Why there is no Newtonian backreaction

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Conventional Framework for Cosmological Dynamics

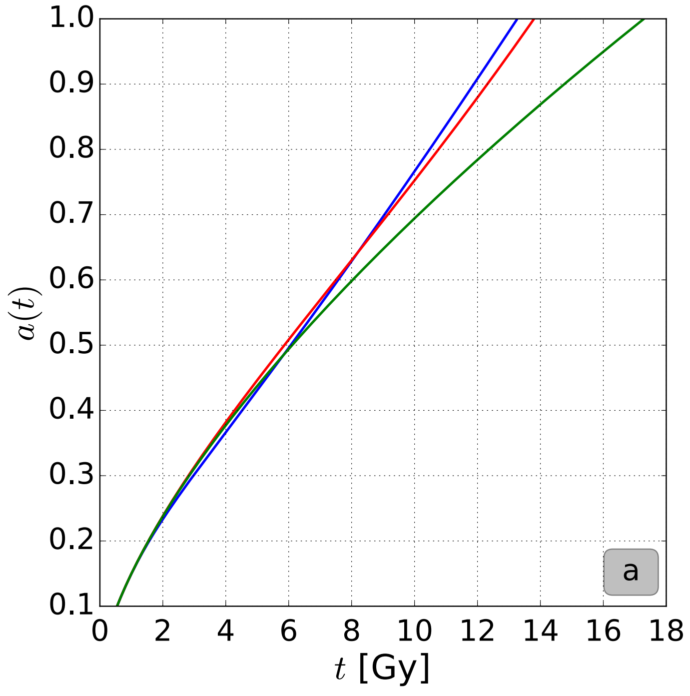
- Homogeneous "background" with scale factor a(t)
  - $a'' = -(4\pi/3)\, G\, \rho_b\, a$ (' = d/dt) Friedmann eq
- Structure (in e.g. N-body calc.) obeys
  - $\mathbf{x}'' + 2\,(a'/a)\,\mathbf{x}' + \boldsymbol{\nabla}\phi \,/\, a^2 = 0$ where
    - $\mathbf{x} = \mathbf{r} \,/\, a$ are "conformal" coords, and
    - $\boldsymbol{\nabla}^2\phi = 4\pi\, G\, (\rho - \rho_b)\, a^2$
  - Analogous effects of background on e.g. Maxwell
- But no feedback (or "backreaction") of $\delta\rho$ on evolution of a(t)
- G.F.R. Ellis (1984...): is this legitimate?
  - Fun fact: who first obtained these equations?

The underlying “world view”

- FRW metric - with expansion factor a(t) - is determined by the *global* properties of the universe
- This involves *strong-field* GR physics
    - which we don't fully understand, except in highly idealised (e.g. homogeneous) situations
    - a(t) is determined by averaging of some "source"
        - Should reduce to density in homogeneous limit
        - But structure may change this
    - a(t) - the "expansion of space" - then affects the small-scale dynamics of structure

Racz et al 2017: Modified N-body calculations

- They assume the conventional DZ <u>structure</u> equations:
  - $\mathbf{x}'' + 2\,(a'/a)\,\mathbf{x}' + \boldsymbol{\nabla}\phi\,/\,a^2 = 0$
  - $\nabla^2\phi = 4\pi\,G\,(\rho - \rho_b)\,a^2$



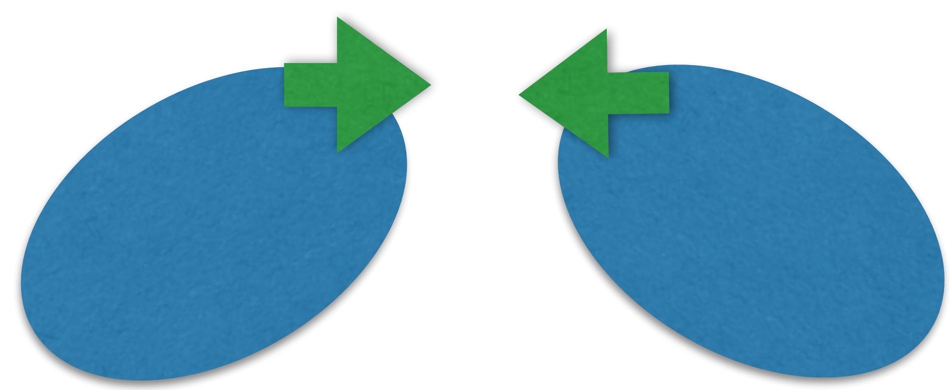
- but evolve a(t) according to $a \rightarrow a + a'\delta t$
  - with a' obtained by averaging <u>local expansion</u>:<a'/a> invoking "separate universe" approximation
- Big effect: a(t) very similar to ΛCDM concordance model
  - And this would change the growth rate also, hence:
  - "<u>concordance cosmology without dark energy</u>"

Is it legitimate to modify the Friedmann equation?

- Does emergence of structure really "backreact" on a(t)?
- Can address this in <u>Newtonian</u> gravity. Relevant as:
  - Accurate description of the local universe ($v << c$)
    - aside from effects from BHs
  - this is where we observe e.g. H_0 = 70 km/s/Mpc!
    - not H_0 ~ 35 km/s/Mpc expected w/o dark energy, Ω_k
  - At $z = 0.1$ relativistic corrections ~ 0.01
- If backreaction is important Newtonian gravity should show it

A candidate effect - tidal torques

- Neighbouring structures exert torques on each other
  - happens as structures reach $\delta \sim 1$
  - essentially non-linear (2nd order) effect
  - Interaction between large scale and internal motions
  - explains spin of galaxies
- can this affect expansion?
  - it would in the local group
  - so why not?



Inhomogeneous Newtonian cosmology

- Lay down particles on a uniform grid in a big uniformly expanding sphere ($\mathbf{v} = H\mathbf{r}$)

- Perturb the particles off the grid $\mathbf{r} \rightarrow \mathbf{r} + \delta\mathbf{r}$

  - plus related velocity perturbations to generate "growing mode" of structure

- $\mathbf{g}(\mathbf{r})$ can be decomposed into:

  - homogenous field sourced by mean density ρ

  - inhomogeneous field sourced by $\delta\rho$ (little dipoles)

- equations of motions $\mathbf{r}'' = \mathbf{g}$ can be re-scaled

  - -> equations that are solved in N-body codes

Newtonian gravity in re-scaled coordinates

N-particles of mass m: $$\ddot{\mathbf{r}}_i = Gm \sum_{j \neq i} \frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|^3}.$$

With **r** = a(t) **x** for arbitrary a(t)

$$\ddot{\mathbf{x}}_i + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}_i = \frac{Gm}{a^3} \sum_{j \neq i} \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|^3} - \frac{\ddot{a}}{a}\mathbf{x}_i.$$

initial conditions: $\mathbf{x} = \mathbf{r}/a \quad \text{and} \quad \dot{\mathbf{x}} = ((H - \dot{a}/a)\mathbf{r} + \delta\dot{\mathbf{r}})/a$

Defining $n(\mathbf{x}) \equiv \sum_i \delta(\mathbf{x} - \mathbf{x}_i)$ and $\delta n \equiv n - \overline{n}$

$$\ddot{\mathbf{x}}_i + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}_i - \frac{Gm}{a^3} \int d^3x \; \delta n(\mathbf{x}) \frac{\mathbf{x} - \mathbf{x}_i}{|\mathbf{x} - \mathbf{x}_i|^3} = -\left(\frac{\ddot{a}}{a} + \frac{4\pi Gm\overline{n}}{3a^3}\right)\mathbf{x}_i.$$

Exactly equivalent to the usual equations in **r**-coords

Newtonian cosmology:

$$\ddot{\mathbf{x}}_i + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}_i - \frac{Gm}{a^3}\int d^3x\, \delta n(\mathbf{x}) \frac{\mathbf{x}-\mathbf{x}_i}{|\mathbf{x}-\mathbf{x}_i|^3} = -\left(\frac{\ddot{a}}{a} + \frac{4\pi G m \overline{n}}{3a^3}\right)\mathbf{x}_i.$$

with ICs

$$\mathbf{x} = \mathbf{r}/a \quad \text{and} \quad \dot{\mathbf{x}} = ((H - \dot{a}/a)\mathbf{r} + \delta\dot{\mathbf{r}})/a$$

- 3N equations for N particles
  - <u>there is no extra equation of motion for a(t)</u>
- But we may <u>choose</u> a(t) to obey Friedmann equation
  - an "auxiliary relation"
- Gives conventional expansion + structure equations
  - <u>a(t) suffers no backreaction from structure emergence</u>
  - a(t) is just a "book-keeping" factor - no physical effect

Is there relativistic backreaction?

- Buchert etc: "GR backreaction" → non-zero - and large
- But local universe should be accurately Newtonian
  - errors ~ v^2/c^2 → ~1% accuracy within $z = 0.1$
  - and that's where we measure H_0
- Are there even very small effects on expansion history?

Is there *relativistic* backreaction?

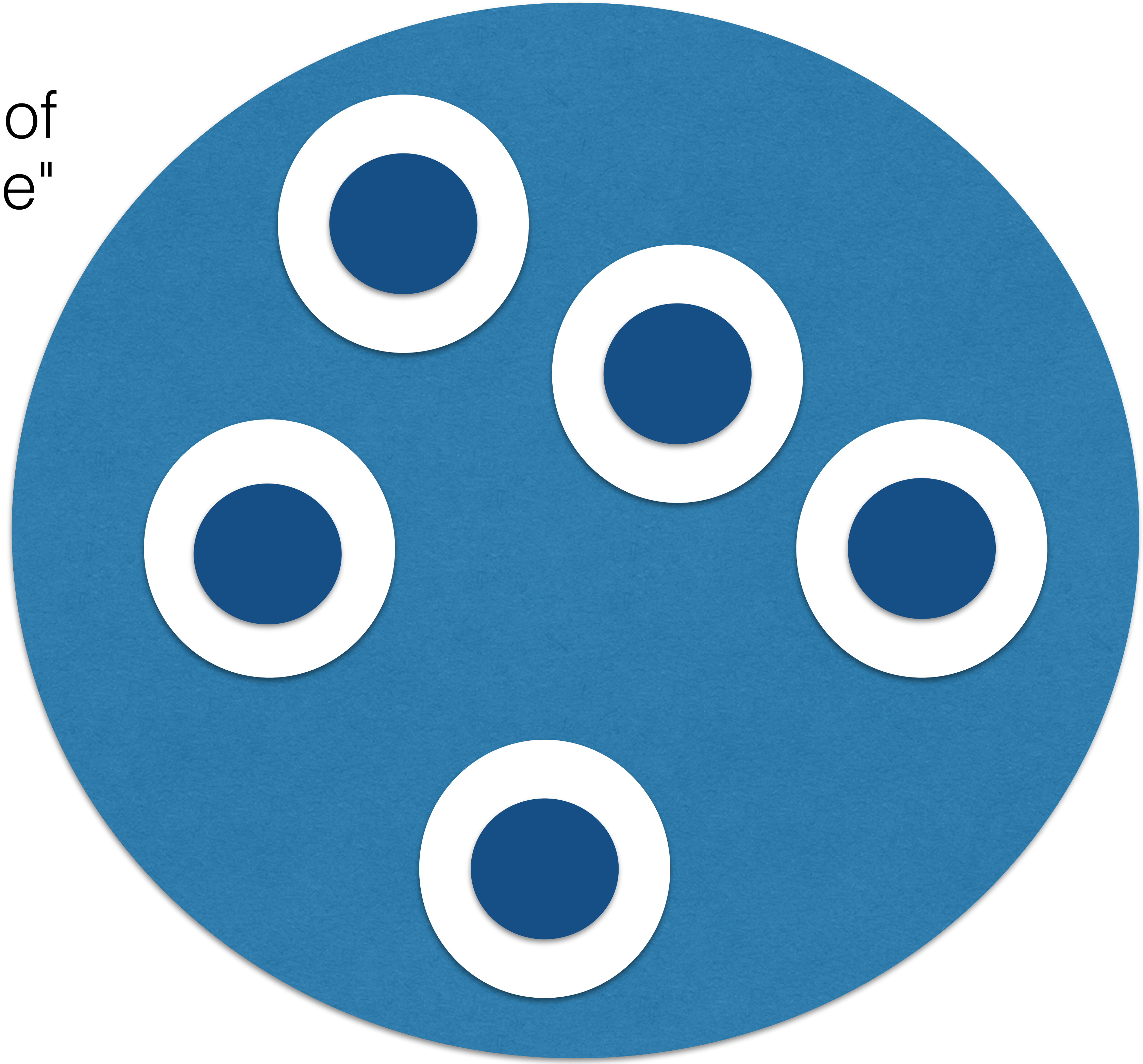
- Averaging of Einstein equations: **G** = **T**
- homogeneous models: metric **g** -> **G** = diagonal
  - **T** = diag(ρ, P, P, P) and **∇.T** = 0 -> Friedmann equations
- with inhomogeneity < **G** > = < **T** >?
  - "averaging problem" widely discussed in BR literature
- what about internal pressure P of clusters?
  - As invoked in EFT perturbation theory
  - or strong internal pressure in stars, compact objects
- Does that give rise to F-equations with non-zero P?

Is there *relativistic* backreaction?

- Averaging of Einstein equations: $< \mathbf{G} > = < \mathbf{T} >$?
- what about e.g. stars with strong internal pressure P
  - does that give rise to F-equations with non-zero P?
- No. Relativistic stars have Schwarzschild external geom
  - mass parameter m
    - space integral of the effective stress tensor
    - independent of time
- Conservation of stars implies $\rho \sim a^{-3}$
  - which demands $P = 0$ in F-equations

Relativistic BR from large-scale structure?

- Einstein-Straus '45
  - "What is the effect of expansion of space"
- -> Swiss-cheese
- Fully non-linear
- Interesting pert^n to e.g. proper mass
- but background expansion is exactly unperturbed
- small effects on D(z)



Backreaction from inter-galactic pressure

- Stars (or DM) ejected from galaxies by merging BHs
    - intergalactic pressure $P = n\ m\ \sigma_v^2$
- Homogeneous (in conformal coords) pressure is a flux of energy with non-zero divergence in real space
    - 1st law ... PdV work .... : $\rho' = - (\rho + P/c^2)\ V' / V$
        - but a very small effect
- relies on pressure being extended throughout space
    - no effect from internal pressure in bound systems that are surrounded by empty space

Summary

- A different perspective on the DZ equations. There is no dynamical equation for a(t). a(t) is arbitrary. There is no freedom to modify F-equation w/o changing structure eqs. Conventional system of equations is exact. Tidal torques might affect expansion of the local group, but can't affect the universal expansion.

- Comments on relativistic backreaction. Averaging of stress-energy for systems with internal pressure does not introduce non-zero P in Freidmann equations. Exact non-linear solutions show no backreaction. Intergalactic P - from grav-waves, high-v stars - does backreact at a weak level, but is positive.