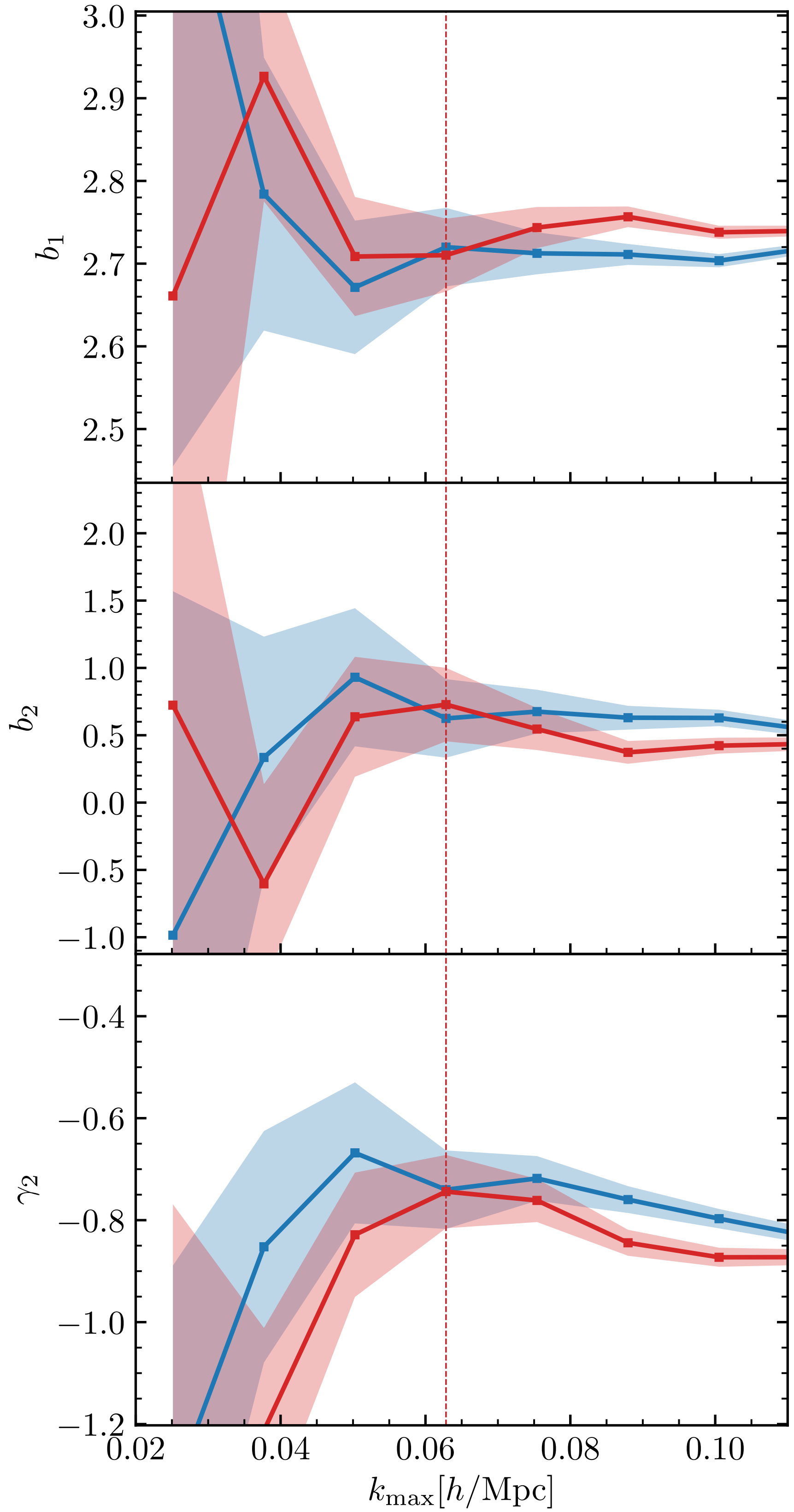
small bins

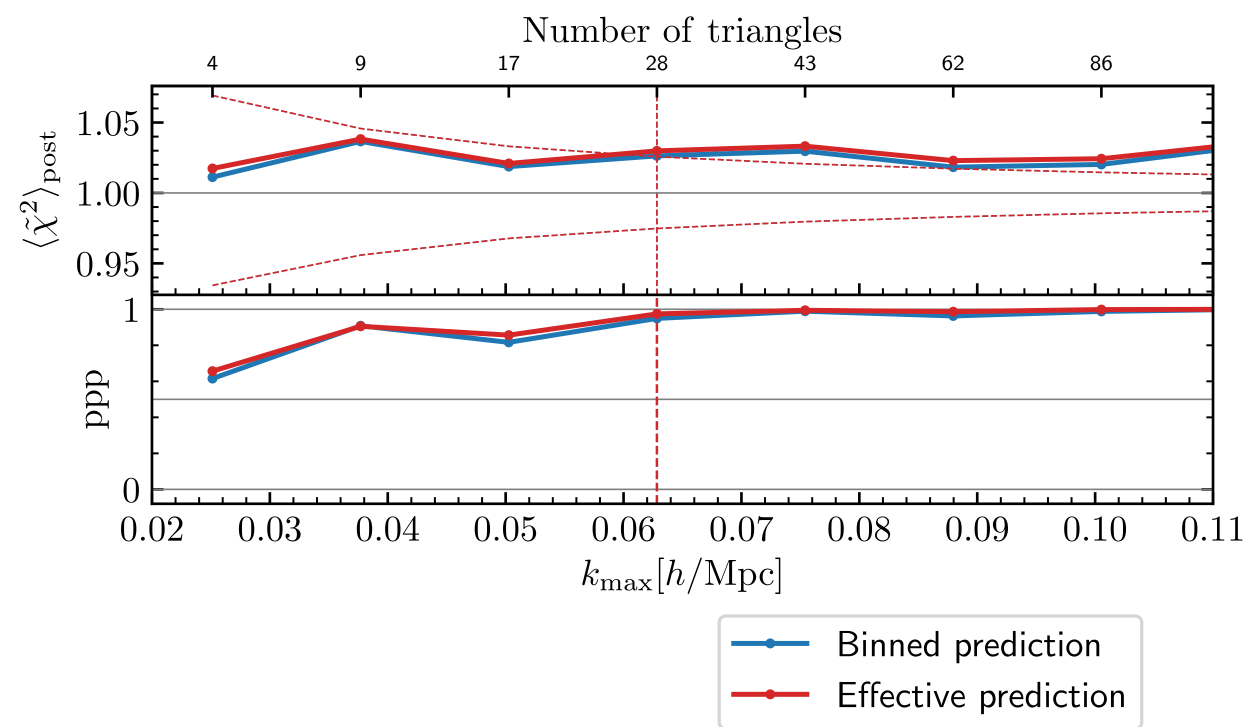
$$B_{binned} = \sum_{\vec{q}_1\in k_1}\sum_{\vec{q}_2\in k_2}\sum_{\vec{q}_3\in k_3} B_{th}(q_1,q_2,q_3)\,\delta_K(\vec{q}_1+\vec{q}_2+\vec{q}_3)$$

$$B_{eff} = B_{th}(k_1^{eff},k_2^{eff},k_3^{eff})$$

with $$k_1^{eff}(k_1;k_2,k_3) = \sum_{\vec{q}_1\in k_1}\sum_{\vec{q}_2\in k_2}\sum_{\vec{q}_3\in k_3} q_1\,\delta_K(\vec{q}_1+\vec{q}_2+\vec{q}_3)$$

Binning (we are almost there)





large bins

$$B_{binned} = \sum_{\vec{q}_1 \in k_1} \sum_{\vec{q}_2 \in k_2} \sum_{\vec{q}_3 \in k_3} B_{th}(q_1, q_2, q_3)\, \delta_K(\vec{q}_1 + \vec{q}_2 + \vec{q}_3)$$

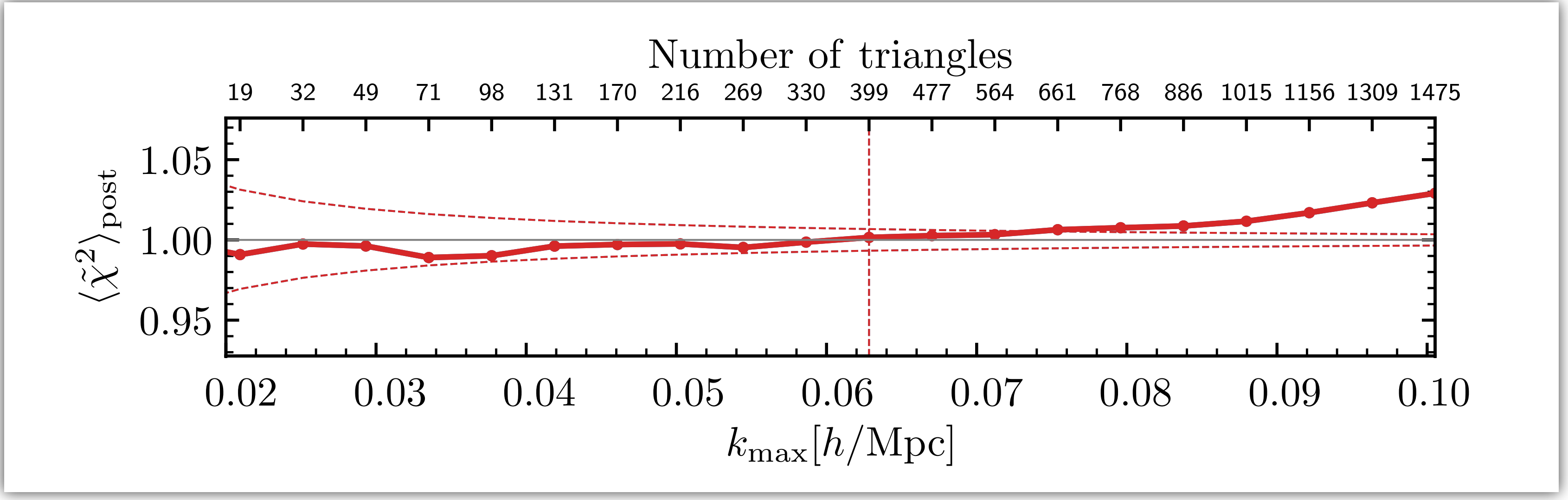
$$B_{eff} = B_{th}(k_1^{eff}, k_2^{eff}, k_3^{eff})$$

with $\quad k_1^{eff}(k_1; k_2, k_3) = \sum_{\vec{q}_1 \in k_1} \sum_{\vec{q}_2 \in k_2} \sum_{\vec{q}_3 \in k_3} q_1\, \delta_K(\vec{q}_1 + \vec{q}_2 + \vec{q}_3)$

but these can be much worse!!

4

Conclusions



A decent χ^2 for the bispectrum requires:

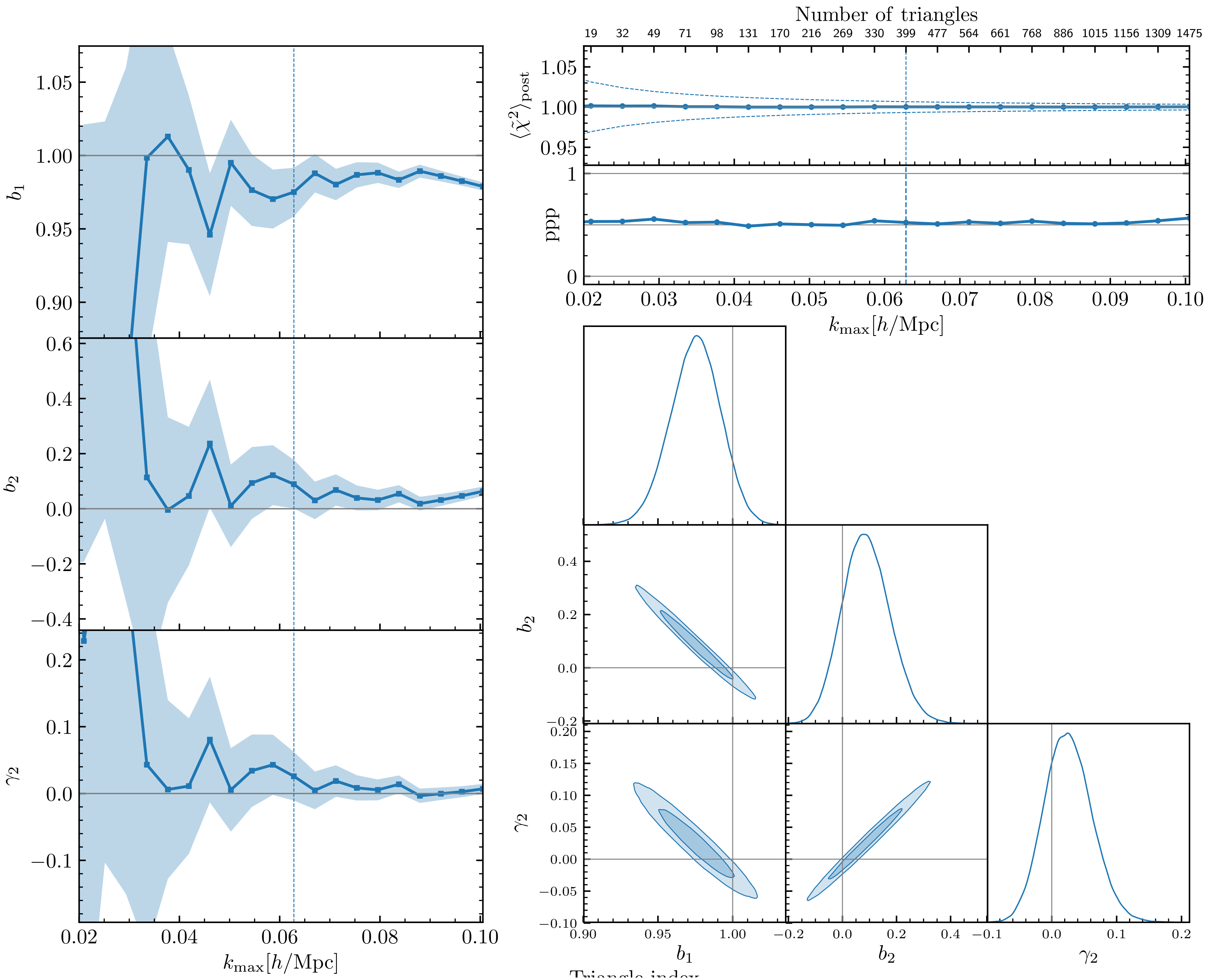
1. Sufficiently small statistical errors on the covariance
2. Sufficiently small systematic errors on the covariance
3. The fitting of all realisations (not the mean)
4. Proper binning of theoretical predictions

… and lots of patience

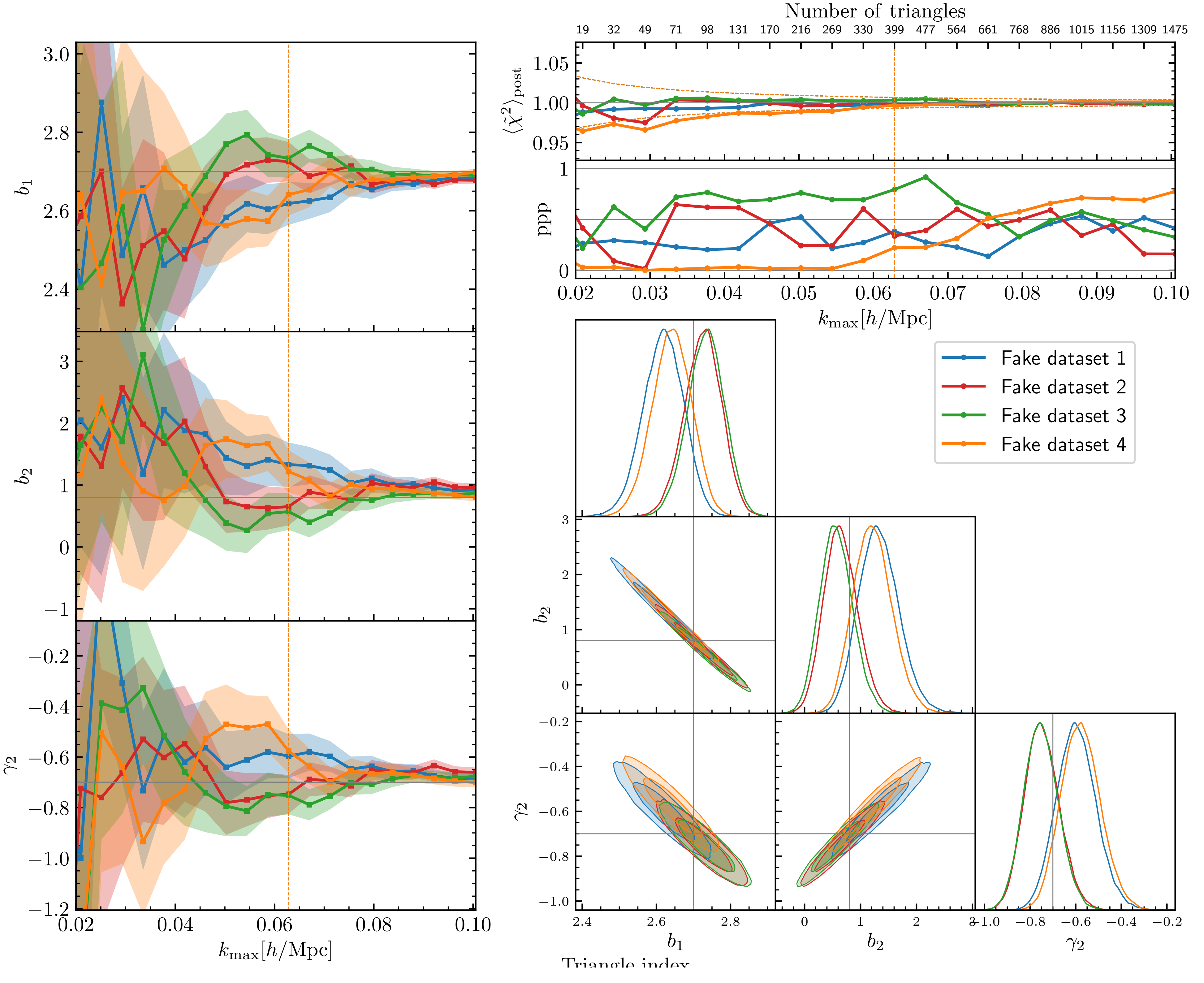
Next steps:
loop corrections, power spectrum, redshift space, neutrinos, $f_{\rm NL}$ (!!!) …

Andrea Oddo, Davit Alkhanishvili, Andrei Lazanu, Titouan Lazeyras, Victoria Yankelevich, Andrea Pezzotta, Azadeh Moradinezhad, Matteo Biagetti, Vincent Desjacques, Martin Crocce, Cristiano Porciani, ES

Test 1: matter



Test 2: fake data



Test 3: 300 realisations sets